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# A HYPERSPACE FOR CONVERGENCE SPACES

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ABSTRACT. The purpose of this note is to introduce a convergence structure $h(t)$ on the collection $C(E)$ of nonempty, compact subsets of a Hausdorff convergence space $(E, t)$. It is shown that if $(E, t)$ is topological, then $h(t)$ agrees with the Vietoris topology on $C(E)$. It is proved that $(C(E), h(t))$ is Hausdorff, that it inherits regularity from $(E, t)$ and that it is compact whenever $(E, t)$ is compact and regular.

1. **Introduction.** Let $C(E)$ be the collection of nonempty, compact subsets of a Hausdorff topological space $(E, t)$, and let $v$ be the Vietoris topology on $C(E)$ as a subspace of the collection of nonempty closed subsets of $E$. (See [5] for the definition and properties of $v$.) Using just the definitions, it is easy to see that a filter $\Lambda$ on $C(E)$ converges to $A \in C(E)$ with respect to $v$ if and only if the following are satisfied:

(a) *Whenever* $A \subset G$, $G$ *open in* $E$, *then there exists* $r \in \Lambda$ *such that* $U \subset G$ *for each* $U \in r$.

(b) *Whenever* $G \cap A \neq \varnothing$, $G$ *open in* $E$, *then there exists* $r \in \Lambda$ *such that* $G \cap U \neq \varnothing$ *for each* $U \in r$.

Let us now make two comments. First, we could have started with a notion of convergence of filters in $C(E)$ defined by (a) and (b) above and then recovered the topology $v$ from the convergence. Second, if our object is to study properties of $C(E)$ which can be defined in terms of convergence, then it is immaterial whether we discover $v$ from the convergence or not. In fact, there are numerous important notions of convergence of filters (or nets) which are not induced by any topology. A few examples are (except for special cases): the convergence of closed sets of Choquet (see p. 87 of [2]), continuous convergence of maps from a Hausdorff space to a Hausdorff uniform space (see [3] and its references), the convergence of a net $(x_n : n \in D)$, in a complete lattice, defined by $\liminf x_n = x = \limsup x_n$. Examples like the ones above led Fischer to formalize the concept of a convergence space in [4]. For the remainder of this paper the reader is assumed to be familiar with the very basic notions of [4].

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Our object here is to show that there is a reasonable convergence structure $h(t)$ on the collection of nonempty, compact subsets of an arbitrary Hausdorff convergence space $(E, t)$. That $h(t)$ is reasonable means that it has the properties mentioned in the Abstract.

Filters on a set $E$ will be denoted by Greek letters with two exceptions: The filter of supersets of $b \in E$ is $[b]$ and if $x: D \to E$ is a net in $E$, the filter of sections of $x$ is $S(x)$. The set of all $x(d)$, $d \geqq e$, is written $S(x, e)$.

If $(E, t)$ is a convergence space and $\alpha$ is a filter on $E$, $\alpha$ accumulates at $b \in E$ if it is coarser than a filter which $t$-converges to $b$. The filter generated by the $\mathrm{cl}(F, t)$, $F \in \alpha$, is $\mathrm{cl}(\alpha)$. The space $(E, t)$ is regular if $\mathrm{cl}(\alpha) \to b$ whenever $\alpha \to b$. See **[1]** for an equivalent formulation of regularity. $(E, t)$ is compact if each ultrafilter on $E$ $t$-converges and is Hausdorff if no filter $t$-converges to two distinct points.

If $(D(b): b \in B)$ is a set of directed sets, the product set $\times\ (D(b): b \in B)$ is always directed by the product order. That is, $f \geqq g$ if and only if $f(b) \geqq g(b)$ for each $b \in B$. Furthermore, if $\alpha$ is a filter on a set $E$ then $\alpha$ will be considered as a directed set with $F \geqq G$ if and only if $F \subset G$, $F$, $G \in \alpha$. These two conventions will be used without explicit mention.

