HIGHER DERIVATIONS OF WILDLY RAMIFIED v-RINGS

MARTIN N. HEINZER[1]

I. **Introduction and results.** In a number of recent papers Heerema [2], [4], Neggers [6], and the author [5], have used techniques involving the lifting of derivations and (infinite) higher derivations from the residue field k, of a local ring R, to derivations and higher derivations of R. These papers are concerned with the automorphism structure of complete regular local rings. In this paper we will identify the group of higher derivations of k which lift to R when R is a wildly ramified v-ring with ramification p over an unramified v-subring. In particular let R_e be a ramified v-ring with ramification e. That is, R_e is a complete, discrete, rank one, valuation ring having characteristic zero with residue field k of characteristic p (p prime$\neq 2$) and pR_e is the eth power of the maximal ideal M of R_e. If $e=1$ or $(e, p)=1$ then all higher derivations of k lift to higher derivations of R_e [**1**, Theorem 1, p. 575], [**7**, Theorem 3.4, p. 24]. We determine which higher derivations of k lift in the simplest wildly ramified case, when $e=p$, and generalize a theorem first proved by Wishart [**7**, Theorem 3.18, p. 44] characterizing those rings R_p with the property that all higher derivations of k lift.

The symbol π will always denote a prime element of R_p and $\bar{a}$ the residue of $a\in R_p$. We have, $\pi^p+pu=0$, $\bar{u}\neq 0$, and if $\bar{u}\in k^p$, π can be chosen so that $\pi^p+p(1+\pi^t w)=0$, $t>0$ and $\bar{w}\neq 0$, or so that $\pi^p+p=0$. It is not difficult [**5**, Lemma 2.3] to see that π can be chosen to satisfy precisely one of the following:

(1.1) $$\pi^p + pu = 0, \quad \bar{u} \notin k^p.$$

(1.2, t) $$\pi^p + p(1 + \pi^t w) = 0, \quad 1 \leqq t \leqq p,\ \bar{w} \neq 0, \quad \text{and} \quad \bar{w} \notin k^p \ \text{when } t = p.$$

(1.3) $$\pi^p + p(1 + \pi^{p+1} w) = 0.$$

We may identify now two parameters, expo R_p and res R_p [**5**], which will have a decisive role in this study.

(1.4)
$$\begin{aligned} \text{expo } R_p &= 0 && \text{if } \pi \text{ satisfies 1.1.}\\ &= t && \text{if } \pi \text{ satisfies 1.2, t.}\\ &= p+1 && \text{if } \pi \text{ satisfies 1.3.}\end{aligned}$$

Received by the editors January 27, 1969.

[1] This research was supported in part by NSF grant GP 8424.

$$\begin{aligned} \text{res } R_p &= \bar{u} \quad \text{if } \pi \text{ satisfies 1.1.} \\ &= \bar{r} \quad \text{if } \pi \text{ satisfies 1.2, t or 1.3 and } \bar{r}^{p^n} = \bar{w} \text{ with } \bar{r} \notin k^p. \\ &= 0 \quad \text{otherwise.} \end{aligned} \tag{1.5}$$

Equation 1.5 does not uniquely determine res R_p. However, res R_p is uniquely determined, mod k^p, either with respect to addition or multiplication, and this will be sufficient for our purposes. The quantity expo R_p is uniquely determined by 1.4.

We let G_D be the group of derivation automorphisms of R_p, and G_S the group of strongly convergent derivation automorphisms of R_p [**5**]. The symbols u and w will always be used as above.

We now state the results of this study.

THEOREM. *The higher derivation* $\{\delta_i\}$ *of* k *lifts to a higher derivation of* R_p *if and only if* δ_i (res R_p) $=0$ *for each* i *with the following exceptions. If* expo $R_p=0$ *then* $\{\delta_i\}$ *lifts if and only if* (res $R_p)^{-1}\delta_{ip}$ (res $R_p)\in k^p$ *for each* i *and* δ_i (res R_p) $=0$ *when* $p\nmid i$. *If* expo $R_p=p$ *then* $\{\delta_i\}$ *lifts if only if* δ_{ip}(res $R_p)\in k^p$ *for each* i *and* δ_i(res R_p) $=0$ *when* $p\nmid i$.

COROLLARY A. *All higher derivations of* k *lift to higher derivations of* R_p *if and only if* res $R_p=0$.

