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LIE AND JORDAN STRUCTURE IN PRIME RINGS WITH DERIVATIONS

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ABSTRACT. In this paper Lie ideals and Jordan ideals of a prime ring R together with derivations on R are studied. The following results are proved: Let R be a prime ring, U be a Lie ideal or a Jordan ideal of R and d be a nonzero derivation of R such that $ud(u)-d(u)u$ is central in R for all u in U. (i) If the characteristic of R is different from 2 and 3, then U is central in R. (ii) If R has characteristic 3 and U is a Jordan ideal then U is central in R; further, if U is a Lie ideal with $u^2 \in U$ for all u in U, then U is central in R. The case when R has characteristic 2 is also studied. These results extend some due to Posner [2].

1. **Introduction.** A theorem of Posner [2] states that if R is a prime ring, and d is a nonzero derivation of R such that, for all $r \in R$, $rd(r)-d(r)r$ is in the centre of R, then R is commutative. Our object is to generalize this theorem to Lie and Jordan ideals of R.

All rings considered here are associative. Let R be a ring and Z be its centre. For $x, y \in R$, $[x, y]=xy-yx$. For $a \in R$, let I_a denote the inner derivation of R by a; i.e., $I_a(x)=ax-xa$ for all $x \in R$. Throughout the paper d denotes a nonzero derivation of R. For definitions see [1].

2. **Basic lemmas.** We begin with some preliminary lemmas.

LEMMA 1. *If R is a prime ring of characteristic different from 2 and U is a Lie ideal of R such that for all $u \in U$, $[u, d(u)] \in Z$, and $u^2 \in U$, then $[u, d(u)]=0$ for all $u \in U$.*

PROOF. First observe that linearizing the relation $[u, d(u)] \in Z$ on $u=u+u^2$, we obtain $[u^2, d(u)]+[u, ud(u)+d(u)u] \in Z$. That is, $4[u, d(u)]u \in Z$ for all $u \in U$. Hence, $[u, d(u)][u, r]=0$ for all $u \in U$, $r \in R$. If for some u in U, $[d(u), u]\neq 0$, then, as it is in the centre Z, we get $[u, r]=0$ for all $r \in R$, in particular $[u, d(u)]=0$. Hence $[u, d(u)]=0$ for all $u \in U$. □

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LEMMA 2. *Let R be a prime ring and U a Lie ideal of R. Suppose that $[u, d(u)] \in Z$ for all $u \in U$. Then $[[d(r), u], u] \in Z$ for all $u \in U$, $r \in R$. Further, if for all $u \in U$, $[u, d(u)]=0$ then $[[d(r), u], u]=0$ for all $r \in R$, $u \in U$.*

PROOF. Let $u \in U$ and $r \in R$, then $[u, r] \in U$, so that $[u+[u, r], d(u+[u, r])] \in Z$. That is, $[[u, r], d(u)]+[u, [d(u), r]]+[u, [u, d(r)]] \in Z$. Now, $[[u, r], d(u)]+[u, [d(u), r]]=[r, [d(u), u]]$ for any $r \in R$, $u \in U$. Since $[d(u), u] \in Z$, we get $[[u, r], d(u)]+[u, [d(u), r]]=0$. Hence

$$[[d(r), u], u] \in Z \quad \text{for all } r \in R, u \in U.$$

The last part can be obtained similarly. □

The following lemma may have some independent interest.

LEMMA 3. *Let R be a prime of characteristic not 2 and let U be a Jordan ideal of R with $ud(u)=d(u)u=0$ for all $u \in U$. Then $U=0$.*