2. **Definition of** $h(t)$. Let $C(E)$ be the collection of nonempty compact subsets of a convergence space $(E, t)$. If $\Lambda$ is a filter on $C(E)$, a cofinal segment of $\Lambda$ is a pair $(D, f)$, where $D$ is a directed set, $f: D \to \Lambda$ is a function, and for each $r \in \Lambda$ there exists $d \in D$ such that $f(e) \subset r$ whenever $e \geqq d$. If $(D, f)$ is a cofinal segment of $\Lambda$, then a selection of $(D, f)$ is a pair $(x, g)$, where $x: D \to E$, $g: D \to C(E)$ are functions and $x(d) \in g(d)$, $g(d) \in f(d)$ for all $d \in D$.

Definition 2.1. Let $C(E)$ be the collection of nonempty compact subsets of a Hausdorff convergence space $(E, t)$. Let $\Lambda$ be a filter on $C(E)$ and $A \in C(E)$. Then $\Lambda$ $h(t)$-converges to $A$ (written $\Lambda \to A$) if and only if the following are satisfied:

(1) For each cofinal segment $(D, f)$ of $\Lambda$ and each selection $(x, g)$ of $(D, f)$, the section filter $S(x)$ accumulates at some point of $A$.

(2) For each $a \in A$ there is some filter $\alpha$, with $\alpha \to a$, and some cofinal segment $(D, f)$ of $\Lambda$ with the following property: For each $F \in \alpha$ there exists $d \in D$ such that, whenever $e \geqq d$ and $U \in f(e)$ then $F \cap U \neq \varnothing$.

The reader should compare Definition 2.1 with the Vietoris convergence of §1 and see what alterations have been made in the convergence space case. That $h(t)$ is indeed a convergence structure on $C(E)$ is shown below.

Theorem 2.1. *If $C(E)$ is the collection of nonempty, compact subsets of a Hausdorff convergence space $(E, t)$, then $h(t)$ is a convergence structure on $C(E)$.*

PROOF. If $A \in C(E)$ then $[A] \to A$ because of the following:

(1) If $(x, g)$ is a selection of a cofinal segment $(D, f)$ of $[A]$, then $x(d) \in g(d) = A$ for $d$ sufficiently large. This means that $A$ contains some $S(x, d)$. The compactness of $A$ now shows that $S(x)$ accumulates at some point of $A$.

(2) If $a \in A$ define $\alpha = [a]$ and, for $r \in [A]$, put $f(r) = r$. It is clear that $([A], f)$ is a cofinal segment of $[A]$ and that $\alpha$ and $([A], f)$ satisfy (2) of Definition 2.1 for $[A]$.

Suppose now that $\Lambda \to A$ and $\Psi \geqq \Lambda$. We show that $\Psi \to A$.

(1) Let $(x, g)$ be a selection of a cofinal segment $(D, f)$ of $\Psi$. On $D \times \Lambda$ define functions as follows: $F(d, r) = f(d) \cup r$, $G(d, r) = g(d)$, $X(d, r) = x(d)$. Then $(D \times \Lambda, F)$ is a cofinal segment of $\Lambda$ and $(X, G)$ is a selection of $(D \times \Lambda, F)$. Thus $S(X)$ accumulates at some point of $A$. But $S(x) \leqq S(X)$ so $S(x)$ accumulates at a point of $A$.

(2) Let $a \in A$ and let $\alpha$, $(D, f)$, be the filter and cofinal segment which satisfy (2) of Definition 2.1 for $\Lambda$. On $D \times \Psi$ define $F(d, r) = f(d) \cap r$. It is clear that $\alpha$ and $(D \times \Psi, F)$ satisfy (2) of Definition 2.1 for $\Psi$.

Finally, suppose $\Phi$, $\Psi$ $h(t)$-converge to $A$. We prove that $\Phi \wedge \Psi \to A$.

(1) Let $(x, g)$ be a selection of a cofinal segment $(D, f)$ of $\Phi \wedge \Psi$. Let $K$ be any well ordered, cofinal subset of $D$. It is asserted that $B(K) = (g(d) : d \in K)$ is in some ultrafilter finer than $\Phi \wedge \Psi$. If this is not so, $C(E) - B(K)$ is in each ultrafilter finer than $\Phi \wedge \Psi$ and hence in $\Phi \wedge \Psi$. The properties of $(D, f)$, $K$ now yield the contradiction that $g(d) \in C(E) - B(K)$ for some $d \in K$.