COROLLARY B. *All higher derivations of* k *lift to higher derivations of* R_p *if and only if* $G_D=G_S$ *and* $p\nmid$ expo R_p.

II. **Proofs and an application.** Throughout this paper R will denote an unramified v-subring of R_p with $[R_p:R]=p$, π a prime element of R_p assumed to satisfy one of the equations 1.1, (1.2, t,) 1.3, $H(R, R_p)$ the set of higher derivations of R into R_p, $H(k)$ the group of higher derivations of k, k^0 the maximal perfect subfield of k, and v the exponential valuation on R_p.

Let $\{D_i\}\in H(R, R_p)$ and denote the unique extension of $\{D_i\}$ to the quotient field of R_p by $\{D_i\}$. Note that $\{D_i\}\in H(R_p)$, the set of higher derivations of R_p, if and only if $D_i(\pi)\in(\pi)$ for each i. Let

$$f(x) = x^p + pa_{p-1}x^{p-1} + \cdots + pa_1x + pa_0 \tag{2.1}$$

be the Eisenstein polynomial of π over R. Let (r, s) denote an ordered set of r nonnegative integers whose sum is s and let $|(r, s)|$ represent the largest integer in (r, s). We let $\sum_{(q,s)}D(a_1, \cdots, a_q)$ denote the sum of all products $D_{i_1}(a_1)D_{i_2}(a_2)\cdots D_{i_q}(a_q)$ such that $i_1+\cdots+i_q=s$ and $i_j\geqq 0$. We also let $f'(x)$ and $f^{D_i}(x)$ represent respectively the ordinary derivative of f and the polynomial obtained by replacing each coefficient in f with its image under D_i. With these conventions we may write the expression for $D_i(\pi)$ derived from $D_i(f(\pi))=0$ as follows:

$$(2.2,\text{ i})\qquad \begin{aligned}(-f'(\pi))D_i(\pi) &= f_i{}^D(\pi) + \sum_{(p,i);\,|(p,i)|<i} D(\pi,\cdots,\pi)\\ &+ \sum_{j=1}^{p-1} p \sum_{(j+1,i),\,|(j+1,i)|<i} D(a_j,\pi,\cdots,\pi).\end{aligned}$$

We list some observations.

I. $p \leqq v(f'(\pi)) \leqq 2p-1$.

II. If $a \in R$ then $D_i(a^{p^n}) = (D_{i/p^n}(a))^{p^n} \bmod(\pi)^p$ where $D_{i/p^n}(a)=0$ if $p^n \nmid i$.

III. If $\pi^p + pu = 0$ then $\bar{u} = \bar{a}_0$.

IV. If $\pi^p + p(1+\pi^t w) = 0$, $1 \leqq t < p$, and $\bar{w} \neq 0$ then $\bar{w} = \bar{a}_t$ where t is the least positive integer i such that $p \nmid a_i$, and $a_0 = 1 + pb_0$ for some b_0.

V. If $\pi^p + p(1+\pi^p w) = 0$, $\bar{w} \neq 0$ then $a_0 = 1+pb_0$ for some b_0, $\bar{w} = -\bar{b}_0$, and $p \mid a_i$ for all $i \geqq 1$.

VI. If $\pi^p + p(1+\pi^{p+1}w) = 0$ then $a_0 = 1+pb_0$ for some b_0, $\bar{w} = [(-a_1/p)]^-$, and $p \mid b_0$.

VII. If $1 \leqq \text{expo } R_p \leqq p$ and $\{D_i\} \in H(R_p)$ then $D_i(\pi) \in (\pi)^2$ for each i.

If expo $R_p = p+1$ then $D_1(\pi) \in (\pi)^3$.

Let $\{D_i\} \in H(R_p)$, $-f'(\pi) = p\pi^{j-1}z$, z a unit, $\pi A_i = D_i(\pi)$ when expo $R_p = 0$, $\pi^2 A_i = D_i(\pi)$ when $1 \leqq \text{expo } R_p \leqq p$, and $\pi^3 A_i = D_i(\pi)$ when expo $R_p = p+1$. We also write $a_0 = 1+pb_0$ when expo $R_p \neq 0$ and $pd_1 = a_1$ when expo $R_p = p+1$.

2.3 Lemma. *If* expo $R_p = 0$ *and* $\{D_i\} \in H(R_p)$ *induces* $\{\delta_i\} \in H(k)$ *then* res $R_p^{-1}\delta_{ip}(\text{res } R_p) \in k^p$ *for each* i *and* $\delta_i(\text{res } R_p) = 0$ *when* $p \nmid i$.

