INDEPENDENCE OF A CERTAIN AXIOMATIC SYSTEM

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To prove the independence of the system of axioms introduced in [1] we exhibit here ten models, each of them satisfying all the axioms but one; e.g. model M_5 fails to satisfy P_5 of [1].

We call model Γ the Euclidean 3-space with the usual vector structure, introduce a suitable order function ϕ and the usual notion of orthogonality. Let $A_i (i=1, 2, 3, 4)$ be the position vectors of four points and define:

$$\phi(A_1, A_2, A_3, A_4) = \text{sign det} \left| A_2 - A_1, A_3 - A_1, A_4 - A_1 \right| .$$

Model Γ shows the relative consistency of the system.

M_1—In Γ, take a new order function: $\phi_1 = |\phi|$.

M_2—In Γ, change the sign of ϕ for exactly one nonsingular tetrad and its opposite.

M_3—Let the ϕ of Γ be identically 0.

M_4—Adjoin one point X to Γ and extend ϕ: $\phi(X, A_1, A_2, A_3) = \phi(O, A_1, A_2, A_3)$ where O is the origin and $A_i \in \Gamma$.

M_5—Vectors of Γ with integral components and ϕ restricted accordingly.

M_6—In the hyperbolic space H^3 take any orientation function for ϕ and keep the usual orthogonality notion.

M_7—Take the vectors of Γ with rational components and restrict ϕ.

M_8—Imbed Γ in the projective space P^3 adding the ideal plane Ω. Let γ be a real conic on Ω and define $l \perp \pi$ (in Γ) to mean $l \cap \Omega$ and $\pi \cap \Omega$ are pole and polar with respect to γ.

M_9—Same as before but $l \perp \pi$ is defined only if $l \cap \Omega$ is a two-tangent point with respect to γ.

M_{10}—Let γ be an elliptic correlation, not a polarity, on Ω, and define $l \perp \pi$ accordingly.

The proof is now complete.

REFERENCE

1. L. Gutierrez-Novoa, *Ten axioms for three-dimensional Euclidean geometry,* Proc. Amer. Math. Soc. **19** (1958), 146–152.

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