PROOF. Linearize the relation $ud(u)=0$ on u to get

$$(1) \qquad ud(v) + vd(u) = 0 \quad \text{for all } u, v \in U.$$

For $u \in U$ and any $r \in R$, $u(ur-ru)+(ur-ru)u \in U$. But $2(ru^2-u^2r)=\{u(ru-ur)+(ru-ur)u\}-\{(ur-ru)u+u(ur-ru)\}$. As the first and second term on the right hand side are in U, $2(ru^2-u^2r) \in U$. As $2u^2 \in U$, $2(u^2r+ru^2) \in U$. It follows that $4u^2r$ and $4ru^2$ are in U. Replacing v by $4ru^2$ where $r \in R$ in (1) and using the hypothesis, we get $ud(r)u^2=0$ for all $u \in U$, $r \in R$. If in (1) we replace v by $ur+ru$ where $r \in R$, then $u^2d(r)+ud(r)u+2urd(u)=0$; and hence $u^2d(r)u+ud(r)u^2=0$. Therefore, $u^2d(r)u=0$ for all $u \in U$ and $r \in R$. Again, put $v=4uru=2\{u(ur+ru)+(ur+ru)u\}-\{2u^2 \cdot r+r \cdot 2u^2\}$ in (1) where $r \in R$; then $0=ud(u)ru+u^2d(r)u+u^2rd(u)=u^2d(r)u+u^2rd(u)$. Hence, $u^2rd(u)=0$ for all $r \in R$, $u \in U$. Lastly, replace v by $4u^2r$ in (1), for $r \in R$; then $0=ud(4u^2r)+4u^2rd(u)=4u^3d(r)$. Hence, $u^3d(r)=0$ for all $u \in U$ and $r \in R$. Then by Lemma 1 of [**2**], $u^3=0$ for all $u \in U$. For $u \in U$ and $r \in R$, $2(u^2r+ru^2) \in U$, so that $0=2^3(u^2r+ru^2)^3$. Multiply on the right by u^2r, to obtain $2^3(u^2r)^4=0$. Hence, $(u^2r)^4=0$. If for some u in U, $u^2 \neq 0$, then u^2R is a nonzero right ideal of R in which the quartic of every element is zero. By Levitzki's theorem [**1**, Lemma 1.1] R would have a nilpotent ideal; which is impossible for a prime ring. Hence $u^2=0$ for all $u \in U$. By repeating the above argument we can show that $u=0$ for all $u \in U$. □

3. **The main theorems.**

THEOREM 1. *Let R be a prime ring of characteristic different from 2 and 3. Let d be a nonzero derivation of R, and U a Lie ideal of R with $[u, d(u)] \in Z$ for all u in U. Then $U \subset Z$.*

PROOF. By Lemma 2, $[[d(r), u], u] \in Z$ for all $u \in U$, $r \in R$. Now, proceeding on the same lines as in Posner [2] (cf. equations (16) to (27)), we have $[d(u), u]=0$ for all $u \in U$. Again by Lemma 2,

$$[[d(r), u], u] = 0 \quad \text{for all } u \in U,\ r \in R. \tag{2}$$

Replace u by $u+w$ with $w \in U$ in (2).

$$[[d(r), u], w] + [[d(r), w], u] = 0 \quad \text{for all } r \in R,\ u,\ w \in U. \tag{3}$$

Suppose now that w, $v \in U$ are such that wv is also in U. By replacing w by wv in (3), where $v \in U$, and expanding we get

$$w[[d(r), u], v] + [[d(r), u], w]v + [d(r), w][v, u] + [[d(r), w], u]v + w[[d(r), v], u] + [w, u][d(r), v] = 0.$$

In view of (3) the last equation reduces to $[d(r), w][v, u]+[w, u][d(r), v]=0$. For any $t \in R$, $w \in U$, the element $v=tw-wt$ satisfies the criterion $wv \in U$, hence by above

$$[d(r), w][[t, w], u] + [w, u][d(r), [t, w]]=0 \quad \text{for } t, r \in R;\ u, w \in U. \tag{4}$$

Putting $u=w$ in (4), we have

$$[d(r), w][[t, w], w] = 0 \quad \text{for } r,\ t \in R \text{ and } w \in W. \tag{5}$$

Substitution of $td(a)$ for r in (5) with $a \in R$ yields on expansion

$$[d(r), w]\{2[t, w][d(a), w] + [[t, w], w]d(a) + t[[d(a), w], w]\} = 0.$$

By (5) the second term is zero and by (2) the third term is zero, so that

$$[d(r), w][t, w][d(a), w] = 0 \quad \text{for all } r,\ t,\ a \in R,\ w \in U. \tag{6}$$

Put $u=[t, w]$ in (4). Then $[[t, w], w][[t, w], d(r)]=0$. Its linearization on $t=t+d(a)$ where $a \in R$ together with (2) yields

$$[[t, w], w][[d(a), w], d(r)] = 0 \quad \text{for all } a,\ t,\ r \in R \text{ and } w \in U. \tag{7}$$

