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SMOOTH SEQUENCE SPACES AND ASSOCIATED NUCLEARITY

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ABSTRACT. We characterize λ-nuclear smooth sequence spaces where λ itself is a smooth sequence space of infinite type. We also show that there is a duality between smooth sequence spaces of finite and infinite type.

1. **Introduction.** Smooth sequence spaces were introduced in **[9]** as generalizations of power series spaces. In a later work **[10]** nuclear and strongly nuclear smooth sequence spaces were investigated, but some of the results obtained depended on a characterization of strongly nuclear sequence spaces by Brudovskiĭ (**[1]**, **[2]**), which was shown to be incomplete by Köthe **[6]**.

In **[8]** Ramanujan defined $\Lambda(\alpha)$-nuclear spaces, where $\Lambda(\alpha)$ is a nuclear power series space of infinite type. In extending this notion Dubinsky and Ramanujan **[3]** replaced the power series space $\Lambda(\alpha)$ by a nuclear smooth sequence space $\lambda(P)$ of infinite type where the power set P is assumed to be countable. In the present paper we remove this restriction and consider this type of nuclearity for smooth sequence spaces and their duals. It turns out that for smooth sequence spaces uniform $\lambda(P)$-nuclearity is equivalent to $\lambda(P)$-nuclearity. Therefore the results of §2 of **[10]** are correct (see Theorems (3.1), (3.2) and their corollaries).

2. **Smooth sequence spaces and associated nuclearity.** For two sequences of scalars $x=(x_n)$ and $y=(y_n)$ we write $x\leqq y$ if $x_n\leqq y_n$ for all $n\in N=\{0,1,2,\cdots\}$. We denote by φ the space of all sequences with finitely many nonzero terms and by ω the space of all sequences. A set of sequences of nonnegative real numbers K will be called a *Köthe set* if it satisfies the following conditions:

(K1) If a and b are two elements of K there is a $c\in K$ with $a\leqq c$ and $b\leqq c$.

(K2) For every integer $r\in N$ there is $a\in K$ with $a_r>0$.

The space of all sequences $x=(x_n)$, such that $p_a(x)=\sum_{n=0}^{\infty}|x_n|a_n<\infty$ for every $a\in K$, is called the *Köthe space generated by* K and denoted by

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$\lambda(K)$. The seminorms $p_a(\ )$, $a \in K$, define a locally convex Hausdorff topology on $\lambda(K)$.

A Köthe set P will be called a *power set of infinite type* if it satisfies the following additional conditions:

(A1) For each $a \in P$, $0 < a_n \leqq a_{n+1}$ for every $n \in N$.

(A2) For each $a \in P$ there is a $b \in P$ with $(a_n)^2 \leqq b_n$ for all $n \in N$.

It is easily checked that (A2) can be replaced by

(A2)′ For $a \in P$ and $b \in P$ there is a $c \in P$ with $a_n b_n \leqq c_n$ for all $n \in N$.

If P is a power set of infinite type the Köthe space $\lambda(P)$ is called a *smooth sequence space of infinite type* or a G_∞-space (cf. [**9**, III, 3] and [**3**, (2.5)]).

If $\alpha = (\alpha_n)$ is a nondecreasing sequence of nonnegative real numbers, the power series space of infinite type $\Lambda(\alpha)$ is the G_∞-space generated by the power set $\{(k^{\alpha_n}) : k = 1, 2, 3, \cdots\}$. For $\alpha_n = \log(n+1)$ we obtain the space of all rapidly decreasing sequences s, which can also be generated by the Köthe set $\{(n+1)^k : k = 1, 2, \cdots\}$.

A Köthe set Q will be called a *power set of finite type* if it satisfies the following conditions:

(B1) For each $q \in Q$, $0 < q_{n+1} \leqq q_n$ for all $n \in N$.

(B2) For each $q \in Q$ there is an $r \in Q$ with $\sqrt{q_n} \leqq r_n$ for all $n \in N$.

