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CORRECTION TO A THEOREM OF MINE

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ABSTRACT. One of the theorems published by me in a previous paper turned out to be incorrect. That theorem is replaced in this note by a corrected one.

In 1966 I published the following theorem [4, Theorem 3, p. 882]: (In what follows C_n denotes an n-dimensional conformally-flat Riemannian space and C_n' a C_n of class one.)

"The coordinates of any C_n' may be so chosen that its metric assumes the normal form

$$ds^2 = \sum_i (dx^i)^2/[f(\theta)]^2, \qquad \theta = \sum_i (x^i)^2,$$

where f is any real analytic function of θ subject to the restriction

$$(n-1)ff' + \theta ff'' - (n-1)\theta f'^2 \neq 0, \qquad (f' = df/d\theta, \text{ etc.})\text{''}$$

In a recent paper [1] G. M. Lancaster has proved that this theorem is incorrect by showing that the above metric does not cover a certain type of C_n'. The purpose of this note is to point out that the metric covers a type of C_n' although it does not cover all C_n', and also to give a correct form of the theorem. Before doing so I have to say that on checking an error in a previous paper of mine [2, equations (1.8), p. 107], the referee of the present note has drawn my attention to the fact that the restriction in the above theorem applies when $f'' \neq 0$; and at the same time he has given a straightforward solution of

$$(n-1)ff' + \theta ff'' - (n-1)\theta f'^2 = 0, \quad \text{where} \quad f'' \neq 0,$$

as

$$f(\theta) = \theta/[K_1 + (n-2)K_2\theta^{n-2}]^{1/(n-2)}, \qquad K_1 > 0.$$

I heartily thank the referee for the pains and the interest he has taken in the paper. A correct form of the theorem which must replace the above theorem may then be stated as follows ($f' = df/d\theta$, etc.):

A C_n $(n > 3)$ *with metric*

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$$ds^2 = \sum_i (dx^i)^2/[f(\theta)]^2, \qquad \theta = \sum_i (x^i)^2,$$

where $f(\theta)$ *is any real analytic function of* θ *subject to the restriction that when* $f'' \neq 0$,

$$f(\theta) \neq \theta/[K_1 + (n-2)K_2\theta^{n-2}]^{1/(n-2)}, \qquad K_1 > 0, \quad K_2 \text{ being constants,}$$

is a C_n'. *The metric covers the case of space of constant curvature when* $f''=0$.

That the C_n is a C_n' is proved straightway by showing that it satisfies the Gauss-Codazzi equations. In fact, referring to my paper [4, p. 882] mentioned at the outset, it is not difficult to see that the C_n satisfies equations (7), namely

$$R_{hijk} = \frac{R_{hj}R_{ik} - R_{hk}R_{ij}}{(n-2)\{(n-2)\rho^2 + \rho\bar{\rho}\}} - \frac{\rho\bar{\rho}}{n-2}(g_{hj}g_{ik} - g_{hk}g_{ik}),$$

where ρ^2 and $\rho\bar{\rho}$ are given by equations (10), namely

$$\rho^2 = 4f'(f - \theta f'), \qquad \rho\bar{\rho} = 4(ff' + \theta ff'' - \theta f'^2).$$

We have only to take now the second fundamental tensor b_{ij} as

$$b_{ij} = -\frac{1}{n-2}(R_{ij}/\rho + \bar{\rho}g_{ij})$$

and establish the Gauss-Codazzi equations in exactly the same way as they have been done in a previous paper of mine [3, equations (3.8), (5.1)].

References

1. G. M. Lancaster, *Note on a theorem by Sen*, Proc. Amer. Math. Soc. **19** (1968), 1119–1122. MR **38** #634.

2. R. N. Sen, *On a type of Riemannian space conformal to a flat space*, J. Indian Math. Soc. **21** (1957), 105–114. MR **20** #4868.

3. ———, *Conformally euclidean space of class one*, Indian J. Math. **6** (1964), 93–103. MR **31** #3977.

4. ———, *On conformally-flat Riemannian space of class one*, Proc. Amer. Math. Soc. **17** (1966), 880–883. MR **34** #753.

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