Replace t by $d(t)p$ with $p \in R$ in (7) and expand; then

$$\{2[d(t), w][p, w] + d(t)[[p, w], w] + [[d(t), w], w]p\}[[d(a), w], d(r)] = 0.$$

By (7) the second term is zero, while by (2) the third term is zero. Hence

$$[d(t), w][p, w][[d(a), w], d(r)] = 0.$$

In view of (6), the last equation reduces to

$$[d(t), w][p, w]d(r)[d(a), w] = 0 \quad \text{for all } a,\ r,\ p,\ t \in R \text{ and } w \in U.$$

In (6) replace t by $td(p)$, where $p \in R$ and using the last equation to get

$$[d(r), w]R[d(p), w][d(a), w] = 0 \quad \text{for all } r,\ p,\ a \in R \text{ and } w \in U.$$

Now, if $[d(r), w]=0$ for all $r \in R$, $w \in U$, that is for all $r \in R$, $w \in U$, $(I_w d)r=0$, then by [**2**, Theorem 1] $w \in Z$ for all $w \in U$. Thus assume that there exists a $w \in U$ such that for some $r \in R$, $[d(r), w] \neq 0$. That is $w \notin Z$. Then for all $a, p \in R$

$$(8) \qquad [d(p), w][d(a), w] = 0.^{2}$$

Replace a by bc where $c, b \in R$ and expand to get

$$[d(p), w][d(b), w]c + [d(p), w]d(b)[c, w] + [d(p), w]b[d(c), w] + [d(p), w][b, w]d(c) = 0.$$

Replace b by $[t, w]$ where $t \in R$. By (8) the first term is zero, while by (6) the third term is zero, and by (5) the fourth term is zero. Therefore,

$$[d(p), w]d([t, w])[w, c] = 0.$$

Since, $d([t, w])=[d(t), w]+[t, d(w)]$ and using (8) we get

$$[d(p), w][t, d(w)][w, c] = 0 \quad \text{for all } p, c, t \in R \text{ and } w \in U.$$

Replace c by cr_1 where $r_1 \in R$, then $[d(p), w][t, d(w)]R[w, c]=0$. Since R is prime and $w \notin Z$, we get $[d(p), w][t, d(w)]=0$ for $p, t \in R$, $w \in U$. Therefore, $[d(p), w]R[t, d(w)]=0$ for $p, t \in R$ and $w \in U$; which together with $[d(r), w] \neq 0$ implies that $d(w) \in Z$.

Now suppose that $u \in U$ and $u \in Z$. Then $0=d[u, a]=[d(u), a]+[u, d(a)]$ and hence $d(u) \in Z$. Therefore, $d(u) \in Z$ for all $u \in U$, so that $d([w, a]) \in Z$ for all $a \in R$. That is, $[d(w), a]+[w, d(a)] \in Z$ for all $a \in R$. Thus $[w, d(a)] \in Z$ for all $a \in R$. In particular,

$$(9) \qquad [w, d(aw)] = [w, d(a)]w + [w,a]d(w) \in Z.$$

Commute (9) with w to get $[w, [w, a]]d(w)=0$ for $a \in R$. If $d(w) \neq 0$, and as it is in the centre Z, $[w, [w, a]]=0$ for all $a \in R$. By [**1**, Sublemma, p. 5] $w \in Z$, a contradiction. Hence, $d(w)=0$. Thus, by (9), $[w, d(a)]w \in Z$ for all $a \in R$; that is $[d(a), w][w, b]=0$ for $a, b \in R$. Replace b by bc, where $c \in R$, then $[d(a), w]R[w, b]=0$. Since R is prime, either $w \in Z$ or $[d(a), w]=0$ for all $a \in R$. So, in both cases $w \in Z$; a contradiction. Hence the conclusion is that $w \in Z$ for all $w \in U$. This proves the theorem. □

Now we should like to settle the problem when R has characteristic 3. Note that the assumption that the characteristic is different from 3 does not enter the proof of Theorem 1 onwards of equation $[u, d(u)]=0$ for all $u \in U$. Therefore, if $[u, d(u)]=0$ holds for all $u \in U$, we can show that $U \subset Z$ even if R has characteristic 3. In view of Lemma 1, if R has characteristic different from 2 and U is a Lie ideal of R such that for all $u \in U$,

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[2] Onward proof of this theorem is given by the referee.

