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LOCALLY-TRIVIAL BUNDLES AND MICROBUNDLES WITH INFINITE-DIMENSIONAL FIBERS

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ABSTRACT. The object of this note is to establish trivialization theorems for locally-trivial bundles and microbundles in which the fiber is any one of a large class of infinite-dimensional topological vector spaces and the base space is any paracompact space. These theorems generalize results of Henderson and Wong.

1. **Introduction.** In this paper we prove that certain locally-trivial bundles and microbundles with infinite-dimensional fibers are trivial. Our first result concerns the triviality of locally-trivial bundles whose fibers are certain infinite-dimensional topological vector spaces (TVS's).

THEOREM 1. *Let ξ be a locally-trivial bundle with fiber F and base B any paracompact space. If F is any TVS which is homeomorphic ($\cong$) to its own countable-infinite product F^ω, then ξ is trivial.*

This is clearly false for finite-dimensional fibers (i.e. Euclidean spaces) and it is the product structure of F^ω that makes this result possible. We remark that there are many examples of infinite-dimensional TVS's F which satisfy the condition $F \cong F^\omega$; for example any separable infinite-dimensional Fréchet space **[1]**, any infinite-dimensional Hilbert space **[2]**, or any infinite-dimensional reflexive Banach space **[2]**. In fact there is no known example of an infinite-dimensional Fréchet space F for which the condition $F \cong F^\omega$ is not satisfied.

A special case of Theorem 1 was established by Wong **[9]** in which $F = l_2$ (separable infinite-dimensional Hilbert space) and B is any countable locally-finite simplicial complex. The proof we give of Theorem 1 uses techniques which are completely different from those of Wong.

In our second result we establish a version of Theorem 1 for microbundles.

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THEOREM 2. *Let ξ be a microbundle with fiber F and base B any paracompact space. Then ξ is trivial provided that F is a metric TVS which satisfies the condition $F \cong F^{\omega}$.*

In [4] Henderson proved Theorem 2 with the same conditions on the fiber F but with more restrictive conditions on the base B; namely that B is a paracompact space which has the homotopy type of a simplicial (or CW) complex. This result was one of the crucial steps in proving the open embedding theorem of [6] (and various other results about infinite-dimensional manifolds).

The proof we give of Theorem 2 is completely independent of the microbundle techniques used by Henderson in [4]. Our idea (for both Theorems 1 and 2) is to make use of a parametric version of Wong's coordinate-switching technique which was used to prove that homeomorphisms on certain infinite-dimensional spaces are isotopic to the identity [8]. This approach enables us to completely ignore the homeomorphism group of the fiber, which is particularly necessary when the fiber is an infinite-dimensional TVS (as the compact-open topology is not even jointly continuous).

2. **Lemmas for Theorem 1.** We first introduce some notation. For finite products $\prod_{i=1}^{n} X_i$ we will use p_i to denote projection onto X_i, for $1 \leqq i \leqq n$. If each X_i is itself a finite product, then $p_j \circ p_i$ will denote the projection of $\prod_{i=1}^{n} X_i$ onto the jth factor of X_i. If we have functions $f_i : X_i \to Y_i$, for $1 \leqq i \leqq n$, then $f_1 \times \cdots \times f_n$ will denote the function from $\prod_{i=1}^{n} X_i$ to $\prod_{i=1}^{n} Y_i$ defined by $f_i \circ p_i = p_i \circ (f_1 \times \cdots \times f_n)$. If $g_i : X \to Y_i$ is a function, for $1 \leqq i \leqq n$, then $(g_1, \cdots, g_n)$ will denote the function from X to $\prod_{i=1}^{n} Y_i$ defined by $g_i = p_i \circ (g_1, \cdots, g_n)$. The above definitions have obvious analogues for countable products.

For spaces X, Y, and Z we say that a function $f: X \times Y \to X \times Z$ is X-*preserving* provided that $p_1 \circ f = p_1$. For each $x \in X$ we let f_x denote the function from Y to Z defined by $f_x(y) = f(x, y)$, for all $y \in Y$. We also make analogous definitions of X-*preserving* functions $f: Y \times X \to Z \times X$. We use I to denote the closed interval $[0, 1]$ and id_X to denote the identity function of X (where the subscript is suppressed when there is no ambiguity). We also use the term *map* to mean a continuous function.

