PROCEEDINGS OF THE
AMERICAN MATHEMATICAL SOCIETY
Volume 32, Number 1, March 1972

## SHORTER NOTES

The purpose of this department is to publish very short papers of an unusually elegant and polished character, for which there is no other outlet.

# THE TANGENT MICROBUNDLE OF A SUITABLE MANIFOLD

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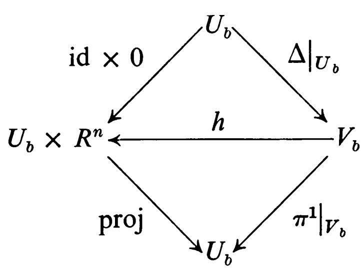
ABSTRACT. The purpose of this note is to generalize to the topological category the fact that a suitable differentiable manifold is parallelizable (Theorem 4 of [1]). This result has a "folk-theorem" status in some quarters, but I believe that in view of the recent interest in $H$-manifolds [2], it would be desirable to have the result on record.

Let $M$ be an $n$-manifold. Define $\Delta: M \to M \times M$ to be the diagonal map, and $\pi^1, \pi^2: M \times M \to M$ to be the projections on the first and second factor respectively. Milnor [3] calls the diagram $\Delta: M \rightleftarrows M \times M: \pi^1$ the *tangent microbundle* of $M$, where for each point $b \in M$ there exists an open set $U_b$ in $M$ containing $b$, an open set $V_b$ in $M \times M$ containing $\Delta(b)$, and a homeomorphism $h: V_b \to U_b \times R^n$ such that the following diagram commutes:




Received by the editors May 18, 1971.

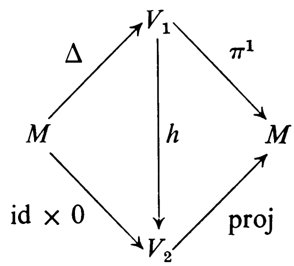
*AMS* 1970 *subject classifications*. Primary 57A55.

*Key words and phrases*. Tangent microbundle, topological parallelizability, suitable manifolds, $H$-spaces.

[1] Work done while author was on an NSF Traineeship at the University of California, Los Angeles.

An $n$-manifold $M$ is *topologically parallelizable* if there exists an open set $V_1$ in $M\times M$ containing $\Delta(M)$, an open set $V_2$ in $M\times R^n$ containing $M\times 0$, and a homeomorphism $h: V_1\to V_2$ so that the following diagram commutes:

(1)




Pick $e\in M$. $M$ is *suitable* if there is a continuous map $\Phi: M\to G(M)$ such that $\Phi(x)(x)=e$ and $\Phi(e)=$identity, where $G(M)$ is the group of all homeomorphisms of $M$ onto itself with the compact-open topology. By Theorem 2 of [**1**], $M$ is suitable iff there exists a $\theta\in G(M\times M)$ such that

$$\theta(M\times(M-e))=\{(x,y)\in M\times M: x\neq y\} \quad \text{and} \quad \pi^1\theta=\pi^1.$$

Note that a suitable manifold supports an $H$-space structure [**1**].

THEOREM. *A suitable n-manifold M is topologically parallelizable.*

PROOF. Let $U_b$ and $V_b$ be as in the definition of the tangent microbundle of $M$. Let $W$ be an open set in $M$ such that $e\in W\subset \text{cl } W\subset U_e$. Choose the $V_b$'s so that $\pi^2\theta^{-1}(x,y)\in W$ for $(x,y)\in V_b$. Let $k: U_e\to R^n$ be a co-ordinate map such that $k(e)=0$. Define $\lambda: M\to[0,1]$ so that $\lambda$ is 1 on a neighborhood of cl $W$ and 0 on a neighborhood of $M-U_e$.

Let $V=\bigcup_{b\in M} V_b$ and define $h: V\to M\times R^n$ by

$$h(x,y)=(x,\lambda(\pi^2\theta^{-1}(x,y))k(\pi^2\theta^{-1}(x,y))).$$

$h$ is a local homeomorphism, i.e. for $b\in M$, $h: V_b\to\text{image } h|_{V_b}$ is a homeomorphism, for define $h'$: image $h|_{V_b}\to V_b$ by

$$h'(x,r)=(x,\pi^2\theta(x,k^{-1}(r))).$$

Then on $V_b$, $\pi^2\theta^{-1}(x,y)\in W$ so $h'h=\text{id}$, and on image $h|_{V_b}$, $k^{-1}(r)\in W$ so $hh'=\text{id}$.

However, $h:\Delta(M)\to M\times 0$ homeomorphically, so by Lemma 4.1 of [**4**], there is a neighborhood $V_1$ in $M\times M$ of $\Delta(M)$ and a neighborhood $V_2$ in $M\times R^n$ of $M\times 0$ such that $h: V_1\to V_2$ is a homeomorphism. As $h$ commutes in (1) this proves our result.

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