Proof. By 2.2,i we have $(p\pi^{j-1}z)D_i(\pi) = pD_i(a_0) + D_{i/p}(\pi)^p$, $\bmod(\pi)^{p+j}$ or $(\pi^{j-1}z)D_i(\pi) = D_i(a_0) - uA_{i/p}^p$, $\bmod(\pi)^j$ where $D_{i/p}(\pi) = 0 = A_{i/p}$ if $p \nmid i$. Since $\bar{a}_0 = \bar{u} = \text{res } R_p$ and $D_i(\pi) \in (\pi)$ the result follows.

2.4 Lemma. *If* $1 \leqq \text{expo } R_p < p$ *and* $\{D_i\} \in H(R_p)$ *induces* $\{\delta_i\} \in H(k)$ *then* δ_i (res R_p) $= 0$ *for each* i.

Proof. If res $R_p \neq 0$ then res $R_p = \bar{r}$ where $\bar{r}^{p^n} = \bar{a}_j$, $r \in R$, $\bar{r} \notin k^p$, expo $R_p = j$, and $v(f'(\pi)) = p + (j-1)$. Using (2.2, i) and II we get

$$\begin{aligned}(p\pi^{j-1}z)D_i(\pi) &= pD_i(a_j)\pi^j, &&\bmod(\pi)^{p+j+1}\\ &= p(D_{i/p^n}(r))^{p^n}\pi^j, &&\bmod(\pi)^{p+j+1}.\end{aligned}$$

Again, since $D_i(\pi) \in (\pi)^2$ the result follows.

2.5 LEMMA. *If* expo $R_p=p$ *and* $\{D_i\}\in H(R_p)$ *induces* $\{\delta_i\}\in H(k)$ *then* δ_{ip} (res $R_p)\in k^p$ *for each* i *and* δ_i(res $R_p)=0$ *when* $p\nmid i$.

PROOF. We have $\text{res}R_p=-\bar{b}_0$ and $(p\pi^{p-1}z)D_i(\pi)=p^2D_i(b_0)+D_{i/p}(\pi)^p$, mod π^{2p+1}. Thus $zD_i(\pi)=-u_1^{-1}\pi D_i(b_0)-u_1\pi A_{i/p}^p$, mod $(\pi)^2$ where $\pi^p+pu_1=0$. But $D_i(\pi)\in(\pi)^2$, $\bar{u}_1=1$ and the proof is complete.

2.6 LEMMA. *If* expo $R_p=p+1$ *and* $\{D_i\}\in H(R_p)$ *induces* $\{\delta_i\}\in H(k)$ *then* δ_i (res $R_p)=0$ *for each* i.

PROOF. Omitted.

We now show that if $\{\delta_i\}\in H(k)$ satisfies the conditions of the theorem then $\{\delta_i\}$ is induced. We assume $\{D_i\}_{i=1}^n$ is a finite higher derivation of R into R_p, $A_0=1$, $f'(\pi)$ and A_i, $i\geqq 1$ are as before. We list some observations which follow from (2.2, i), $1\leqq i\leqq n$.

VIII. Let expo $R_p=0$, $v(f'(\pi))<2p-1$ and $D_i(\pi)\in(\pi)$ for $i=1,\cdots,n-1$. If $D_n(a_0)-uA_{n/p}^p=\pi^jc_n$ then

$$(p\pi^{j-1}z)D_n(\pi) = pD_n(a_j)\pi^j + pD_n(a_0) + (D_{n/p}(\pi))^p + p\sum_{(j+1,n);\,|(j+1,n)|<n} D(a_j\pi\cdots\pi), \quad \text{mod}(\pi)^{p+j+1}$$

where $v(f'(\pi))=p+(j-1)$. Thus if we write $\sum_{i_m<n}D_{i_0}(a_j)A_{i_1}\cdots A_{i_j}$ for the sum of all products $D_{i_0}(a_j)A_{i_1}\cdots A_{i_j}$ where $i_0+i_1+\cdots+i_j=n$, $0\leqq i_m<n$, $m=0,\cdots,j$ then

$$A_n = z^{-1}(D_n(a_j) + c_n + \sum_{i_m<n} D_{i_0}(a_j)A_{i_1}\cdots A_{i_j}), \quad \text{mod}(\pi).$$