The basic notion of this section is the following: A space X has the *reflective isotopy property* if there exists an I-preserving homeomorphism f of $X^{\omega} \times I$ onto itself such that $f_0 = \mathrm{id}$ and f_1 interchanges the first and second coordinates, i.e. $p_1 \circ f_1 = p_2$, $p_2 \circ f_1 = p_1$, and $p_i \circ f_1 = p_i$, for all $i \geqq 3$. The following lemma is due to J. E. West [7] and it identifies some spaces which have the reflective isotopy property.

LEMMA 2.1 [7, p. 579]. *Every TVS has the reflective isotopy property.*

Moreover the I-preserving homeomorphism of Lemma 2.1 which switches coordinates can be required to fix the origin at each level. That is for a TVS F there is an I-preserving homeomorphism f of $F^\omega \times I$ onto itself such that $f_0=\mathrm{id}$, f_1 interchanges the first and second coordinates, and $f_t(0, 0, \cdots)=(0, 0, \cdots)$, for all $t \in I$. We now apply Lemma 2.1 to obtain the following result.

LEMMA 2.2. *If F is a TVS, then there exists an I-preserving map $\varphi: F^\omega \times F^\omega \times I \to F^\omega \times I$ such that*

(1) *$\varphi | F^\omega \times F^\omega \times (0, 1)$ is a homeomorphism of $F^\omega \times F^\omega \times (0, 1)$ onto $F^\omega \times (0, 1)$,*

(2) *$\varphi_0=p_1$ (projection of $F^\omega \times F^\omega$ on to its first factor F^ω),*

(3) *$\varphi_1=p_2$,*

(4) *$\varphi_t((0, 0, \cdots), (0, 0, \cdots))=(0, 0, \cdots)$, for $0 \leqq t \leqq 1$,*

(5) *for each integer $n \geqq 1$ there is an ε, $0<\varepsilon<1$, such that $p_i \circ p_1 = p_i \circ \varphi_t$, for $0 \leqq t \leqq \varepsilon$ and $1 \leqq i \leqq n$,*

(6) *for each integer $n \geqq 1$ there is an ε, $0<\varepsilon<1$, such that $p_i \circ p_2 = p_i \circ \varphi_t$, for $1-\varepsilon \leqq t \leqq 1$ and $1 \leqq i \leqq n$.*

PROOF. Define $\varphi_{1/2}: F^\omega \times F^\omega \to F^\omega$ by

$$\begin{aligned} p_i \circ \varphi_{1/2} &= p_{(i+1)/2} \circ p_1, && \text{for } i \text{ odd}, \\ &= p_{i/2} \circ p_2, && \text{for } i \text{ even}. \end{aligned}$$

For all integers $n \geqq 2$ define $\varphi_{1/(n+1)}: F^\omega \times F^\omega \to F^\omega$ by

$$\begin{aligned} p_i \circ \varphi_{1/(n+1)} &= p_i \circ p_1, && \text{for } 1 \leqq i \leqq n, \\ &= p_{i-n} \circ p_2, && \text{for } n+1 \leqq i \leqq 2n, \\ &= p_i \circ \varphi_{1/2}, && \text{for } 2n+1 \leqq i. \end{aligned}$$

Similarly for all integers $n \geqq 2$ define $\varphi_{n/(n+1)}: F^\omega \times F^\omega \to F^\omega$ by

$$\begin{aligned} p_i \circ \varphi_{n/(n+1)} &= p_i \circ p_2, && \text{for } 1 \leqq i \leqq n, \\ &= p_{i-n} \circ p_1, && \text{for } n+1 \leqq i \leqq 2n, \\ &= p_i \circ \varphi_{1/2}, && \text{for } 2n+1 \leqq i. \end{aligned}$$

This defines φ on the set $\bigcup \{F^\omega \times F^\omega \times \{t\} | t=1/(n+1) \text{ or } t=n/(n+1)\}$. To extend φ to all of $F^\omega \times F^\omega \times I$ put $\varphi_0=p_1$, $\varphi_1=p_2$, and use Lemma 2.1 to extend φ to the entire interval joining any two consecutive points of $\{1/(n+1) | n \geqq 1\} \cup \{n/(n+1) | n \geqq 1\}$.

