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SHORTER NOTES

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ON THE HYPERSPACE OF SUBCONTINUA OF AN ARC-LIKE CONTINUUM

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ABSTRACT. It is shown that the hyperspace of each arc-like continuum can be embedded in E^3.

W. R. R. Transue [1] in a beautiful note gave a positive answer to A. Connor's [2, p. 152] question "Can the hyperspace of subcontinua of the pseudoarc (with the Hausdorff metric) be embedded in E^3." This note extends this result to arc-like continua, i.e. inverse limits on arcs originally called snake-like continua by R. H. Bing [3].

Here $\{W, f_i\}$ will denote the inverse limit system with indexing set the nonnegative integers and with each factor space W. The associated inverse limit space will be denoted by $\lim\{W, f_i\}$. See [4, p. 87] for a discussion of inverse limits. The hyperspace of continua of a space X, denoted by $C(X)$, is studied in [5]. The closed interval $[0, 1]$ will be called I.

THEOREM. *The hyperspace of continua of the inverse limit space* $X=\lim\{I, f_i\}$ *embeds in* E^3.

PROOF. There is no loss to assume that none of the maps f_i is constant on an open set. Since a continuum in I is either a closed interval or a point, $C(I)$ will be identified with

$$D = \{(x, y, z) \subset E^3 \mid 0 \leqq x \leqq y \leqq 1, z = 0\}.$$

Take $F_i: D \to D$ by

$$F_i(x, y, 0) = (\min f_i(t), \max f_i(t), 0), \qquad t \in [x, y].$$

Now F_i is the natural map from $C(I)$ to $C(I)$ induced by f_i.

J. Segal [6] proved that the hyperspace of continua of the inverse limit space X is homeomorphic to $\lim\{C(I), F_i\}$. The proof of the theorem will be completed by embedding $\lim\{D, F_i\}$ in E^3. Now if each of the maps F_i could be *approximated by embeddings in* E^3 in the

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sense of McCord [7, Theorem 2] then $\lim\{D, F_i\}$ embeds in E^3 by McCord's Theorem. To construct these approximations one must prove the following:

LEMMA. *If $\epsilon > 0$, then there is a homeomorphism h_i of E^3 onto E^3 such that $\|h_i | D, F_i\| < \epsilon$.*

PROOF. For each $(x, y, 0) \in D$ take

$$G_i(x, y, 0) = F_i(x, y, 0) + \left(0, 0, \frac{\epsilon}{4} \cdot \frac{x+y}{2}\right) \quad \text{(vector addition)}.$$

As f_i is not locally constant, a check will show that each point inverse of G_i is a point or a closed interval which does not separate D. Thus $G_i(D)$ is a topological disk and consequently there is a homeomorphism H_i of D onto $G_i(D)$ such that $\|H_i, G_i\| < \epsilon/4$ by Radó [8, Theorem 2.17]. Next $G_i(D)$ is tamed by an approximation theorem of R. H. Bing [9] which constructs a homeomorphism J_i of D into E^3, $\|J_i, H_i\| < \epsilon/4$ and with $J_i(D)$ a polyhedron. So there is a homeomorphism h_i of E^3 onto E^3 with $h_i | D = J_i$ since $J_i(D)$ is tame.

REMARK. With more care one can construct this embedding so that the intersection of the embedded $C(X)$ and the plane $x = y$ is exactly the set of degenerate subcontinua of X.

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