IX. Let expo $R_p=0$, $v(f'(\pi))=2p-1$ and $D_i(\pi)\in(\pi)$ for $i=1,2,\cdots n-1$. If $D_n(a_0)-uA_{n/p}^p=\pi^pc_n$ then

$$(p\pi^{p-1}z)D_n(\pi) = pD_n(a_0) + \sum_{(p,n);\,|(p,n)|<n} D(\pi,\cdots,\pi), \quad \text{mod}(\pi)^{2p+1}.$$

Thus if in $\sum_{i_m<n}A_{i_1}\cdots A_{i_p}$ we also require that for some m $i_m\neq n/p$ we have

$$A_n = z^{-1}\left[c_n + \frac{1}{p}\sum_{i_m<n} A_{i_1}\cdots A_{i_p}\right], \quad \text{mod}(\pi).$$

X. Let expo $R_p=p$, $\pi^p+pu_1=0$, and $D_i(\pi)\in(\pi)^2$ for $i=1,2,\cdots,n-1$. If $D_n(b_0)+u_1^2A_{n/p}^p=c_n\pi$ then

$$(p\pi^{p-1}z)D_n(\pi) = p^2D_n(d_1)\pi + p^2D_n(b_0) + (D_{n/p}(\pi))^p, \quad \text{mod}(\pi)^{2p+2}.$$

Thus $A_n=z^{-1}[-u_1^{-1}D_n(d_1)-u_1^{-1}c_n]$, mod (π).

We may now proceed with the lifting of higher derivations of k to R_p. The following theorem will be used repeatedly:

2.7 THEOREM [3, THEOREM 4, p. 38]. *Let $\overline{S}$ be a p-basis for k and let $S\subset R$ be a set of representatives of the elements of $\overline{S}$. If I is the set of positive integers and f is a mapping from $S\times I$ into R_p then there is one and only one $\{D_i\}\in H(R, R_p)$ (finite higher derivation $\{D_i\}_{i=1}^n$ of length n) such that $D_i(s)=f(s, i)$ for all $s\in S$ and all $i\in I$ (all $s\in S$ and all $i=1, 2, \cdots, n$).*

2.8 LEMMA. *Let* expo $R_p=0$ *and* $v(f'(\pi))=p+(j-1)<2p-1$. *If* $\{\delta_i\}\in H(k)$, $\delta_i(\text{res } R_p)=0$ *when* $p\nmid i$ *and* res $R_p^{-1}\delta_{ip}(\text{res } R_p)\in k^p$ *for each* i, *then* $\{\delta_i\}$ *lifts to a higher derivation of* R_p.

PROOF. Note that $\bar{a}_0=\bar{u}=\text{res } R_p$. Let $B_0=1$, $B_i\in R_p$ so that $\overline{B}_i^p=\bar{u}^{-1}\delta_{ip}(\bar{a}_0)$, and $C_i\in R_p$ so that

$$\overline{C}_i = \bar{z}\overline{B}_i - \delta_i(\bar{a}_j) - \sum_{i_m<i} \delta_{i_0}(\bar{a}_j)\overline{B}_{i_1} \cdots \overline{B}_{i_j}.$$

Choose a p-basis $\overline{S}$, $\bar{a}_0\in\overline{S}$, for k and let $S\subset R$, $a_0\in S$ be a set of representatives for $\overline{S}$. For $s\in S-\{a_0\}$ define D_1 so that D_1 will induce δ_1. Let $D_1(a_0)=\pi^jC_1$. Then $D_1(\pi)\in(\pi)$ so D_1 extends to R_p. Also $\pi^{-1}D_1(\pi)=z^{-1}[D_1(a_j)+C_1]$, mod (π) or

$$\overline{A}_1 = \bar{z}^{-1}[\delta_1(\bar{a}_j) + \bar{z}\overline{B}_1 - \delta_1(\bar{a}_j)] = \overline{B}_1.$$