We now state and prove the main result of this section. It will be needed in the proof of Theorem 1.

LEMMA 2.3. *Let X be a topological space, $\alpha: X \to I$ be a map, and let F be a TVS. If f is an X-preserving homeomorphism of $X \times F^\omega$ onto itself, then* *there is an X-preserving homeomorphism* f_α *of* $X\times F^\omega$ *onto itself such that* $(f_\alpha)_x=f_x$, *for all* $x\in\alpha^{-1}(0)$, *and* $(f_\alpha)_x=\mathrm{id}$, *for all* $x\in\alpha^{-1}(1)$.

PROOF. Let $i:X\times F^\omega\to X\times F^\omega\times I$ be defined by $i(x,a)=(x,a,\alpha(x))$, for all $(x,a)\in X\times F^\omega$. Then we define f_α to be the function which makes the following diagram commute:

$$\begin{tikzcd}
X\times F^\omega\times F^\omega\times I \arrow[r, "f\times\mathrm{id}\times\mathrm{id}"] & X\times F^\omega\times F^\omega\times I \arrow[d, "\mathrm{id}\times\varphi"] \\
X\times F^\omega\times I \arrow[u, "\mathrm{id}\times\varphi^{-1}"] & X\times F^\omega\times I \arrow[d, "(p_1,p_2)"] \\
X\times F^\omega \arrow[u, "i"] \arrow[r, "f_\alpha"'] & X\times F^\omega
\end{tikzcd}$$

Here φ is the function of Lemma 2.2 and (p_1,p_2) is projection. To avoid ambiguity we remark that $(\mathrm{id}\times\varphi)\circ(f\times\mathrm{id}\times\mathrm{id})\circ(\mathrm{id}\times\varphi^{-1})(x,a,0)=(f(x,a),0)$ and $(\mathrm{id}\times\varphi)\circ(f\times\mathrm{id}\times\mathrm{id})\circ(\mathrm{id}\times\varphi^{-1})(x,a,1)=(x,a,1)$, for all $(x,a)\in X\times F^\omega$. It can be routinely verified that f_α fulfills our requirements.

3. **Proof of Theorem 1.** We are given a locally-trivial bundle $\xi=(E,p,B)$ with fiber F^ω, where F is a TVS. That is $p:E\to B$ is a map and for each $b\in B$ there is an open set $U\subset B$ containing b and a homeomorphism h_U of $U\times F^\omega$ onto $p^{-1}(U)$ such that $p\circ h_U=p_1$. We want to prove that ξ is trivial, i.e. construct a homeomorphism h of $B\times F^\omega$ onto E such that $p\circ h=p_1$.

Since B is paracompact there exists a locally-finite open cover $\{U_\sigma|\sigma\in\Sigma\}$ of B such that for each $\sigma\in\Sigma$ there exists a homeomorphism h_σ of $U_\sigma\times F^\omega$ onto $p^{-1}(U_\sigma)$ such that $p\circ h_\sigma=p_1$. Choose a closed cover $\{C_\sigma|\sigma\in\Sigma\}$ of B such that $C_\sigma\subset U_\sigma$, for all $\sigma\in\Sigma$. Let $\mathscr{G}$ be the set of all pairs $(\Gamma, h(\Gamma))$, where $\Gamma\subset\Sigma$ and $h(\Gamma)$ is a homeomorphism of $(\bigcup\{C_\gamma|\gamma\in\Gamma\})\times F^\omega$ onto $p^{-1}(\bigcup\{C_\gamma|\gamma\in\Gamma\})$ satisfying $p\circ h(\Gamma)=p_1$. We can partially order $\mathscr{G}$ by defining $(\Gamma_1,h(\Gamma_1))\leqq(\Gamma_2,h(\Gamma_2))$ if and only if $\Gamma_1\subset\Gamma_2$ and $h(\Gamma_2)$ agrees with $h(\Gamma_1)$ on

$$[(\bigcup\{C_\gamma\mid\gamma\in\Gamma_1\})\backslash(\bigcup\{U_\gamma\mid\gamma\in\Gamma_2\backslash\Gamma_1\})]\times F^\omega.$$

Let $\mathscr{H}$ be any chain in $\mathscr{G}$. We will show that $\mathscr{H}$ has an upper bound.