Let $\Lambda(K)$ be an ultrafilter finer than $\Phi \wedge \Psi$ which contains $B(K)$.

*Case* 1. For some $K$, $\Lambda(K)$ is not a $[g(d)]$, $d \in K$. Then, by transfinite induction, $\Lambda(K)$ contains $B(K, d) = (g(e) : e \geqq d,\ e \in K)$ for each $d \in K$. On $K \times \Lambda(K)$ define functions as follows: $F(d, r) = B(K, d) \cap r$; $G(d, r) = g(p)$ for some $p \geqq d$, $p \in K$, with $g(p) \in B(d) \cap r$; $X(d, r) = x(p)$. ($G$ and $X$ exist by the axiom of choice.) It is clear that $(K \times \Lambda(K), F)$ is a cofinal segment of $\Lambda(K)$ and that $(X, G)$ is a selection of $(K \times \Lambda(K), F)$. Now $\Lambda(K)$ is an ultrafilter so it is finer than one of $\Phi$, $\Psi$. Whence $\Lambda(K) \to A$ and $S(X)$ accumulates at some point of $A$. From $S(x) \leqq S(X)$ it follows that $S(x)$ accumulates at some point of $A$.

*Case* 2. For each well ordered, cofinal subset $K$ of $D$, $\Lambda(K)$ is some $[g(d(K))]$, $d(K) \in K$. Then $[g(d(K))]$ is finer than one of $\Phi$, $\Psi$ and $[g(d(K))] \to g(d(K))$. Thus, for each $K$, $g(d(K)) = A$ and $[A] = \Lambda(K)$. But, if $K$ is one well ordered, cofinal subset of $D$ then $K(d) = (e \in K : e \geqq d)$ is another for each $d \in K$. Then the set $(g(e) : e \geqq d,\ e \in K) \in \Lambda(K(d)) = [A]$ and, from this, the set $M = (d \in K : g(d) = A)$ is cofinal in $K$. Hence $(x(d) : d \in M) \subset A$ so, since $A$ is compact, $S(x)$ accumulates at a point of $A$. It has been shown that (1) of Definition 2.1 holds for $\Phi \wedge \Psi$ and $A$.

(2) Let $a \in A$. By assumption there are filters $\alpha$, $\beta$ which $t$-converge to $a$, and cofinal segments $(f, D)$, $(p, H)$ of $\Phi, \Psi$ respectively which satisfy (2) of Definition 2.1. Put $\gamma = \alpha \wedge \beta$ and $F(d, h) = f(d) \cup p(h)$. Then $\gamma$, $(D \times H, F)$, satisfy (2) of Definition 2.1 for $\Phi \wedge \Psi$ and $A$. This completes the proof.

Next it will be shown that $h(t)$ indeed is a generalization of the Vietoris topology.

THEOREM 2.2. *If $C(E)$ is the set of nonempty, compact subsets of a Hausdorff topological space $(E, t)$, then $h(t)$ is the Vietoris topology on $C(E)$.*

PROOF. Suppose that $\Lambda$ $v$-converges to $A \in C(E)$. Let $(x, g)$ be a selection of a cofinal segment $(D, f)$ of $\Lambda$. Suppose $S(x)$ fails to accumulate at any point of $A$. A standard compactness argument shows that there exists a finite subset $T$ of $A$, points $d(a) \in D$, and open sets $G(a)$ containing $a$, such that $A \subset \bigcup (G(a) : a \in T)$ and $B = \bigcap (S(x, d(a)) : a \in T) \cap \bigcup (G(a) : a \in T) = \varnothing$. By (a) of §1 there exists $r \in \Lambda$ such that $U \subset \bigcup (G(a) : a \in T)$ for each $U \in r$. But $(D, f)$ is a cofinal segment of $\Lambda$ so $g(d) \in f(d) \subset r$ for $d$ sufficiently large. Hence, if $d \geqq d(a)$ for all $a \in T$ then $x(d) \in B$, a contradiction. It follows that (1) of Definition 2.1 holds for $\Lambda$ and $A$.