Suppose we have defined $\{D_j\}_{j=1}^{i-1}$ a finite higher derivation of R into R_p with the properties $D_j(\pi)\in(\pi)$, D_j induces δ_j, and $\overline{A}_j=\overline{B}_j$. Define D_i on $S-\{a_0\}$ so that D_i induces δ_i and let $D_i(a_0)=\pi^jC_i+uA_{i/p}^p$. Then $[D_i(a_0)]^-=\bar{u}\overline{A}_{i/p}^p=\bar{u}\overline{B}_{i/p}^p=\bar{u}(\bar{u}^{-1}\delta_i(\bar{a}_0))=\delta_i(\bar{a}_0)$. Thus D_i induces δ_i. Also

$$\begin{aligned}(p\pi^{j-1}z)D_i(\pi) &= pD_i(a_j)\pi^j + p\pi^jC_i + upA_{i/p}^p + (D_{i/p}(\pi)^p) \\ &\quad + p\sum_{(j+1,i);\,|(j+1,i)|<i} D(a_j, \pi \cdots \pi), \quad \text{mod}(\pi)^{p+j+1}\end{aligned}$$

so $D_i(\pi)\in(\pi)$. Using this equation we have

$$\begin{aligned}\overline{A}_i = \bar{z}^{-1}[\delta_i(\bar{a}_j) + \bar{z}\overline{B}_i - \delta_i(\bar{a}_j) - \sum_{i_m<i} \delta_{i_0}(\bar{a}_j)\overline{B}_{i_1} \cdots \overline{B}_{i_j} \\ + \sum_{i_m<i} \delta_{i_0}(\bar{a}_j)\overline{A}_{i_1} \cdots \overline{A}_{i_j}] = \overline{B}_i.\end{aligned}$$

The lemma follows from 2.7 and mathematical induction.

2.9 LEMMA. *Let* expo $R_p=0$ *and* $v(f'(\pi))=2p-1$. *If* $\{\delta_i\}\in H(k)$, $\delta_i(\text{res } R_p)=0$ *when* $p\nmid i$, *and* res $R_p^{-1}\delta_{ip}$ (res $R_p)\in k^p$ *for each* i *then* $\{\delta_i\}$ *lifts to a higher derivation of* R_p.

PROOF. Let $B_i\in R_p$ be such that $\bar{B}_i^p=\bar{u}^{-1}\ \delta_{ip}(\bar{a}_0)$ and $C_i=zB_i-p^{-1}\sum_{i_m<i}B_{i_1}\cdots B_{i_p}$, where in the summation, $\sum_{i_m<i}B_{i_1}\cdots B_{i_p}$, we also require that for some $m=1,\cdots,p$ $i_m\neq i/p$ (as in IX). Choose a p-basis $\bar{S}$, $\bar{a}_0\in\bar{S}$, for k and let $S\subset R$, $a_0\in S$ be a set of representatives for S. Define D_1 on $S-\{a_0\}$ so that D_1 will induce δ_1. Let $D_1(a_0)=\pi^pC_1$. Then D_1 induces δ_1, $D_1(\pi)\in(\pi)$, and $A_1=B_1$, mod(π). Suppose $\{D_j\}_{j=1}^{i-1}$ is a finite higher derivation of R into R_p with D_j inducing δ_j, $D_j(\pi)\in(\pi)$, and $A_j=B_j$, mod (π). Define D_i on $S-\{a_0\}$ so that D_i induces δ_i and let $D_i(a_0)-uA_{i/p}^p=\pi^pC_i$. Then D_i induces δ_i, $D_i(\pi)\in(\pi)$, and by IX

$$A_i = z^{-1}\left(C_i+\frac{1}{p}\sum_{i_m<i}A_{i_1}\cdots A_{i_p}\right),\qquad \text{mod }(\pi)$$

$$= z^{-1}\left(zB_i-\frac{1}{p}\sum_{i_m<i}B_{i_1}\cdots B_{i_p}+\frac{1}{p}\sum_{i_m<i}A_{i_1}\cdots A_{i_p}\right),\quad \text{mod }(\pi).$$

Thus, again $\bar{A}_i=\bar{B}_i$ and the proof is complete.

2.10 LEMMA. *Let* $1\leqq$expo $R_p=j<p$. *If* $\{\delta_i\}\in H(k)$ *and* δ_i(res $R_p)=0$ *for each* i *then* $\{\delta_i\}$ *lifts to a higher derivation of* R_p.