$u^2 \in U$ and $[u, d(u)] \in Z$, then $[u, d(u)]=0$ for all $u \in U$. Hence, we get the following weaker result.

THEOREM 2. *Let R be a prime ring of characteristic 3 and d a nonzero derivation of R. If U is a Lie ideal of R with $[u, d(u)] \in Z$ and $u^2 \in U$ for all $u \in U$, then $U \subset Z$.*

Now we will show that the conclusion of Theorems 1 and 2 holds even if U is a Jordan ideal of R. In this regard, we prove the following.

THEOREM 3. *Let R be a prime ring of characteristic not equal to 2. Let d be a nonzero derivation of R and U be a Jordan ideal of R, such that $[u, d(u)] \in Z$ for all $u \in U$. Then $U \subset Z$.*

PROOF. For $u \in U$, $2u^2 \in U$. Therefore by Lemma 1, $[u, d(u)]=0$ for all $u \in U$. Replace u by $u+v$, where $v \in U$, then

$$[u, d(v)] + [v, d(u)] = 0 \quad \text{for all } u, v \in U. \tag{10}$$

In (10), replace v by $ur+ru$, $r \in R$, and expand to get

$$u[u, d(r)] + [u, d(r)]u + d(u)[u, r] + [u, r]d(u) + u[r, d(u)] + [r, d(u)]u = 0,$$

i.e.,

$$2urd(u) - 2d(u)ru + u^2d(r) - d(r)u^2 = 0 \quad \text{for } r \in R, u \in U. \tag{11}$$

Replace r by ur in (11), then

$$d(u)(u^2r - ru^2) = 0 \quad \text{for all } r \in R, u \in U \tag{12}$$

that is, $d(u)I_{u^2}(r)=0$ for all $r \in R$, $u \in U$; hence by [**2**, Lemma 1], either

$$u^2 \in Z \quad \text{or} \quad d(u) = 0 \quad \text{for all } u \in U. \tag{13}$$

For $u \in U$ and any $r \in R$, $ur+ru \in U$. But

$$4uru = 2\{u(ur + ru) + (ur + ru)u\} - \{2u^2 \cdot r + r \cdot 2u^2\}.$$

The first and the second term on the right are in U. Hence $4uru \in U$. Therefore, if we replace v by $4uru$ in (10), where $r \in R$, then

$$d(u)[u, r]u + u[u, d(r)]u + u[u, r]d(u) + u[r, d(u)]u = 0,$$

i.e.,

$$u^2rd(u) - d(u)ru^2 + u^2d(r)u - ud(r)u^2 = 0 \quad \text{for } r \in R, u \in U. \tag{14}$$

Replace r by ur in (14) and use (14) to get $ud(u)(uru-ru^2)=0$. However in view of (12), this equation reduces to $ud(u)u(ur-ru)=0$. That is, $ud(u)u \cdot I_u(r)=0$. By [**2**, Lemma 1], either

$$ud(u)u = 0 \quad \text{or} \quad u \in Z \quad \text{for all } u \in U. \tag{15}$$

In (12), replace u by $u+v$ where $v \in U$ and use (12). Then

$$\{d(u) + d(v)\}[uv + vu, r] + d(u)[v^2, r] + d(v)[u^2, r] = 0.$$

Replace u by $-u$, then

$$\{-d(u) + d(v)\}[-uv - vu, r] - d(u)[v^2, r] + d(v)[u^2, r] = 0.$$

Adding last two equations and dividing by 2, we have $d(u)[uv+vu, r]+d(v)[u^2, r]=0$ for all $r \in R$ and $u, v \in U$. By Lemma 3, $ud(u)\neq 0$, for some u in U, $d(u)\neq 0$, hence by (13) $u^2 \in Z$. The net result of this is

$$d(u)[uv + vu, r] = 0.$$

That is, $d(u)I_{uv+vu}(r)=0$ for all $r \in R$ and $v \in U$. By [**2**, Lemma 1] $uv+vu \in Z$ for all $v \in U$. If $u^2=0$, then $0=d(u^2)=ud(u)+d(u)u=2ud(u)$ so that $ud(u)=0$, a contradiction. Hence $u^2\neq 0$. Suppose that $ud(u)u=0$ then $u^2d(u)=0$ which implies that $d(u)=0$, a contradiction. Hence $ud(u)u\neq 0$, so (15) gives $u \in Z$. Hence $2uv \in Z$; so that $uv \in Z$ for all $v \in U$. As $u\ (\neq 0) \in Z$, we have $v \in Z$ for all $v \in U$. Hence $U \subset Z$. This completes the proof of Theorem 3. □

We should like to settle the problem even when R has characteristic 2. In this case Lie ideals and Jordan ideals will coincide. We are proving now the following weaker result.