The Köthe space $\lambda(Q)$ is then called a *smooth sequence space of finite type* or a G_1-space [**9**, III, 3].

A power series space of finite type $\Lambda_1(\alpha)$ is the G_1-space generated by the Köthe set $\{((1-1/k)^{\alpha_n}) : k = 1, 2, 3, \cdots\}$.

PROPOSITION (2.1). *A G_∞-space $\lambda(P)$ is nuclear if and only if for every $k \in N$ there exists a $b \in P$ and $\rho > 0$ such that $(n+1)^k \leqq \rho b_n$ for all $n \in N$.*

PROOF. $\lambda(P)$ is nuclear if and only if there is an element a of P with $(1/a_n) \in l^1$ [**9**, III, 3, (1)]. Hence the condition is clearly sufficient. On the other hand if there is an $a \in P$ with $(1/a_n) \in l^1$ we have by (A1)

$$(n+1)\frac{1}{a_n} \leqq \sum_{i=0}^{n} \frac{1}{a_i} \leqq \sum_{i=0}^{\infty} \frac{1}{a_i} < \infty.$$

Therefore $(n+1) \leqq \mu a_n$ for some $\mu > 0$. For $k \in N$ we choose $b \in P$ with $(a_n)^{2^k} \leqq b_n$ for all $n \in N$. Then $(n+1)^k \leqq \mu^{2^k} b_n$.

From now on we will assume that $\lambda(P_0)$ is a fixed nuclear G_∞-space. By the proposition we have proved $\lambda(P_0) \subset s \subset l^1$. A linear mapping T of a normed space E into another normed space F is called *$\lambda(P_0)$-nuclear* ([**3**], [**8**]) if it has a representation in the form $Tx = \sum_{n=0}^{\infty} \alpha_n v_n(x) y_n$ where $(\alpha_n) \in \lambda(P_0)$ and (v_n), (y_n) are bounded sequences in E' and F respectively.

A locally convex space E is called *$\lambda(P_0)$-nuclear* [**3**] if for every absolutely convex and closed neighbourhood U there is another such neighbourhood

V contained in U such that the canonical mapping of the associated Banach space $\hat{E}_v$ into the associated Banach space $\hat{E}_u$ is $\lambda(P_0)$-nuclear.

The next two results can be proved in the usual way (cf. [3], [6], [7]).

THEOREM (2.1). *A locally convex space E is $\lambda(P_0)$-nuclear if and only if for every absolutely convex and closed neighbourhood U there exists another such neighbourhood V such that $(d_n(V, U)) \in \lambda(P_0)$, where $d_n(V, U)$ is the nth diameter of V with respect to U* (cf. [7], [9]).

THEOREM (2.2). *A Köthe space $\lambda(K)$ is $\lambda(P_0)$-nuclear if and only if for each $a \in K$ there exists a $b \in K$ with $a \leqq b$ and an injection $\sigma: N \to N$ such that $\sigma(N) = \{n \in N : a_n \neq 0\}$ and $(a_{\sigma(n)}/b_{\sigma(n)}) \in \lambda(P_0)$.*

Following Köthe [6] we call a Köthe space $\lambda(K)$ *uniformly $\lambda(P_0)$-nuclear* if there is a "universal" permutation σ such that for every $a \in K$ there are a $b \in K$ and an $x \in \lambda(P_0)$ with $a_{\sigma(n)} \leqq x_n b_{\sigma(n)}$ for all $n \in N$.

3. **$\lambda(P_0)$-nuclearity of smooth sequence spaces.** We consider first the question when a G_∞-space is $\lambda(P_0)$-nuclear. This way we generalize the result in [10] about strongly nuclear ($\equiv s$-nuclear) G_∞-spaces.