PROOF. If res $R_p\neq 0$ then res $R_p=\bar{r}$ where $\bar{r}^{p^n}=\overline{a_j}$, $r\in R$, and $\bar{r}\notin k^p$. Let $\bar{S}$, $\bar{r}\in\bar{S}$, be a p-basis for k and $S\subset R$, $r\in S$, a set of representatives for $\bar{S}$. For each i define D_i on $S-\{r\}$ so that D_i induces δ_i. Let $D_i(r)=0$ for all i. Then $\{D_i\}$ induces $\{\delta_i\}$ and by checking (2.2, i) $\{D_i\}$ extends to R_p. If res $R_p=0$ choose any p-basis $\bar{S}$ for k and $S\subset R$ a set of representatives for $\bar{S}$. Define $\{D_i\}$ on S so that D_i induces δ_i for each i. Then 2.2, i and II imply $\{D_i\}$ extends to R_p.

2.11 LEMMA. *Let* expo $R_p=p$ *and* $\pi^p=pu_1=0$. *If* $\{\delta_i\}\in H(k)$, δ_{ip}(res $R_p)\in k^p$ *for each* i, *and* δ_i(res $R_p)=0$ *when* $p\nmid i$, *then* $\{\delta_i\}$ *lifts to a higher derivation of* R_p.

PROOF. By V $a_0=1+pb_0$ and res $R_p=-\bar{b}_0$. Let $\bar{B}_i^p=-\delta_{ip}(\bar{b}_0)$ and $\bar{C}_i=-\delta_i(\bar{d}_1)-\bar{z}\bar{B}_i$, where again $a_1=pd_1$. Choose a p-basis $\bar{S}$, $\bar{b}_0\in\bar{S}$, for k and let $S\subset R$, $b_0\in S$, be a set of representatives for $\bar{S}$. Define D_1 on $S-\{b_0\}$ so that D_1 induces δ_1 and let $D_1(b_0)=\pi C_1$. Then D_1 induces δ_1, $D_1(\pi)\in(\pi)^2$, and $\bar{A}_1=\bar{B}_1$. Suppose we have defined $\{D_j\}_{j=1}^{i-1}$, a finite higher derivation of R into R_p, with D_j inducing δ_j, $D_j(\pi)\in(\pi)^2$ and $\bar{A}_j=\bar{B}_j$. Define D_i on $S-\{b_0\}$ so that D_i induces

δ_i and let $D_i(b_0)+u_1^2A_{i/p}^p=\pi C_i$. Then D_i induces δ_i, $D_i(\pi)\in(\pi)^2$, and by X $\overline{A}_i=\overline{B}_i$.

2.12 LEMMA. *Let* expo $R_p=p+1$. *If* $\{\delta_i\}\in H(k)$ *and* $\delta_i(\text{res } R_p)=0$ *for each* i *then* $\{\delta_i\}$ *lifts to a higher derivation of* R_p.

PROOF. Omitted.

In view of the fact that higher derivations of k are completely determined by their action on a p-basis for k corollary A is clear. Corollary B follows from [5].

Let S be a ring and τ the natural map of $S[[x]]$ onto S. The inertial embedding group of S is the group of all automorphisms η of $S[[x]]$ such that $\tau\eta(a)=a$, for each $a\in S$, and $\eta(x)=x$. The higher derivation $\{D_i\}$ of S determines an (inertial) embedding η of S where $\eta(a)=a+\sum_{i=1}^{\infty} D_i(a)x_i$ for $a\in S$ and $\eta(x)=x$. Conversely an embedding of S determines a higher derivation of S. The correspondence is an isomorphism of the higher derivation group of S onto the inertial embedding group of S. An application of our theorem will identity the subgroup of the embedding group of k consisting of those embeddings which are induced by embeddings of R_p.

REFERENCES

1. N. Heerema, *Embeddings of a p-adic field and its residue field in their power series rings*, Proc. Amer. Math. Soc. **14** (1963), 574–580.

2. ———, *Derivations and intertial automorphisms of complete regular local rings*, Amer. J. Math. **88** (1966), 33–42.

3. ———, *Convergent higher derivations on local rings*, Trans. Amer. Math. Soc. **132** (1968), 31–44.

4. ———, *Intertial automorphisms of a class of wildly ramified v-rings*, Trans. Amer. Math. Soc. **132** (1968), 45–54.

5. M. Heinzer, *Strongly convergent derivation automorphisms on a class of wildly ramified v-rings*, Duke Math. J. (to appear).

6. J. Neggers, *Derivations on p-adic fields*, Trans. Amer. Math. Soc. **115** (1965), 496–504.

7. E. Wishart, *Higher derivations on p-adic fields*, Dissertation, Florida State University, Tallahassee, Fla., 1965.

FLORIDA STATE UNIVERSITY