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GENERATING CLASSES OF PERFECT BANACH SEQUENCE SPACES

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ABSTRACT. A perfect sequence space λ is said to be a *step* if $l^1 \subset \lambda \subset l^\infty$ and λ is a Banach space in its strong topology from $\lambda^\times$. In this paper a method is given to generate additional steps from a step λ. Precisely, λ^p is a step where $\lambda^p \equiv \{x=(x_i) \mid x_i \in C$ and $|x|^p=(|x_i|^p) \in \lambda\}$, for $1 \leqq p < \infty$, with norm $\|x\|_{\lambda^p}=(\| \, |x|^p \|_\lambda)^{1/p}$. It is shown that λ^p, $1<p<\infty$, is reflexive iff λ has a Schauder basis. The space of diagonal maps of λ^p into λ is characterized, as is the space of diagonal nuclear maps of λ into λ^p when λ has a Schauder basis.

If λ is a perfect sequence space which is a Banach space under the strong topology from $\lambda^\times$, and contains l^1 and is contained in l^∞, we say that λ is a *step*. Examples of steps in general include the Köthe dual of the usual sequence space associated with a Banach space possessing a normalized unconditional basis; see [**4**]. More specifically the l^p spaces, the spaces $\mu_{a,p}$ and $\nu_{a,p}$ of Garling [**5**], and the spaces $m(\phi)$ and $n(\phi)$ of Sargent [**13**], are steps. In this paper we generate additional steps from a step λ by paralleling a method of generating the l^p spaces from l^1. In the cases where the usual coordinate vectors form a basis for λ, these generated spaces are reflexive. Others results paralleling the known properties of l^p spaces are obtained under this additional hypothesis.

1. **Definitions and preliminary results.** The general terminology of this paper is as in [**9**]. Throughout we will assume that the sequence spaces λ are normal and equipped with the topology $\mathscr{T}_b(\lambda^\times)$, unless we specifically state otherwise.

For sequences $x=(x_i)$, $y=(y_i)$ we denote by xy the sequence (x_iy_i). Using the notation of Ruckle [**12**], we denote by μ^λ the $\{u \mid ux \in \lambda$ for each

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$x \in \mu\}$, where μ and λ are sequence spaces. Since μ^λ is a normal sequence space, $\mu^{\lambda\lambda} \equiv (\mu^\lambda)^\lambda$ has meaning. If $\mu^{\lambda\lambda} = \mu$, we say that μ is *λ-perfect*. If $\lambda = l^1$, we use the standard terminology of **[9]** and write $\mu^\times$ for μ^λ and say that μ is *perfect* if $\mu^{\times\times} = \mu$.

Let x be the sequence (x_i). We denote by $x^{(n)}$ the sequence (y_i), where $y_i = x_i$ for $i \leqq n$, and $y_i = 0$ for $i > n$.

Suppose λ is a step and u and v elements in λ. We may choose the norm, $\|\cdot\|_\lambda$, on λ so that $\|u\|_\lambda \leqq \|v\|_\lambda$ whenever $|u_i| \leqq |v_i|$ for each i; see **[1]**. Also, for λ a step, $\lambda^\times$ is a step (see **[2]**). We denote by λ_r the $\{x \in \lambda \mid x^{(n)}$ converges to x in the $\mathscr{T}_b(\lambda^\times)$-topology$\}$. Under our general hypothesis it is easy to see that the topological dual of λ_r is $\lambda^\times$. If $\lambda = \lambda_r$, we say that λ is *regular*.

To eliminate some writing in the following sections we will always mean that λ is a step when this symbol is used.

As a final preliminary we give the following parallel of the definition of l^p:

DEFINITION. For λ a step and $1 \leqq p < \infty$, define λ^p to be the $\{x \mid |x|^p \equiv (|x_i|^p) \in \lambda\}$.