THEOREM (3.1). *For a G_∞-space $\lambda(P)$ the following conditions are equivalent:*

(i) *$\lambda(P)$ is $\lambda(P_0)$-nuclear.*

(ii) *$\lambda(P)$ is uniformly $\lambda(P_0)$-nuclear.*

(iii) *For every $b \in P$ there is an $a \in P$ with $b \leqq a$ and $(1/a_n) \in \lambda(P_0)$.*

(iv) *There is an $a \in P$ with $(1/a_n) \in \lambda(P_0)$.*

PROOF. If $\lambda(P)$ is $\lambda(P_0)$-nuclear then for each $b \in P$ there is an $a \in P$ with $b \leqq a$ and $(d_n(U_a, U_b)) \in \lambda(P_0)$ by Theorem (2.1). Here $U_a = \{x \in \lambda(P) : p_a(x) \leqq 1\}$. Let L_n be the linear span of $e_0, e_1, \cdots, e_n$. For $x \in L_n$ we have

$$p_a(x) = \sum_{i=0}^{n} |x_i| \, b_i \frac{a_i}{b_i} \leqq \frac{a_n}{b_0} p_b(x).$$

Hence

$$(b_0/a_n)(U_b \cap L_n) \subset U_a.$$

By a result due to Tikhomirov [9, I, 2, (1)] it follows that $b_0/a_n \leqq d_n(U_a, U_b)$ for all $n \in N$. Therefore $(1/a_n) \in \lambda(P_0)$.

If there is an $a \in P$ with $(1/a_n) \in \lambda(P_0)$ then for $b \in P$ we choose $d \in P$ with $a \leqq d$ and $b \leqq d$. Hence $(1/d_n) \in \lambda(P_0)$ also. We choose $c \in P$ with $(d_n)^2 \leqq c_n$ for all $n \in N$. From the inequality

$$b_n/c_n \leqq b_n/(d_n)^2 \leqq 1/d_n$$

it follows that $(b_n/c_n) \in \lambda(P_0)$. Therefore $\lambda(P)$ is uniformly $\lambda(P_0)$-nuclear.

Since a uniformly $\lambda(P_0)$-nuclear Köthe space is $\lambda(P_0)$-nuclear and since condition (iv) follows trivially from (iii) the proof is finished.

REMARK. If a G_∞-space $\lambda(P)$ is $\lambda(P_0)$-nuclear then by condition (iv) there is an $a \in P$ with $(1/a_n) \in \lambda(P_0)$. This means that for each $c \in P_0$ there is a constant $\rho > 0$ with $c_n \leqq \rho a_n$ for all $n \in N$. Therefore $\lambda(P) \subset \lambda(P_0)$. Further the inclusion is strict, because $\lambda(P_0)$ itself is not $\lambda(P_0)$-nuclear, again by (iv).

THEOREM (3.2). *For a G_1-space $\lambda(Q)$ the following conditions are equivalent*:

(i) *$\lambda(Q)$ is $\lambda(P_0)$-nuclear.*

(ii) *$\lambda(Q)$ is uniformly $\lambda(P_0)$-nuclear.*

(iii) *$Q \subset \lambda(P_0)$.*

PROOF. If $\lambda(Q)$ is $\lambda(P_0)$-nuclear then for every $a \in Q$ there is a $b \in Q$ with $a \leqq b$ such that $(d_n(U_b, U_a)) \in \lambda(P_0)$ by Theorem (2.1). We proceed as in the proof of the previous theorem by considering $x \in L_n$. From

$$p_b(x) = \sum_{i=0}^{n} |x_i|\, a_i \frac{b_i}{a_i} \leqq \frac{b_0}{a_n}\, p_a(x)$$

we get

$$(a_n/b_0)(U_a \cap L_n) \subset U_b.$$

By Tikhomirov's theorem we have $a_n \leqq b_0\, d_n(U_b, U_a)$ and so $(a_n) \in \lambda(P_0)$.

If $Q \subset \lambda(P_0)$ then for $a \in Q$ we choose $b \in Q$ such that $a_n \leqq (b_n)^2$ for all n. Since $b \in \lambda(P_0)$ the G_1-space $\lambda(Q)$ is uniformly $\lambda(P_0)$-nuclear.