THEOREM 4. *Let R be a prime ring of characteristic 2, and let d be a nonzero derivation of R. Let U be a Lie (Jordan) ideal and a subring of R. Suppose that $[u, d(u)] \in Z$ for all $u \in U$. Then U is commutative.*

PROOF. By Lemma 2, $[[d(r), u], u] \in Z$ i.e.,

$$d(r)u^2 + u^2d(r) \in Z \quad \text{for all } r \in R,\ u \in U. \tag{16}$$

Commuting (16) with $d(r)$ and u^2 respectively, we get

$$u^2d(r)^2 = d(r)^2u^2 \quad \text{for all } r \in R,\ u \in U \tag{17a}$$

and

$$u^4d(r) = d(r)u^4 \quad \text{for all } r \in R,\ u \in U \tag{17b}$$

where $d(r)^2$ stands for $(d(r))^2$.

In (17a) replace r by $v+u^2v$ where $v \in U$ and use (17a). Then

$$(u^2d(v))^2 + u^2d(v)d(u^2)v + u^2d(u^2)vd(v) + u^4d(v)^2$$
$$= (d(v)u^2)^2 + d(v)d(u^2)vu^2 + d(u^2)vd(v)u^2 + u^2d(v)^2u^2.$$

For $u \in U$, $d(u^2)=ud(u)+d(u)u \in Z$, so that in view of (17b) the last equation reduces to $(u^2d(v)+d(v)u^2)^2=0$ for $u, v \in U$. Since R is prime, by using (16) we get

$$(18)\qquad u^2d(v) = d(v)u^2 \quad \text{for all } u, v \in U.$$

Replace u by $u+w$ where $w \in U$ in (18). Then

$$(uw + wu)d(v) = d(v)(uw + wu).$$

Replace w by wu, then $(uw+wu)ud(v)=d(v)(uw+wu)u=(uw+wu)d(v)u$. Therefore, $(uw+wu)(ud(v)+d(v)u)=0$ for all $u, v, w \in U$. Linearize the last equation on $u=u+u_1^2$, where $u_1 \in U$ and put $v=u$. Then using (18) we get

$$(u_1^2w + wu_1^2)(ud(u) + d(u)u) = 0 \quad \text{for all } u, v, w \in U.$$

If $[d(u), u]\neq 0$ for some u in U, then $u_1^2w=wu_1^2$ for all $u_1, w \in U$; so that, $u^2(wr+rw)=(wr+rw)u^2$ for all $r \in R$, $u, w \in U$. That is, $w(u^2r+ru^2)=(u^2r+ru^2)w$ for all $r \in R$, $u, w \in U$. Replace r by ru, then $(u^2r+ru^2)\times(wu+uw)=0$ for all $r \in R$, $u, w \in U$. Replacing w by $[u, t]$ we get

$$(u^2r + ru^2)(u^2t + tu^2) = 0 \quad \text{for all } r, t \in R, u \in U.$$

Replace t by tp where $p \in R$; then $(u^2r+ru^2)R(u^2t+tu^2)=0$. Since R is prime, we get $u^2 \in Z$ for all $u \in U$. Thus assume that $[d(u), u]=0$ for all $u \in U$. By Lemma 2, $[[d(r), u], u]=0$ i.e., $u^2 d(r)=d(r)u^2$ for all $r \in R$, $u \in U$. Replace r by ra where $a \in R$, then