To prove that (2) holds let $a \in A$. Let $N$ be the collection of open subsets of $E$ which contain $a$. Order $N$ by reverse inclusion. If $(r, G) \in \Lambda \times N$ then, by (b) of §1, there is an $f(r, G) \subset r$ with $U \cap G \neq \varnothing$ for $U \in f(r, G)$. Define $\alpha$ to be the neighborhood filter at $a$. Then $\alpha$, $(\Lambda \times N, f)$, satisfy (2) for $\Lambda$ and $A$. We have shown that $\Lambda \to A$ whenever $\Lambda$ $v$-converges to $A$.

Conversely, let $\Lambda \to A$. Suppose (a) of §1 fails for $\Lambda$. Then there is an open set $G$, $A \subset G$, such that $r \in \Lambda$ implies there exists $g(r) \in r$ for which $g(r) \not\subset G$. If $f$ is the identity map on $\Lambda$ and $x(r) \in g(r) - G$ then $(x, g)$ is a selection of the cofinal segment $(\Lambda, f)$. By assumption $S(x)$ accumulates at some $b \in A$. But each $S(x, r) \subset E - G$, $E - G$ closed, so $b \in E - G \subset E - A$ which is a contradiction. It has been shown that (a) of §1 holds for $\Lambda$. To prove that (b) holds suppose $G \cap A \neq \varnothing$, $G$ open. Let $a \in G \cap A$. There is some filter $\gamma \to a$ and some cofinal segment $(D, f)$ of $\Lambda$ which satisfy (2) of Definition 2.1 for $\Lambda$. Moreover $\gamma$ is finer than the neighborhood filter at the point $a$ since $t$ is topological. Thus $G \in \gamma$ and $U \cap G \neq \varnothing$ for $U \in f(d)$ and some $d \in D$.

## 3. **Separation properties of $h(t)$.**

THEOREM 3.1. *Let $(E, t)$ be a Hausdorff convergence space. Then $(C(E), h(t))$ is Hausdorff.*

PROOF. Assume $\Lambda \to A$ and $\Lambda \to B$. Let $a \in A$ and let $\alpha$, $(D, f)$ be the filter and cofinal segment of $\Lambda$ which satisfy (2) of Definition 2.1 for $\Lambda$ and

$A$. If $H \in \alpha$ there exists $d(H) \in D$ such that whenever $d \geqq d(H)$ and $U \in f(d)$ then $U \cap H \neq \varnothing$. Further, if $(d, H) \in D \times \alpha$ there exists $p(d, H) \geqq d(H), d$. Now there are functions on $D \times \alpha$ with the following properties: $F(d, H) = f(p(d, H))$, $G(d, H)$ is arbitrary in $F(d, H)$, $X(d, H) \in G(d, H) \cap H$. Then $(X, G)$ is a selection of the cofinal segment $(D \times \alpha, F)$ of $\Lambda$ so $S(X)$ accumulates at some point of $B$. However $\alpha \leqq S(X)$ so $\alpha \rightarrow a \in A$ and $\alpha$ accumulates at some point of $B$. Since $(E, t)$ is Hausdorff, $a \in B$. It has been shown that $A \subset B$; similarly $B \subset A$. So $A = B$ and $(C(E), h(t))$ is Hausdorff.

In order to show that $C(E)$ inherits regularity from $E$ two lemmas will be employed. The reader will recognize the second of these as the analogue of a "diagonal net" theorem from topology.

LEMMA 3.2. *Let $(E, t)$ be a Hausdorff convergence space. If $F \subset E$, $r \subset C(E)$ and $U \cap F \neq \varnothing$ for each $U \in r$, then $U \cap \operatorname{cl}(F, t) \neq \varnothing$ for each $U \in \operatorname{cl}(r, h(t))$.*

PROOF. If $U \in \operatorname{cl}(r, h(t))$, there is a filter $\Lambda$ on $C(E)$ with $r \in \Lambda$ and $\Lambda \rightarrow U$. Define functions on $\Lambda$ as follows: $f(s) = s$, $g(s)$ is arbitrary in $s \cap r$ and $x(s) \in g(s) \cap F$. Then $S(x)$ accumulates at some point $b \in U$. But $F \in S(x)$ so $b \in \operatorname{cl}(F, t)$. Thus $b \in U \cap \operatorname{cl}(F, t)$.