In **[9]** it was shown that if P and Q are countable power sets of infinite and finite type respectively, then $\lambda(P)$ is not isomorphic to $\lambda(Q)$, provided that one of the spaces is a Schwartz space. The two theorems we have proved in this section enable us to give a similar result in a simple fashion.

COROLLARY (3.1). *A nuclear G_∞-space $\lambda(P)$ cannot be isomorphic to a G_1-space $\lambda(Q)$.*

PROOF. If $\lambda(P)$ is isomorphic to a G_1-space $\lambda(Q)$ their diametral dimensions would be equal. From [**9**, III, (3) and (6)] we would obtain $(\lambda(P))' = \lambda(Q)$. Therefore $Q \subset \lambda(P)$ and, by Theorem (3.2), $\lambda(Q)$ would be $\lambda(P)$-nuclear. This contradicts Theorem (3.1), since $\lambda(P)$ is not $\lambda(P)$-nuclear.

As another application of Theorem (3.2) we generalize a result about power series spaces [**10**, 2, (4)].

COROLLARY (3.2). *For a G_1-space $\lambda(Q)$ the following conditions are equivalent*:

(i) $\lambda(Q)$ *is nuclear.*

(ii) $\lambda(Q)$ *is s-nuclear.*

(iii) $\lambda(Q)$ *is uniformly s-nuclear.*

(iv) $Q \subset l^1$.

(v) $Q \subset s$.

PROOF. Conditions (i), (iv), and (v) are equivalent by [**9**, III, 3, (7)]. By Theorem (3.2) the conditions (v), (ii), and (iii) are equivalent.

In [**8**, Proposition 5] a sufficient condition for the $\Lambda(\alpha)$-nuclearity of a power series space $\Lambda(\beta)$ is given. We show that this condition is also necessary in the following result.

COROLLARY (3.3). *Let $\Lambda(\alpha)$ be a nuclear power series space of infinite type. The following conditions are equivalent*:

(i) $\Lambda(\beta)$ *is* $\Lambda(\alpha)$-*nuclear.*

(ii) $\Lambda_1(\beta)$ *is* $\Lambda(\alpha)$-*nuclear.*

(iii) $\lim(\beta_n/\alpha_n)=\infty$.

PROOF. If $\Lambda(\beta)$ is $\Lambda(\alpha)$-nuclear by Theorem (3.1) there is a number q with $0<q<1$ and $(q^{\beta_n})\in\Lambda(\alpha)$. This implies that $\lim(q)^{\beta_n/\alpha_n}=0$ by [**8**, Lemma 2]. Hence $\lim(\beta_n/\alpha_n)=\infty$.

If $\lim(\beta_n/\alpha_n)=\infty$, then for every k we have $q^{\beta_n/\alpha_n}\leqq q^k$ for almost all n, where $0<q<1$. This implies that $(q^{\beta_n})\in\Lambda(\alpha)$ and so $\Lambda_1(\beta)$ is $\Lambda(\alpha)$-nuclear by Theorem (3.2).

Finally, if $\Lambda_1(\beta)$ is $\Lambda(\alpha)$-nuclear, then, by Theorem (3.2), $(q^{\beta_n})\in\Lambda(\alpha)$ for some $0<q<1$. By Theorem (3.1) (iv), $\Lambda(\beta)$ is then $\Lambda(\alpha)$-nuclear.

4. **The dual of a smooth sequence space.** The topological dual of a Köthe space $\lambda(K)$ is isomorphic to the space of all sequences u for which $|u_n|\leqq\rho a_n$ for some $a\in K$ and $\rho>0$. In general $\lambda(K)'$ is a proper subspace of the α-dual $\lambda(K)^\alpha$. For every bounded subset B of a nuclear Köthe space $\lambda(K)$ there is a positive element x of $\lambda(K)$ such that $\sup_{y\in B}|y_n|\leqq x_n$ for all $n\in N$ [**5**]. Hence if L denotes the Köthe set $\{x\geqq 0: x\in\lambda(K)\}$ then $\lambda(K)^\alpha$ is set-theoretically equal to $\lambda(L)$ and the strong dual $\lambda(K)'_b$ is a dense subspace of the Köthe space $\lambda(L)$. Our aim is to investigate the duals of smooth sequence spaces and show that there is a certain duality between G_1- and G_∞-spaces.