2. **Norming λ^p.** For $x \in \lambda^p$ we let $\|x\|_{\lambda^p}$ denote the number $(\| |x|^p \|_\lambda)^{1/p}$. We show in this section that $\|\cdot\|_{\lambda^p}$ defines a norm on λ^p which is strictly convex if λ is regular. To simplify notation we let μ denote λ^p.

2.1 *Minkowski inequality.* If $x, y \in \mu$, then $\|x+y_\mu \leqq \|x\|_\mu + \|y\|_\mu$.

PROOF. Let x, $y \in \mu$, and $u \in \lambda^\times$, with $\|u\|_{\lambda^\times} \leqq 1$. By definition, $|x|^p \in \lambda$; so $\sum_{i=1}^\infty |u_i|\,|x_i|^p < \infty$ and $(|u|^{1/p})(x) \in l^p$. Thus $(|u|^{1/p})(x)$ and $(|u|^{1/p})(y)$ are in l^p. Using the familiar Minkowski inequality for the second inequality we have

$$\begin{aligned}
(|\langle u, |x+y|^p\rangle|)^{1/p} &\leqq \left(\sum_{i=1}^\infty (|u_i|^{1/p}(|x_i| + |y_i|))^p\right)^{1/p} \\
&= \left(\sum_{i=1}^\infty (|u_i|^{1/p}\,|x_i| + |u_i|^{1/p}\,|y_i|)^p\right)^{1/p} \\
&\leqq \left(\sum_{i=1}^\infty (|u_i|^{1/p}\,|x_i|)^p\right)^{1/p} + \left(\sum_{i=1}^\infty (|u_i|^{1/p}\,|y_i|)^p\right)^{1/p} \\
&= \langle |u|, |x|^p\rangle^{1/p} + \langle |u|, |y|^p\rangle^{1/p} \\
&\leqq (\| |x|^p \|_\lambda)^{1/p} + (\| |y|^p \|_\lambda)^{1/p}.
\end{aligned}$$

The last inequality follows since $\|u\|_{\lambda^\times} = \| |u| \|_{\lambda^\times}$. Hence

$$\sup\{(|\langle u, |x+y|^p\rangle|)^{1/p} \mid u \in \lambda^\times,\ \|u\|_{\lambda^\times} \leqq 1\} \leqq \|x\|_\mu + \|y\|_\mu,$$

and the conclusion holds. □

Using 2.1 it is clear that $(\lambda^p, \|\cdot\|_{\lambda^p})$ is a normed space.

2.2 PROPOSITION. *The norm of μ is strictly convex if λ is regular.*

PROOF. By way of contradiction assume the existence of a pair of elements x, y in μ with $x \neq y$, $\|x\|_\mu = \|y\|_\mu = 1$ and $\|\frac{1}{2}(x+y)\|_\mu = 1$. Since $\lambda = \lambda_r$, we have $\lambda' = \lambda^\times$, and hence there is an element u in $\lambda^\times$ with $\|u\|_{\lambda^\times} = 1$ and $\langle u, |\frac{1}{2}(x+y)|^p\rangle = 1$. It is clear that u may be chosen with $u_i \geqq 0$ for all i. Thus

$$1 = \left(\sum_{i=1}^{\infty}\left(u_i^{1/p}\left|\frac{x_i}{2} + \frac{y_i}{2}\right|\right)^p\right)^{1/p} = \left(\sum_{i=1}^{\infty}\left|(u_i^{1/p})\left(\frac{x_i}{2}\right) + (u_i^{1/p})\left(\frac{y_i}{2}\right)\right|^p\right)^{1/p}$$
$$= \|\tfrac{1}{2}((u^{1/p})(x) + (u^{1/p})(y))\|_p.$$

l^p is strictly convex, so $\|(u^{1/p})(x)\|_p$ or $\|(u^{1/p})(y)\|_p$ is larger than 1. Assuming $\|(u^{1/p})(x)\|_p > 1$ yields $\langle u, |x|^p\rangle > 1$ and $\|x\|_\mu > 1$. □

3. **λ^p is a step which is λ-perfect.** Again for simplicity of notation we let $\mu = \lambda^p$ and $\Psi = \lambda^q$, where $1/p + 1/q = 1$.

