SHORTER NOTES

The purpose of this department is to publish very short papers of an unusually elegant and polished character, for which there is no other outlet.

ASSOCIATIVE, ABELIAN H-SPACES HAVE TRIVIAL POSTNIKOV INVARIANTS

MARTIN ARKOWITZ

We give a short, homotopy-theoretic proof of this result as a consequence of the Dold-Thom Theorem [1]. A different proof based on the D-T Theorem appears in [1, §7] and several semisimplicial proofs exist (e.g., [3]). All spaces are assumed to be of the homotopy type of connected CW-complexes.

Step 1. If X is an associative, abelian H-space, then the embedding $i: X \to SP^\infty X$ admits a retraction.

Define $r: SP^\infty X \to X$ by $r(x_1, x_2, \cdots, x_n) = x_1 \cdot x_2 \cdot \cdots \cdot x_n$.

Step 2. If $\lambda_n: \pi_n(SP^\infty X) \xrightarrow{\approx} H_n(X)$ is the isomorphism of the D-T Theorem and $i_n: \pi_n(X) \to \pi_n(SP^\infty X)$ is induced by i, then $\lambda_n i_n$ is the Hurewicz homomorphism $h_n: \pi_n(X) \to H_n(X)$.

This is part of the D-T Theorem [1, p. 274].

Step 3. If X is an associative, abelian H-space, then the Hurewicz homomorphism $h_n: \pi_n(X) \to H_n(X)$ is a monomorphism onto a direct summand.

This is a consequence of Steps 1 and 2.

Step 4. If X is a space such that the Hurewicz homomorphism $h_n: \pi_n(X) \to H_n(X)$ is a monomorphism onto a direct summand, then the Postnikov invariant $k^{n+1} \in H^{n+1}(X_{n-1}; \pi_n)$ is zero.

This is a known result [2, p. 273], but we sketch the proof for completeness. The hypotheses implies $h_n': \pi_n(X_n) \to H_n(X_n)$ is a monomorphism onto a direct summand and this in turn implies that $j_*: H_n(F_n) \to H_n(X_n)$ is, where F_n is the Eilenberg-MacLane complex $K(\pi_n, n)$ and $j: F_n \to X_n$ is the inclusion of the fibre in the total space. Let $\alpha(A): H^n(A; \pi_n) \to \operatorname{Hom}(H_n(A), \pi_n)$ be the epimorphism of the universal coefficient theorem. Then $\operatorname{Hom}(j_*, \pi_n) \circ \alpha(X_n) = \alpha(F_n) \circ j^*$. Since $\operatorname{Hom}(j_*, \pi_n)$ and $\alpha(X_n)$ are epimorphisms and $\alpha(F_n)$ is an isomorphism, we have that $j^*: H^n(X_n; \pi_n) \to H^n(F_n; \pi_n)$ is an epimorphism. Thus the basic class $b_n \in H^n(F_n; \pi_n)$ is in Image j^*, and so by exactness $\tau(b_n) = 0$, where $\tau: H^n(F_n; \pi_n) \to H^{n+1}(X_{n-1}; \pi_n)$ is the transgression homomorphism. Since $k^{n+1} = -\tau(b_n)$, the result follows.

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REMARK. The results on Postnikov systems hold if the space X is n-simple for all n. This is of course the case when X is an H-space.

ADDED IN PROOF: A similar proof has been given by H. Haslam in his doctoral dissertation, University of California, Irvine, 1969.

BIBLIOGRAPHY

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DARTMOUTH COLLEGE, HANOVER, NEW HAMPSHIRE 03755