A TAUBERIAN THEOREM FOR THE $(C, 1)(N, 1/(n+1))$ SUMMABILITY METHOD

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ABSTRACT. A Tauberian theorem is proved which infers the $(C, 1)$ summability of a sequence associated with a formally differential Fourier series from its $(C, 1)(N, 1/(n+1))$ summability under suitable conditions.

In the present note we answer in the affirmative a question raised in [**2**, p. 19]. The definitions and notations of [**2**] are used in this note without further explanation.

THEOREM. *If the sequence* $\{s_n\}$ *is summable* $(C, 1)(N, 1/(n+1))$ *and if*

$$\overset{1}{s}_n - \overset{1}{s}_{n-1} = O(n^{-\delta}), \qquad (0 < \delta < 1; n \to \infty),$$

where s_n^1 *is the nth* $(C, 1)$ *mean of* $\{s_n\}$, *then* $\{s_n\}$ *is* $(C, 1)$ *summable.*

It follows from this result that Theorem A of [**2**] is indeed a consequence of Theorem 1 of [**2**].

We need the following results for the proof of the theorem.

LEMMA 1. *If a sequence* $\{s_n\}$ *is summable* $(N, 1/(n+1))$ *to* s *and* $s_n - s_{n-1} = O(n^{-\delta})$, $0<\delta<1$, *then* $\{s_n\}$ *converges to* s.

Lemma 1 is due to Iyengar [**3**, cf. Theorem I].

LEMMA 2. *If for* $n=0, 1, 2, \cdots, p_n>0$ *and* $p_{n+1}/p_n \leqq p_{n+2}/p_{n+1} \leqq 1$, *then*

$$(N, p_n)(C, 1) \text{ is equivalent to } (C, 1)(N, p_n).$$

While proving inclusion relations for the absolute $(C, 1)(N, p_n)$ and $(N, p_n)(C, 1)$ methods, Das has pointed out in [**1**] that the corresponding analogues for ordinary summability also hold. Thus we have Lemma 2 as an analogue of Theorem 5 of [**1**].

To prove the theorem we first observe that the hypotheses of Lemma 2 are satisfied if $p_n = 1/(n+1)$ and therefore the summability $(C, 1)(N, 1/(n+1))$ of $\{s_n\}$ is the same as the $(N, 1/(n+1))$ summability of the sequence of $(C, 1)$ means of $\{s_n\}$. These means satisfy the hypothesis of Lemma 1 and the theorem follows.

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