Note that $(\Gamma, h(\Gamma))\leftrightarrow\Gamma$ sets up a bijection between $\mathscr{H}$ and a collection $\mathscr{D}$ of subsets of Σ. Let $\Gamma^*=\bigcup\{\Gamma|\Gamma\in\mathscr{D}\}$ and for each $\Gamma\in\mathscr{D}\cup\{\Gamma^*\}$ let $C(\Gamma)=\bigcup\{C_\gamma|\gamma\in\Gamma\}$. For any element $x\in C(\Gamma^*)$ we can choose an element $\Gamma_x\in\mathscr{D}$ such that $x\in U_x\subset C(\Gamma_x)$, where U_x is relatively open in

$C(\Gamma^*)$ and $U_x \cap U_\sigma = \varnothing$, for all $\sigma \in \Gamma^* \backslash \Gamma_x$. Then we define $h(\Gamma^*)|U_x \times F^\omega = h(\Gamma_x)|U_x \times F^\omega$. In this manner we obtain a function $h(\Gamma^*): C(\Gamma^*) \times F^\omega \to p^{-1}(C(\Gamma^*))$. It can easily be checked that $h(\Gamma^*)$ is an onto homeomorphism and $p \circ h(\Gamma^*) = p_1$. Moreover $(\Gamma^*, h(\Gamma^*))$ is an upper bound for $\mathscr{H}$.

Using Zorn's lemma it follows that $\mathscr{G}$ has a maximal element $(\Gamma, h(\Gamma))$. We will be done if we can prove that $\Gamma = \Sigma$. Thus assume that there is some element $\sigma \in \Sigma \backslash \Gamma$. We will obtain a contradiction to the fact that $(\Gamma, h(\Gamma))$ is maximal. Let $C = \bigcup \{C_\gamma | \gamma \in \Gamma\}$ and put $h = (h(\Gamma))^{-1} \circ h_\sigma | (C \cap U_\sigma) \times F^\omega$, which is a $(C \cap U_\sigma)$-preserving homeomorphism of $(C \cap U_\sigma) \times F^\omega$ onto itself. Let $\tilde{\alpha}: C \to I$ be a map such that $C \cap C_\sigma \subset \tilde{\alpha}^{-1}(0)$ and $K \subset C \cap U_\sigma$, where K is the closure of $\tilde{\alpha}^{-1}([0, 1))$ in C. Then put $\alpha = \tilde{\alpha} | C \cap U_\sigma$. Using the notation of Lemma 2.3 (with $X = C \cap U_\sigma$) consider the homeomorphism h_α of $(C \cap U_\sigma) \times F^\omega$ onto itself. It follows that $h_\alpha | (C \cap C_\sigma) \times F^\omega = h | (C \cap C_\sigma) \times F^\omega$ and $h_\alpha | ((C \cap U_\sigma) \backslash K) \times F^\omega = \mathrm{id}$. Then $h(\Gamma) \circ h_\alpha : (C \cap U_\sigma) \times F^\omega \to p^{-1}(C \cap U_\sigma)$ is an onto homeomorphism which satisfies $h(\Gamma) \circ h_\alpha | (C \cap C_\sigma) \times F^\omega = h_\sigma | (C \cap C_\sigma) \times F^\omega$ and $h(\Gamma) \circ h_\alpha | ((C \cap U_\sigma) \backslash K) \times F^\omega = h(\Gamma) | ((C \cap U_\sigma) \backslash K) \times F^\omega$. This means that we can define $g: (C \cup C_\sigma) \times F^\omega \to p^{-1}(C \cup C_\sigma)$ as follows:

$$\begin{aligned} g(x, a) &= h(\Gamma)(x, a), && \text{for } (x, a) \in (C \backslash K) \times F^\omega, \\ &= h(\Gamma) \circ h_\alpha(x, a), && \text{for } (x, a) \in (C \cap U_\sigma) \times F^\omega, \\ &= h_\sigma(x, a), && \text{for } (x, a) \in C_\sigma \times F^\omega. \end{aligned}$$

It is clear that $(\Gamma \cup \{\sigma\}, g) \in \mathscr{G}$ and $(\Gamma, h(\Gamma)) < (\Gamma \cup \{\sigma\}, g)$, contradicting the maximality of $(\Gamma, h(\Gamma))$.