3.1 *Hölder inequality.* If $x \in \mu$, $z \in \Psi$, then $\|xz\|_\lambda \leqq \|x\|_\mu \|z\|_\Psi$.

PROOF. For $u \in \lambda^\times$, $x \in \mu$, and $z \in \Psi$, it follows as in the proof of 2.1 that $(|u|^{1/p})(x) \in l^p$ and $(|u|^{1/q})(z) \in l^q$. Using the perfectness of λ, it clearly follows that $xz \in \lambda$.

Now assume $\|u\|_{\lambda^\times} \leqq 1$. Using the standard Hölder inequality for the second inequality, we have

$$|\langle u, xz\rangle| \leqq \sum_{i=1}^{\infty} |u_i x_i z_i| \leqq \left(\sum_{i=1}^{\infty} (|u_i|^{1/p}|x_i|)^p\right)^{1/p}\left(\sum_{i=1}^{\infty}(|u_i|^{1/q}|z_i|)^q\right)^{1/q}$$
$$= \langle |u|, |x|^p\rangle^{1/p}\langle |u|, |z|^q\rangle^{1/q} \leqq \|x\|_\mu \|z\|_\Psi.$$

The conclusion follows as argued in 2.1. □

3.2 LEMMA. *If $z \in \Psi$, then $\|z\|_\Psi$ equals the norm of z as an operator from μ into λ.*

PROOF. By 3.1 we have, for $z \in \Psi$, that z is an operator of μ into λ and that the operator norm of z is no larger than $\|z\|_\Psi$.

For the reverse inequality let $z \in \Psi$ and $\alpha = \| |z|^q \|_\lambda$. We have $\|z\|_\Psi = (\alpha)^{1/q} = (\alpha)(\alpha^{-1/p}) = \| |z|^q/\alpha^{1/p}\|_\lambda = \|(z)(|z|^{q/p})/\alpha^{1/p}\|_\lambda \leqq$ operator norm of z, since $\| |z|^{q/p}/\alpha^{1/p}\|_\mu = 1$. □

3.3 PROPOSITION. $\mu^\lambda = \Psi$.

PROOF. In [1] it is shown that μ^λ is a step and that the norm of μ^λ is equivalent to the norm for operators of μ into λ. For $z \in \mu^\lambda$, $|z_i^{(n)}| \leqq |z_i|$ for each i, so $\|z^{(n)}\|_{\mu^\lambda} \leqq \|z\|_{\mu^\lambda}$ for each n. Thus the sequence $\{z^{(n)}\}$ is norm bounded in μ^λ and thus is a bounded sequence in the operator norm. Applying 3.2 and the fact that $\{z^{(n)}\} \subset \Psi$ we have $\{z^{(n)}\}$ a norm bounded sequence in Ψ. Hence $\{|z^{(n)}|^q\}$ is a norm bounded sequence in λ implying that $\{|z^{(n)}|^q\}$ is $\mathscr{T}_s(\lambda^\times, \lambda)$-bounded. Using the normality of $\lambda^\times$, the sequence $\{\sum_{i=1}^n |u_i|\,|z_i|^q\}$ is a bounded sequence of real numbers for each $u \in \lambda^\times$. Thus $\sum_{i=1}^\infty |u_i|\,|z_i|^q < \infty$ and $z \in \Psi^r$.