LEMMA 3.3. *Let $(E, t)$ be a regular, Hausdorff convergence space. Let $D$ be a directed set and suppose there are filters $\alpha(d)$ on $E$ with $\alpha(d) \rightarrow x(d)$ for each $d \in D$. Define $I = D \times \bigtimes(\alpha(d) : d \in D)$ and suppose $X : I \rightarrow E$ has the property that $X(d, h) \in h(d)$ for each $h$ and each $d$. Then, if $S(X)$ accumulates at $y \in E$ so does $S(x)$.*

PROOF. Suppose $S(X)$ accumulates at $y$. Then $S(X) \leqq \beta \rightarrow y$ for some filter $\beta$. Since $(E, t)$ is regular, $S(x)$ accumulates at $y$ if $\sup(S(x), \operatorname{cl}(\beta))$ exists. Let $S(x, d)$ be an arbitrary section of $S(x)$ and suppose $B \in \beta$. If, for some $p \geqq d$, $B \cap H \neq \varnothing$ for all $H \in \alpha(p)$ then the filter generated by the $B \cap H$, $H \in \alpha(p)$, contains $B$ and is finer than $\alpha(p)$. Since $\alpha(p) \rightarrow x(p)$, this means $x(p)$ is in $\operatorname{cl}(B) \cap S(x, d)$. Hence $\operatorname{cl}(B) \cap S(x, d) \neq \varnothing$, $\sup(S(x), \operatorname{cl}(\beta))$ exists, and $S(x)$ accumulates at $y$. Suppose, contrariwise, that for each $p \geqq d$ there exists $H(p) \in \alpha(p)$ with $H(p) \cap B = \varnothing$. Define a function $h$ by putting $h(p) = H(p)$ if $p \geqq d$ and $h(p) = E$ otherwise. Since $X(d, h) \in h(d)$, it follows that $S(X, (d, h)) \subset E - B$. Then $E - B \in \beta$ which contradicts $B \in \beta$.

THEOREM 3.4. *Let $(E, t)$ be a regular Hausdorff convergence space. Then $(C(E), h(t))$ is regular.*

PROOF. Suppose $\Lambda \rightarrow A$ and let $(x, g)$ be a selection of the cofinal segment $(D, f)$ of $\operatorname{cl}(\Lambda)$. Then, for each $(d, r) \in D \times \Lambda$ there exists $n(d, r) \geqq d$ for which $f(n(d, r)) \subset \operatorname{cl}(r, h(t))$. Thus, there are filters $\Psi(d, r)$ on $C(E)$

with $r \in \Psi(d, r)$ and $\Psi(d, r) \to g(n(d, r))$. Since $x(n(d, r)) \in g(n(d, r))$ it follows that there are filters $\alpha(d, r) \to x(n(d, r))$ and cofinal segments $(D(d, r), j(d, r))$ of $\Psi(d, r)$ which satisfy (2) of Definition 2.1 for $\Psi(d, r)$ and $g(n(d, r))$. This and the fact that $r \in \Psi(d, r)$ shows that there exist functions defined on $I=(D \times \Lambda) \times \bigtimes\ (\alpha(d, r):(d, r) \in D \times \Lambda)$ as follows: $F((d, r), h)=r$, $G((d, r), h) \in r$ with $G((d, r), h) \cap h(d, r) \neq \varnothing$, $X((d, r), h) \in G((d, r), h) \cap h(d, r)$. Now $(X, G)$ is a selection of the cofinal segment $(I, F)$ of $\Lambda$ so $S(X)$ accumulates at a point $y \in A$. By Lemma 3.3, $S(xn)$ accumulates at $y$ and it is clear, then, that $S(x)$ accumulates at $y \in A$. Thus (1) of Definition 2.1 holds for $\mathrm{cl}(\Lambda)$ and $A$.

Next, let $a \in A$ and let $\alpha$, $\alpha \to a$, $(D, f)$, be the filter and final segment of $\Lambda$ which satisfy (2) of Definition 2.1 for $\Lambda$ and $A$. Define $m: D \to \mathrm{cl}(\Lambda)$ by $m(d)=\mathrm{cl}(f(d), h(t))$. Since $(E, t)$ is regular, $\mathrm{cl}(\alpha) \to a$ and, by Lemma 3.2, $(D, m)$ and $\mathrm{cl}(\alpha)$ satisfy (2) of Definition 2.1 for $\mathrm{cl}(\Lambda)$ and $A$.