4. **Lemmas for Theorem 2.** We will first establish a version of Lemma 2.3 where $f: X \times F^\omega \to X \times F^\omega$ is an X-preserving open *embedding* (i.e. a homeomorphism into), rather than a homeomorphism onto. At first one might think that such a result would follow directly from the proof of Lemma 2.3, but it will not work. Indeed if f is an open embedding in the proof of Lemma 2.3, then f_α does not necessarily have to be open. The problem arises in the upper part of the diagram used there. Let us call f^* the map which makes the following diagram commute:

$$\begin{array}{ccc} X \times F^\omega \times F^\omega \times I & \xrightarrow{f \times \mathrm{id} \times \mathrm{id}} & X \times F^\omega \times F^\omega \times I \\ {\scriptstyle \mathrm{id} \times \varphi^{-1}} \big\uparrow & & \big\downarrow {\scriptstyle \mathrm{id} \times \varphi} \\ X \times F^\omega \times I & \xrightarrow[f^*]{} & X \times F^\omega \times I \end{array}$$

If f is a homeomorphism onto, then so is f^*. However if f is an open embedding, then f^* is an embedding and $f^* | X \times F^\omega \times [0, 1)$ is an open embedding, but f^* might fail to be open at points of $X \times F^\omega \times \{1\}$. Roughly the idea is to use $f^*|X\times F^\omega\times[0,\frac{1}{2}]$ to move "halfway" from $\alpha^{-1}(0)$ to $\alpha^{-1}(1)$. Then we move the rest of the way to $\alpha^{-1}(1)$ by using a modification of Alexander's trick.

LEMMA 4.1. *Let X be a topological space, $\alpha: X\to I$ be a map, and let F be a metric TVS. If $f: X\times F^\omega\to X\times F^\omega$ is an X-preserving open embedding which satisfies $f|X\times\{(0,0,\cdots)\}=\mathrm{id}$, then there is an X-preserving open embedding $f_\alpha: X\times F^\omega\to X\times F^\omega$ such that $f_\alpha|X\times\{(0,0,\cdots)\}=\mathrm{id}$, $(f_\alpha)_x=f_x$, for all $x\in\alpha^{-1}(0)$, and $(f_\alpha)_x=\mathrm{id}_X$, for all $x\in\alpha^{-1}(1)$.*

PROOF. Let $X_1=\alpha^{-1}([0,\frac{1}{2}])$ and using the notation of Lemma 2.3 let $f_1: X_1\times F^\omega\to X_1\times F^\omega$ be the map which makes the following diagram commute:

$$\begin{tikzcd}
X_1\times F^\omega\times F^\omega\times I \arrow[r, "f\times\mathrm{id}\times\mathrm{id}"] & X_1\times F^\omega\times F^\omega\times I \arrow[d, "\mathrm{id}\times\varphi"] \\
X_1\times F^\omega\times I \arrow[u, "\mathrm{id}\times\varphi^{-1}"] & X_1\times F^\omega\times I \arrow[d, "(p_1,p_2)"] \\
X_1\times F^\omega \arrow[u, "i"] \arrow[r, "f_1"'] & X_1\times F^\omega
\end{tikzcd}$$

It is easy to check that f_1 is an open embedding which satisfies $f_1|X_1\times\{(0,0,\cdots)\}=\mathrm{id}$, $(f_1)_x=f_x$, for all $x\in\alpha^{-1}(0)$, and $(f_1)_x(a_1,a_2,\cdots)=(a_1',a_2,a_3',a_4,\cdots)$, where $(a_1',a_3',\cdots)=f_x(a_1,a_3,\cdots)$, for all $x\in\alpha^{-1}(\frac{1}{2})$ and $(a_1,a_2,\cdots)\in F^\omega$. Thus $(f_1)_x(0,a_2,0,a_4,\cdots)=(0,a_2,0,a_4,\cdots)$, for all $(0,a_2,0,a_4,\cdots)\in F^\omega$ and $x\in\alpha^{-1}(\frac{1}{2})$.