The reverse inclusion follows from 3.1. □

3.4 THEOREM. *λ^p is a step which is λ-perfect.*

PROOF. 3.3 and the fact that $\Psi^{r\lambda}$ is a step show that λ^p is a step. The λ-perfectness of λ^p also follows from 3.3. □

The following corollary parallels the result stating that $(l^s)^{l^t} = l^r$, where $1/t = 1/r + 1/s$; see [13]:

3.5 COROLLARY. *$(\lambda^s)^{\lambda^t} = \lambda^r$, where $1/t = 1/r + 1/s$.*

PROOF. $\lambda^s = (\lambda^t)^{s/t}$ and $\lambda^r = (\lambda^t)^{r/t}$ with $t/s + t/r = 1$. The result follows from 3.3 and 3.4. □

The referee has observed the following generalized Hölder inequality:

3.6 COROLLARY. *If $1/t = 1/s + 1/r$ and $x \in \lambda^s$, $y \in \lambda^r$, then $xy \in \lambda^t$ with $\|xy\|_{\lambda^t} \leqq \|x\|_{\lambda^s}\|y\|_{\lambda^r}$.*

PROOF. We need only check the norm inequality since the first conclusion is contained in 3.5. For $x \in \lambda^s$, $y \in \lambda^r$ it follows that $|x|^t \in \lambda^{s/t}$ and $|y|^t \in \lambda^{r/t}$. Using the definition of the λ^p norm and 3.1 we have

$$\begin{aligned}\|xy\|_{\lambda^t} &= (\|\,|xy|^t\|_\lambda)^{1/t} \leqq [(\|\,|x|^t\|_{\lambda^{s/t}})(\|\,|y|^t\|_{\lambda^{r/t}})]^{1/t} \\ &= ((\|\,|x|^s\|_\lambda)^{t/s})^{1/t}((\|\,|y|^r\|_\lambda)^{t/r})^{1/t} = \|x\|_{\lambda^s}\,\|y\|_{\lambda^r}. \quad \square\end{aligned}$$

REMARKS. 1. Essentially repeating the proof that a sequence space $\nu = \nu^\times$ if and only if $\nu = l^2$ (see [9]), we have $\nu = \nu^\lambda$ if and only if $\nu = \lambda^2$.

PROOF. One implication follows from 3.3.

Conversely, suppose $\nu = \nu^\lambda$, and let $x \in \nu$ with $\bar{x} = (\bar{x}_i)$. Then $x\bar{x} = |x|^2 \in \lambda$, so $\nu \subset \lambda^2$. However, this inclusion implies $\lambda^2 = (\lambda^2)^\lambda \subset \nu^\lambda = \nu$.

2. In [3, §3], we find

THEOREM. *If ν and ζ are arbitrary perfect sequence spaces and ζ is ν-perfect, then each absolutely ν-summing map is absolutely ζ-summing.*

It is observed in [3] that for $\nu = l^t$, $\zeta = l^s$, with $1 \leqq t \leqq s \leqq \infty$, the hypothesis of this theorem is satisfied. Corollary 3.5 above shows that for $\nu = \lambda^t$ and $\zeta = \lambda^s$, $1 \leqq t \leqq s < \infty$, the above hypothesis is satisfied.

4. **Diagonal nuclear maps and reflexivity of λ^p.** The two main objectives of this section are to isolate the diagonal nuclear maps of λ into λ^p when λ is regular, and show that λ^q, $1 < p < \infty$, is reflexive when λ is regular.

4.1 PROPOSITION. *Each diagonal map of λ^p into λ^s, $1 \leqq s < p < \infty$, is compact if and only if λ is regular.*

PROOF. If λ is regular, it clearly follows that λ^t is regular, for $1 \leqq t < \infty$. In §3 of [1] it is shown that, for ν and ζ steps, the set of diagonal compact operators of ν into ζ is represented precisely by the set $(\nu^\zeta)_r$. The one implication now follows from 3.5.

Conversely, suppose $\lambda \neq \lambda_r$. There is then an x in λ, $x=(x_i)$, $x_i \geqq 0$, all i, with $x^{(n)} \not\to x$ in the norm of λ. Then for some $\varepsilon_0 > 0$ we have $\|x-x^{(n)}\|_\lambda \geqq \varepsilon_0$, for all n. By definition $x^{1/q} \in \lambda^q = \mu^\lambda$ and $\|x^{1/q}-(x^{1/q})^{(n)}\|_{\lambda^q} = (\|x-x^{(n)}\|_\lambda)^{1/q} \geqq (\varepsilon_0)^{1/q}$. Using 3.5 and the result from [1] used above we have the conclusion. □

The following lemma is a slight modification of 3.1 of [8]. For the readers benefit we repeat the proof here using our notation.