$$d(r)(u^2a + au^2) + (u^2r + ru^2)d(a) = 0.$$

For $v \in U$, $d(v^2)=vd(v)+d(v)v=0$. Hence $d(r)(u^2v^2+v^2u^2)=0$ for all $r \in R$, $v \in U$. Thus by [2, Lemma 1] $u^2v^2=v^2u^2$ for $u, v \in U$. Therefore $u^2(vw+wv)=(vw+wv)u^2$ for $u, v, w \in U$. Replace v by vw, then $(vw+wv)\times(u^2w+wu^2)=0$; so that $(w^2r+rw^2)(u^2w+wu^2)$, i.e., $I_{w^2}(r)(u^2w+wu^2)=0$ for all $r \in R$, $u, w \in U$. The Lemma 1 of [2] forces that if $w^2 \notin Z$ for some w in U, then for that w, $u^2w=wu^2$ for all $u \in U$. So that, $[[u, v], w]=0$ for all $u, v \in U$. For $w \in U$, then $[[v, w], u]+[[w, u], v] = [[u, v], w]=0$ for all $u, v \in U$. Replace, in $[[v, w], u]+[[w, u], v]=0$, v by vw and expand to obtain $[[v, w], u]w+[v, w][w, u]+[[w, u], v]w=0$. Hence, $[v, w][w, u]=0$ for all $u, v \in U$. Replacing v by $[w, r]$ and u by $[w, t]$, we get

$$(w^2r + rw^2)(w^2t + tw^2) = 0 \quad \text{for all } r, t \in R.$$

Replace t by tp where $p \in R$, then $(w^2r+rw^2)R(w^2t+tw^2)=0$, which implies that $w^2 \in Z$, a contradiction. Hence the conclusion is that $u^2 \in Z$ for all $u \in U$. So in all possible cases $w^2 \in Z$ for all $u \in U$ so that $(uv+vu) \in Z$ and $(uv+vu)u \in Z$ for all $u, v \in U$. If $u \notin Z(U)$, where $Z(U)$ denotes the centre of U, then $uv+vu=0$ for all $v \in U$ and $u \in Z(U)$. Hence U is commutative.

In Theorem 4, if we just assume that U is only a Lie (Jordan) ideal or only a subring of R, then U may not be commutative. This is shown by the following examples.

EXAMPLE 1. Let R be a prime ring of all 2×2 matrices over a noncommutative prime ring. Consider $U=\{\left(\begin{smallmatrix}\alpha & 0\\ 0 & \beta\end{smallmatrix}\right)\in R\}$. It is clear that U is a subring, but not a Lie ideal of R. Let us define $d\colon R\to R$ such that

$$d\begin{pmatrix}\alpha & \beta\\ \gamma & \delta\end{pmatrix}=\begin{pmatrix}0 & -\beta\\ \gamma & 0\end{pmatrix},\quad \text{for all } \begin{pmatrix}\alpha & \beta\\ \gamma & \delta\end{pmatrix}\in R.$$

It is easy to verify that d is a nonzero derivation of R with $[u, d(u)]\in Z$ for all $u\in U$. But U is not commutative.

EXAMPLE 2. Consider the prime ring R of all 2×2 matrices over $GF(2)$. Let $U=\{\left(\begin{smallmatrix}a & b\\ c & a\end{smallmatrix}\right), a, b, c\in GF(2)\}$. It is clear that U is a Lie ideal but not a subring of R. Let us define $d\colon R\to R$ such that

$$d\begin{pmatrix}a & b\\ c & d\end{pmatrix}=\begin{pmatrix}d-c & a-d\\ a-d & b-c\end{pmatrix}\quad \text{for all } \begin{pmatrix}a & b\\ c & d\end{pmatrix}\in R.$$

It can be seen that d is a nonzero derivation of R with $[u, d(u)]\in Z$ for all $u\in U$. However, U is not commutative.

Following example shows that a ring may satisfy all the assumptions of Theorem 4, but U may not be in the centre, even though U is commutative.

EXAMPLE 3. Let R be a ring of all 2×2 matrices with entries from $GF(2)$. Consider $U=\{\left(\begin{smallmatrix}a & b\\ b & a\end{smallmatrix}\right), a, b\in GF(2)\}$. It can easily be verified that U is both a Lie (Jordan) ideal and a subring of R, but it is not an ideal of R. Define $d\colon R\to R$ as in Example 2. Then d satisfies $[u, d(u)]\in Z$ for all $u\in U$. Clearly U is commutative, but U is not in the centre of R.

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REFERENCES

1. I. N. Herstein, *Topics in ring theory*, Univ. of Chicago Press, Chicago, Ill., 1969. MR **42** #6018.

2. E. C. Posner, *Derivations in prime rings*, Proc. Amer. Math. Soc. **8** (1957), 1093–1100. MR **20** #2361.

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