PROPOSITION (4.1). *A nuclear smooth sequence space is also co-nuclear.*

PROOF. Since a nuclear G_1-space is uniformly s-nuclear by Corollary (3.2), its strong dual is also nuclear [**10**, 3, (1)]. For a nuclear G_∞-space

$\lambda(P)$ it is sufficient to show that $((n+1)^2 x_n) \in \lambda(P)$ whenever $(x_n) \in \lambda(P)$ by the result of Köthe mentioned above. By Proposition (2.1), there is a $b \in P$ with $(n+1)^2 \leqq \rho b_n$ for some $\rho > 0$. For $a \in P$ we choose $c \in P$ such that $a_n b_n \leqq c_n$ for all $n \in N$. We have

$$\sum_{n=0}^{\infty} (n+1)^2 |x_n| a_n \leqq \rho \sum_{n=0}^{\infty} |x_n| c_n < \infty.$$

THEOREM (4.1). *If $\lambda(P)$ is a nuclear G_∞-space with the property that there is a $y \in \lambda(P)$ with $y_n \neq 0$ for all $n \in N$, the strong dual of $\lambda(P)$ is a dense subspace of a uniformly $\lambda(P)$-nuclear G_1-space.*

PROOF. Let $Q = \{x \in \lambda(P) : 0 < x_{n+1} \leqq x_n\}$. Since there is a $z \in \lambda(P)$ with $0 < |y_n| \leqq z_n$ and $z_{n+1} \leqq z_n$ for all $n \in N$, Q is not empty [3, Lemma (2.8)]. If $x \in Q$ and $a \in P$ we choose $b \in P$ with $(a_n)^2 \leqq b_n$. Since

$$\sup_n x_n (a_n)^2 \leqq \sup_n x_n b_n < \infty$$

we have that the sequence $(\sqrt{x_n} a_n)$ is bounded for every $a \in P$. By the Grothendieck-Pietsch criterion $(\sqrt{x_n}) \in Q$ also. Therefore Q is a power set of finite type. Let $L = \{z \in \lambda(P) : z \geqq 0\}$. By Lemma (2.8) of [3] for every $z \in L$ there exists $x \in \lambda(P)$ with $0 \leqq z_n \leqq x_n$ and $x_{n+1} \leqq x_n$. By adding some $y \in Q$ to x, if necessary, we may assume that $x_n \neq 0$ for all $n \in N$ and so $x \in Q$. Therefore the Köthe spaces $\lambda(Q)$ and $\lambda(L)$ are set-theoretically equal and the topologies defined by Q and L coincide. Since $Q \subset \lambda(P)$ by Theorem (3.2), the G_1-space $\lambda(Q)$ is uniformly $\lambda(P)$-nuclear.

REMARK. If P is the set of all nondecreasing sequences of positive real numbers the nuclear G_∞-space $\lambda(P)$ is nothing but φ with its usual direct sum topology. Its strong topological dual ω is not a G_1-space; for if $\omega = \lambda(Q)$ for some power set of finite type Q, then $Q \subset \varphi$ which contradicts the condition (B1). On the other hand ω is uniformly φ-nuclear.

COROLLARY (4.1). *If P is a countable power set of infinite type and if $\lambda(P)$ is nuclear, then $\lambda(P)'_b$ is a uniformly $\lambda(P)$-nuclear G_1-space.*

PROOF. If P is countable, then the G_∞-space $\lambda(P)$ contains an element y with $y_n \neq 0$ for all $n \in N$ [3, Lemma (2.9)]. Since $\lambda(P)$ is barrelled its topological dual coincides with its α-dual and so $\lambda(P)'_b = \lambda(Q)$, where Q is the power set of finite type constructed in the proof of the theorem.