4.2 LEMMA. *Let ν and ζ be steps both of which are regular. If all the diagonal maps from ν into ζ are compact, then there is a projection $P: K(\nu, \zeta) \to \nu^\zeta$ for which $\|P\|=1$. ($K(\nu, \zeta)$ denotes the space of all compact maps of ν into ζ and ν^ζ here means the space of diagonal matrices with diagonal from ν^ζ.)*

PROOF. Since $\nu = \nu_r$, all continuous linear maps of ν into ζ can be represented as matrices. Using the hypothesis that $\zeta = \zeta_r$, we have that ζ has a Schauder basis and hence, by [14, p. 114], $\nu' \mathbin{\overset{\approx}{\otimes}} \zeta = K(\nu, \zeta)$.

Let $A=(a_{ij}) \in K(\nu, \zeta)$ and let $x \in \nu$, $u \in \zeta^\times$. xu, as a map of ζ into ν, can be factored as $\zeta \to^u l^1 \to^i c_0 \to^x \nu$ where i is the inclusion map of l^1 into c_0. It is known from [7] that i is an integral map with norm $\leqq 1$. It is easy to see that the operator norms of u and x are respectively $\|u\|_{\zeta^\times}$ and $\|x\|_\nu$. Thus xu is an integral map of ζ into ν with integral norm less than $\|x\|_\nu \|n\|_{\zeta^\times}$ (see [6]). Hence the diagonal matrix with diagonal xu is a continuous linear form on $K(\nu, \zeta)$ [14, p. 168]. This means that $\sum_{i=1}^\infty a_{ii}x_iu_i < \infty$, for each $x \in \nu$, $u \in \zeta^\times$. $\nu\zeta^\times$ is normal by 1.1 of [1]; so $\sum_{i=1}^\infty |a_{ii}x_iu_i| < \infty$ for each $xu \in \nu\zeta^\times$. Thus $(a_{ii}) \in (\nu\zeta^\times)^\times = \nu^\zeta$ (see 1.2 of [1]) implying that the diagonal matrix with diagonal (a_{ii}) is in $K(\nu, \zeta)$.

By 1.5 of [1], the norm of ν^ζ can be chosen so that

$$\begin{aligned}\|(a_{ii})\|_{\nu^\zeta} &= \sup\left\{\left|\sum_{i=1}^\infty a_{ii}x_iu_i\right| \,\middle|\, \|x\|_\nu \leqq 1,\ \|u\|_{\zeta^\times} \leqq 1\right\} \\ &\leqq (\text{integral norm of } (xu))\ (\text{operator norm of } A) \\ &\leqq (\text{operator norm of } A),\end{aligned}$$

by the arguments above. Hence $P: K(\nu, \zeta) \to \nu^\zeta$ given by $P(A) = (a_{ii})$ is the desired projection.

In the context of the above lemma the beneficial conclusion for us is that ν^ζ has a topological complement in $K(\nu, \zeta)$, i.e. $K(\nu, \zeta) = \nu' \mathbin{\overset{\approx}{\otimes}} \zeta = \nu^\zeta \oplus F$ for some subspace F of $K(\nu, \zeta)$. The hypothesis of the lemma and the result from [**1**], given above in the proof of 4.1, yield $\nu^\zeta=(\nu^\zeta)_r$. We now have$(K(\nu, \zeta))'=J(\zeta, \nu^\times)=(\nu^\zeta)'\oplus F'=(\nu^\zeta)'\oplus F'$. Hence each element of $(\nu^\zeta)^\times$ is an integral map of ζ into ν (see [**6**, p. 126]). Analyzing the identifications involved in the duality between $K(\nu, \zeta)$ and $J(\zeta, \nu^\times)$ it can be seen that, for $v \in (\nu^\zeta)^\times$, the operation of v as an integral map of ζ into ν is just coordinatewise multiplication.