Let $X_2=\alpha^{-1}([\frac{1}{2},1])$ and let $A=\{(a_i)\in F^\omega | a_i=0 \text{ for } i \text{ odd}\}$. Using Theorem 3.1 of [3] there exists a homeomorphism u of F^ω onto $F^\omega\times[\frac{1}{2},1)$ such that $u(A)=F^\omega\times\{\frac{1}{2}\}$. Define $g: X_2\times F^\omega\to X_2\times F^\omega$ to be the open embedding which satisfies $g_x(a_1,a_2,\cdots)=(a_1',a_2,a_3',a_4,\cdots)$, where $(a_1',a_3',\cdots)=f(a_1,a_3,\cdots)$, for all $x\in X_2$ and $(a_1,a_2,\cdots)\in F^\omega$. Note that $f_1|\alpha^{-1}(\frac{1}{2})\times F^\omega=g|\alpha^{-1}(\frac{1}{2})\times F^\omega$. Also $\tilde g=(\mathrm{id}_{X_2}\times u)\circ g\circ(\mathrm{id}_{X_2}\times u^{-1})$ is an X_2-preserving open embedding of $X_2\times F^\omega\times[\frac{1}{2},1)$ into itself which satisfies $\tilde g_x|F^\omega\times\{\frac{1}{2}\}=\mathrm{id}$, for all $x\in X_2$.

Using Alexander's trick define $\tilde g_2: X_2\times F^\omega\times[\frac{1}{2},1)\to X_2\times F^\omega\times[\frac{1}{2},1)$ by the following formula:

$$\begin{aligned}
&(\tilde g_2)_x((a_i),t)\\
&\quad=((a_i),t), && \text{for } \tfrac{1}{2}\leqq t\leqq\alpha(x),\\
&\quad=w(x)\circ\tilde g_x((a_i),(t-2\alpha(x)+1)/2(1-\alpha(x))), && \text{for } \alpha(x)\leqq t,
\end{aligned}$$

where $w(x)((a_i),s)=((a_i),(2s-1)(1-\alpha(x))+\alpha(x))$, for $\frac{1}{2}\leqq s\leqq 1$. Note that $(\tilde{g}_2)_x=\tilde{g}_x$, for $x \in \alpha^{-1}(\frac{1}{2})$, and $(\tilde{g}_2)_x=\mathrm{id}$, for $x \in \alpha^{-1}(1)$. It can easily be checked that $\tilde{g}_2$ is an open embedding. Then $f_2=(\mathrm{id}_{X_2}\times u^{-1})\circ\tilde{g}_2\circ(\mathrm{id}_{X_2}\times u):X_2\times F^\omega\to X_2\times F^\omega$ is an X_2-preserving open embedding which satisfies $f_2|\alpha^{-1}(\frac{1}{2})\times F^\omega=f_1|\alpha^{-1}(\frac{1}{2})\times F^\omega$ and $f_2|\alpha^{-1}(1)\times F^\omega=\mathrm{id}$. Then $f_\alpha:X\times F^\omega\to X\times F^\omega$ can be defined by $f_\alpha|X_1\times F^\omega=f_1$ and $f_\alpha|X_2\times F^\omega=f_2$.

We will also need the following result.