In particular, the strong dual of a nuclear power series space $\Lambda(\alpha)$ is a uniformly $\Lambda(\alpha)$-nuclear G_1-space [8, Proposition 6].

COROLLARY (4.2). *Let $\lambda(P)$ be a nuclear G_∞-space such that there is a $y \in \lambda(P)$ with $y_n \neq 0$ for all $n \in N$. If $\lambda(P)$ is $\lambda(P_0)$-nuclear then $\lambda(P)'_b$ is also $\lambda(P_0)$-nuclear.*

PROOF. Let Q be the power set of finite type constructed in the proof of the theorem. By the remark following Theorem (3.1) we have $\lambda(P)\subset\lambda(P_0)$. Hence $Q\subset\lambda(P_0)$ and by Theorem (3.2). $\lambda(Q)$ is a $\lambda(P_0)$-nuclear G_1-space. Since $\lambda(P)'_b$ is a subspace of $\lambda(Q)$, then it is also $\lambda(P_0)$-nuclear.

REMARK. Since the strong dual of a nuclear G_∞-space $\lambda(P)$ is $\lambda(P)$-nuclear but $\lambda(P)$ itself is not $\lambda(P)$-nuclear, the converse of this result is false.

THEOREM (4.2). *The strong dual of a nuclear G_1-space is a dense subspace of a nuclear G_∞-space.*

PROOF. Let $P=\{x\in\lambda(Q):0<x_0\leqq x_1\leqq\cdots\}$ where $\lambda(Q)$ is a nuclear G_1-space. By Corollary (3.2) every bounded sequence is an element of $\lambda(Q)$ and so P is not empty. If $x\in P$ and $a\in Q$ we choose $b\in Q$ with $\sqrt{a_n}\leqq b_n$ for all $n\in N$. Since (x_nb_n) is a bounded sequence, it follows that $(x_n^2a_n)$ is also bounded for every $a\in Q$ and hence $(x_n^2)\in\lambda(Q)$. Therefore P is a power set of infinite type. Our aim is to show that if $L=\{y\in\lambda(Q):y\geqq 0\}$ then $\lambda(L)$ is set-theoretically equal and topologically isomorphic to the G_∞-space $\lambda(P)$. Let $y\in L$, $x_n=\sup\{y_i:0\leqq i\leqq n\}$ and $\rho=\sup\{y_na_n:n\in N\}$ where $a=(a_n)\in Q$. From

$$x_n\leqq\rho\sup_{0\leqq i\leqq n}(1/a_i)\leqq\rho/a_n$$

it follows that $x\in\lambda(Q)$ and $y_n\leqq x_n$ for all $n\in N$. By adding a sequence from P to x, if necessary, we may assume that $x_n>0$ for all $n\in N$. Hence $x\in P$ and so $\lambda(L)=\lambda(P)$. The nuclearity of $\lambda(Q)'_b$ follows from Proposition (4.1).

REMARK. If Q is a countable power set of finite type, then $\lambda(Q)'_b$ coincides with $\lambda(P)$. Hence the strong dual of a nuclear power series space of finite type is a nuclear G_∞-space.

COROLLARY (4.3). *Let $\lambda(Q)$ be a nuclear G_1-space. If $\lambda(Q)'_b$ is $\lambda(P_0)$-nuclear then $\lambda(Q)$ is also $\lambda(P_0)$-nuclear.*

PROOF. Since $\lambda(Q)'_b$ is a dense subspace of $\lambda(P)$, the G_∞-space $\lambda(P)$ is also $\lambda(P_0)$-nuclear, where $P=\{x\in\lambda(Q):0<x_{n+1}\leqq x_n\}$. Therefore $Q\subset\lambda(P)$ and $\lambda(P)$ is contained in $\lambda(P_0)$ by the remark following Theorem (3.1). Hence $\lambda(Q)$ is $\lambda(P_0)$-nuclear by Theorem (3.2).

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