We recall a theorem of Grothedieck [**6**, p. 134]:

THEOREM. *Each integral linear map of a locally convex space E into a Banach space F is nuclear, if F is separable and the strong dual of a Banach space.*

Using this theorem we can now obtain a result similar to, but not the same as, 3.2 of [**8**].

4.3 PROPOSITION. *Let ν and ζ be steps which are regular and such that each diagonal map of ν into ζ is compact. Then a diagonal map u of ζ into ν is nuclear if and only if $u \in (\nu^\zeta)^\times$. In addition, the nuclear norm of u is equivalent to the norm of u in $(\nu^\zeta)^\times$, and $(\nu^\zeta)^\times$ is regular.*

PROOF. Suppose $u \in (\nu^\zeta)^\times$. From the above discussion we know that u is an integral map of ζ into ν. By hypothesis $\nu=\nu_r$ so ν is separable; ν is perfect, so $\nu=((\nu^\times)_r)'$. u is a nuclear map of ζ into ν by the quoted theorem of Grothendieck.

Conversely, it is proved in [**12**] that the diagonal of A is in $((\nu^\zeta)^\times)_r$, if A is a nuclear matrix map of ζ into ν. Hence $u \in ((\nu^\zeta)^\times)_r \subset (\nu^\zeta)^\times$ whenever u is a diagonal nuclear map of ζ into ν.

It is proved in [**6**, p. 179] that the nuclear norm agrees with the dual norm from $K(\zeta, \nu)$. Thus by the above discussion the nuclear norm of $u \in (\nu^\zeta)^\times$ is equivalent to $\|u\|_{(\nu^\zeta)^\times}$.

The regularity of $(\nu^\zeta)^\times$ is clear from the earlier part of this proof. □

The following corollary may be anticipated from Tong's result that the space of diagonal nuclear maps of l^p into l^t, for $1 \leqq p < t < \infty$, is l^s, where $1/p=1/t+(s-1)/s$; see [**15**]:

4.4 COROLLARY. *If λ is regular, then the diagonal nuclear maps of λ^p into λ^t, for $1 \leqq p < t < \infty$, are $(\lambda^r)^\times$, where $1/p=1/t+1/r$.*

PROOF. Use 3.5 and 4.3.

REMARK. It is shown in 3.4 of [**8**] that for $t \leqq p$ the diagonal nuclear maps of λ^p into λ^t are just l^1.

4.5 PROPOSITION. *Let ν and ζ be steps which are regular. Then each diagonal map of ν into ζ is compact if and only if ν^ζ is a reflexive space.*

PROOF. If each diagonal map of ν into ζ is compact then $(\nu^\zeta)_r = \nu^\zeta$, so $(\nu^\zeta)' = (\nu^\zeta)^\times$. By 4.3, $(\nu^\zeta)^\times = ((\nu^\zeta)^\times)_r$, giving $((\nu^\zeta)^\times)' = \nu^\zeta$.

Conversely, if ν^ζ is reflexive, then $((\nu^\zeta)^\times)_r$ is reflexive giving $(\nu^\zeta)^\times = ((\nu^\zeta)^\times)_r$. Thus $(\nu^\zeta)^\times$ is reflexive yielding $(\nu^\zeta)_r$ reflexive. Hence $\nu^\zeta = (\nu^\zeta)_r$, and each diagonal map of ν into ζ is compact.

4.6 COROLLARY. *For $1 < p < \infty$, λ^p is reflexive if and only if λ is regular.*

PROOF. It is clear that λ is regular if and only if λ^t is regular, for $1 \leqq t < \infty$. The conclusion follows from 3.3 and 4.5.

We conclude with the following problem:

If λ is regular, are the diagonal λ-nuclear maps of λ^p into λ^t, for $1 \leqq p < t < \infty$, given by λ^s, where $1/p = 1/t + (s-1)/s$? (See [10].)

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