LEMMA 4.2. *Let X be a paracompact space and let F be a metric TVS. If $A\subset X$ is closed and $U\subset A\times F^\omega$ is a relatively open subset of $A\times F^\omega$ which contains $A\times\{(0,0,\cdots)\}$, then there exists an X-preserving open embedding $f:X\times F^\omega\to X\times F^\omega$ such that $f|X\times\{(0,0,\cdots)\}=\mathrm{id}$ and $f(A\times F^\omega)\subset U$.*

PROOF. Let V be an open subset of $X\times F^\omega$ which satisfies $V\cap(A\times F^\omega)=U$ and put $W=V\cup[(X\backslash A)\times F^\omega]$, an open set containing $X\times\{(0,0,\cdots)\}$. As an immediate consequence of Lemma 1.2 of **[4]** and Lemma 2 of **[5]** there is an X-preserving open embedding $f:X\times F^\omega\to X\times F^\omega$ such that $f|X\times\{(0,0,\cdots)\}=\mathrm{id}$ and $f(X\times F^\omega)\subset W$. Clearly $f(A\times F^\omega)\subset U$.

5. **Proof of Theorem 2.** We are given a microbundle $\xi=(E,i,j,B)$ with fiber F^ω, where F is a metric TVS. That is we are given a diagram $B\xrightarrow{i}E\xrightarrow{j}B$, where B is the base space, E is the total space, and i,j are maps such that $j\circ i=\mathrm{id}_B$ and the following local triviality condition is satisfied: for each $b\in B$ there exists an open neighborhood U of b and an open embedding $h_U:U\times F^\omega\to E$ such that the following diagram commutes:

$$\begin{tikzcd}
& U \times F^\omega \arrow[dd, "h_U"] \arrow[dr, "p_1"] & \\
U \arrow[ur, "x0"] \arrow[dr, "i|U"'] & & U \\
& E \arrow[ur, "j"'] &
\end{tikzcd}$$

Here $x0$ denotes the injection $u\to(u,(0,0,\cdots))$. In the sequel we will write this diagram as $h_U\circ(x0)=i|U$ and $j\circ h_U=p_1$. What we need to do is construct an open embedding $h:B\times F^\omega\to E$ such that $h\circ(x0)=i$ and $j\circ h=p_1$.

The proof now proceeds in a manner similar to the proof of Theorem 1. Zorn's lemma is used in an analogous fashion and we will only prove a result which is analogous to the last part of the proof of Theorem 1. Thus let $C \subset B$ be closed and assume that there exists an embedding $h_C: C \times F^\omega \to E$ such that $h_C(C \times F^\omega)$ is a relatively open subset of $j^{-1}(C)$, $h_C \circ (x0) = i|C$, and $j \circ h_C = p_1$. Let $A \subset U \subset B$, where A is closed and U is open. Assume that there exists an open embedding $h_U: U \times F^\omega \to E$ such that $h_U \circ (x0) = i|U$ and $j \circ h_U = p_1$. What we want to do is construct an embedding $h^*: (A \cup C) \times F^\omega \to E$ such that $h^*((A \cup C) \times F^\omega)$ is a relatively open subset of $j^{-1}(A \cup C)$, $h^* \circ (x0) = i|A \cup C, j \circ h^* = p_1$, and h^* agrees with h_C on $(C \backslash U) \times F^\omega$.

Using Lemma 4.2 we can assume that $h_U(U \times F^\omega) \cap j^{-1}(C) \subset h_C(C \times F^\omega)$. Thus (as in the proof of Theorem 1) let $h = h_C^{-1} \circ h_U|(C \cap U) \times F^\omega$, which is a $(C \cap U)$-preserving open embedding of $(C \cap U) \times F^\omega$ into itself such that $h|(C \cap U) \times \{(0, 0, \cdots)\} = \mathrm{id}$. Now we use Lemma 4.1 (as Lemma 2.3 was analogously used in the proof of Theorem 1) to obtain a $(C \cap U)$-preserving open embedding h_α of $(C \cap U) \times F^\omega$ into itself such that $h_\alpha|(C \cap U) \times \{(0, 0, \cdots)\} = \mathrm{id}$, $h_\alpha|(C \cap A) \times F^\omega = h|(C \cap A) \times F^\omega$, and $h_\alpha|((C \cap U) \backslash K) \times F^\omega = \mathrm{id}$, where (as in the proof of Theorem 1) K is the closure of $\tilde{\alpha}([0, 1))$ in C. Then h_α is used in a manner similar to the proof of Theorem 1 to obtain our required h^*.

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