It has been demonstrated that $\mathrm{cl}(\Lambda) \to A$ whenever $\Lambda \to A$ so $(C(E), h(t))$ is regular and the proof is complete.

LEMMA 3.5. *Let $(x, g)$ be a selection of a cofinal segment $(D, f)$ of a filter $\Lambda$ on $C(E)$ and suppose $S(x)$ accumulates at a point $b \in E$. Then, there is a selection $(X, G)$ of a cofinal segment $(I, F)$ of $\Lambda$ with $S(X) \to b$.*

PROOF. By assumption $S(x) \leqq \alpha \to b$ and we may define functions on $I=D \times \alpha$ as follows: $F(d, A)=f(n(d, A))$ where $n(d, A) \geqq d$ and $x(n(d, A)) \in A$; $G(d, A)=g(n(d, A))$; $X(d, A)=x(n(d, A))$. It follows easily that $(X, G)$ is a selection of $(I, F)$ and that $S(X) \to b$.

LEMMA 3.6. *Let $\Lambda$ be a filter on $C(E)$ with $(E, t)$ compact, regular and Hausdorff. Let $A$ be the set of all $z$ for which there exists a selection $(x, g)$ of a cofinal segment of $\Lambda$ such that $S(x)$ accumulates at $z$. Then $A$ is a compact subset of $E$.*

PROOF. Since $(E, t)$ is compact, Hausdorff, it suffices to prove that $\mathrm{cl}(A, t) \subset A$. Toward this end let $b \in \mathrm{cl}(A, t)$. There is a filter $\alpha \to b$ with $A \in \alpha$. So, for each $H \in \alpha$ there exists $y(H) \in H \cap A$. By definition of $A$ and lemma 3.5 there are selections $(x(H), g(H))$ of cofinal segments $(D(H), f(H))$ with $S(x(H)) \to y(H)$. Put $I=\alpha \times \bigtimes\ (D(H): H \in \alpha)$ and define functions on $I$ as follows: $F(H, p)=f(H)(p(H))$, $G(H, p)=g(H)(p(H))$, $X(H, p)=x(H)(p(H))$. Then $(X, G)$ is a selection of the cofinal segment $(I, F)$. Since $(E, t)$ is compact $S(X) \leqq \beta \to z$ for some $z \in E$. Since $(E, t)$ is regular, an argument similar to the one given in the first part of Theorem 3.4 shows that $S(y)$ accumulates at $z$. But surely $S(y) \to b$ so $z=b$. Thus $S(X)$ accumulates at $b$ and $b \in A$ by definition of $A$.

THEOREM 3.7. *Let $(E, t)$ be a compact, Hausdorff, regular convergence space. Then $(C(E), h(t))$ is compact.*

PROOF. Let $\Lambda$ be an ultrafilter on $C(E)$. Define $A$ to be the set of all points $z$ for which there exists a selection $(x, g)$ of a cofinal segment of $\Lambda$ such that $S(x)$ accumulates at $z$. By Lemma 3.6, $A \in C(E)$. We prove that $\Lambda \to A$.

(1) That (1) of Definition 2.1 is satisfied for $\Lambda$ and $A$ is clear from the definition of $A$ and the compactness of $(E, t)$.

(2) Let $a \in A$. From definition and Lemma 3.5 there is some selection $(x, g)$ of some cofinal segment $(D, f)$ of $\Lambda$ with $S(x) \to a$. If $S(x, d)$ is an arbitrary generator of $S(x)$ define $r$ to be the set of all $B \in C(E)$ such that $B \cap S(x, d) \neq \varnothing$. Since $\Lambda$ is an ultrafilter either $r \in \Lambda$ or $C(E) - r \in \Lambda$. But $C(E) - r \in \Lambda$ leads to $f(e) \subset C(E) - r$ for $e$ sufficiently large. This in turn leads to the contradiction $g(e) \cap S(x, d) = \varnothing$ for $e$ sufficiently large. Thus $r \in \Lambda$ and it follows that $(D, f)$ and $S(x)$ satisfy (2) of Definition 2.1 for $\Lambda$ and $A$.

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