

\mathcal{L} -REALCOMPACTIFICATIONS AS EPIREFLECTIONS

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ABSTRACT. If \mathcal{L} is a countably productive normal base on a Tychonoff space X , then $\eta(X, \mathcal{L})$ is an \mathcal{L}_* -realcompact extension of X . R. A. Alo and H. L. Shapiro thus generalized the Hewitt realcompactification of X . In the following paper, we extend this construction to T_1 -spaces and show that it is an epireflection functor on an appropriate category. We are thus concerned with the question of the extendibility of a continuous map $f: X \rightarrow Y$ to a continuous map $g: \eta(X, \mathcal{L}_X) \rightarrow \eta(Y, \mathcal{L}_Y)$. We derive necessary and sufficient conditions therefor in the case when \mathcal{L}_Y is a nest generated intersection ring on Y .

All of the topological spaces which we consider are assumed to satisfy the T_1 -separation axiom. A family \mathcal{L} of closed subsets of a space X is called a countably productive separating base if the following are satisfied:

(i) \mathcal{L} is a *ring*, i.e. it is closed under finite unions and finite intersections and $\emptyset, X \in \mathcal{L}$.

(ii) \mathcal{L} is *countably productive*, i.e. closed under countable intersections.

(iii) \mathcal{L} is *separating*, i.e. whenever $x \in X - S$, S closed in X , there exist $L_1, L_2 \in \mathcal{L}$ such that $x \in L_1$, $S \subset L_2$, and $L_1 \cap L_2 = \emptyset$ (E. F. Steiner [9]).

If in addition to the above properties,

(iv) \mathcal{L} is *nest generated*, i.e. $L \in \mathcal{L}$ implies the existence of $E_n, F_n \in \mathcal{L}$, $n \in \mathbb{N}$, such that $F_{n+1} \subset X - E_n \subset F_n$ and $L = \bigcap \{F_n : n \in \mathbb{N}\}$ (A. K. Steiner and E. F. Steiner [8], [10]), then \mathcal{L} is called a *nest generated intersection ring*. (R. A. Alo and H. L. Shapiro [2] use the term strong delta normal base.) In [8], A. K. Steiner and E. F. Steiner prove that a nest generated intersection ring \mathcal{L} is normal, i.e. $L_1, L_2 \in \mathcal{L}$, and $L_1 \cap L_2 = \emptyset$ implies the existence of $L'_1, L'_2 \in \mathcal{L}$ such that $L_1 \cap L'_1 = \emptyset$, $L_2 \cap L'_2 = \emptyset$, and $L'_1 \cup L'_2 = X$.

A well-known example of a nest generated intersection ring is $Z(X)$, the family of all zero sets of a Tychonoff space X .

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Any countably productive separating base \mathcal{L} on X determines a T_1 -extension of X which we now proceed to describe. These extensions have been studied previously by R. A. Alo and H. L. Shapiro [1], [2], A. K. Steiner and E. F. Steiner [8], and M. S. Gagrut and S. A. Naimpally [5] with the additional hypothesis that \mathcal{L} is normal, in which case X is Tychonoff. In the first part of this paper we weaken this to the T_1 -axiom. In addition, we take a formally different approach to the subject at hand by using the technically more convenient contiguity clusters (a concept introduced in our earlier work [4]) instead of ultrafilters. We introduce the following notation: if \mathcal{L} is a countably productive separating base on X and $A_i \subset X$ for $i \in N$, we write $\bar{C}_X(A_i: i \in N)$ iff there exist $L_i \in \mathcal{L}$ such that $A_i \subset L_i$ and $\bigcap \{L_i: i \in N\} = \emptyset$. We write $C_X(A_i: i \in N)$ iff not $\bar{C}_X(A_i: i \in N)$. In words, we are thinking of $C_X(A_i: i \in N)$ as meaning that the sets in the family $\{A_i: i \in N\}$ are contiguous. We extend the above notations by writing $C_X(A_1, \dots, A_n)$ iff $C_X(B_i: i \in N)$, where $B_i = A_i$ for $i = 1, \dots, n$, and $B_i = X$ for $i > n$.

Clusters play a key role in studying Hausdorff compactifications of Tychonoff spaces (for the definition of a cluster see [7]). However, for T_1 -compactifications of T_1 -spaces the concept of contiguity cluster [4] is useful. A collection σ of subsets of X is called a *contiguity cluster* iff σ is maximal with respect to the property that $A_1, \dots, A_n \in \sigma$ implies $C_X(A_1, \dots, A_n)$. We now go a step further and call a collection σ an *ω -contiguity cluster* iff σ is maximal with respect to the property that $\{A_i: i \in N\} \subset \sigma$ implies $C_X(A_i: i \in N)$.

Any separating base \mathcal{L} on X determines a T_1 -compactification $w(X, \mathcal{L})$ of X (E. F. Steiner [9]). $w(X, \mathcal{L})$ is the space of all contiguity clusters and $\bar{\mathcal{L}} = \{\bar{L}: L \in \mathcal{L}\}$ is a separating base on $w(X, \mathcal{L}) = \bar{X}$, where for $L \subset X$, $\bar{L} = \{\sigma \in w(X, \mathcal{L}): L \in \sigma\}$ (for the equivalence of this to Steiner's see [4]). If we let $e_X(x) = \{A \subset X: x \in A\}$ for each $x \in X$, then the map $e_X: X \rightarrow \bar{X}$ is a topological embedding of X in \bar{X} and for each $L \subset X$, $\bar{L} = \text{Cl}_{\bar{X}}(e_X L)$.

Now let \mathcal{L} be a countably productive separating base on X and let $\eta(X, \mathcal{L}) = \{\sigma \in \bar{X}: \sigma \text{ is an } \omega\text{-contiguity cluster}\}$. $\eta(X, \mathcal{L})$ becomes a topological space by taking it to be a subspace of \bar{X} . For each $L \subset X$, let $L^* = \bar{L} \cap \eta(X, \mathcal{L})$. Then $\mathcal{L}^* = \{L^*: L \in \mathcal{L}\}$ is a countably productive separating base on $\eta(X, \mathcal{L}) = X^*$ and for each $L \subset X$, $L^* = \text{Cl}_{X^*}(e_X L)$.

We now introduce another notion of contiguity of collections. We write $C'_X(A_i: i \in N)$ iff $\bigcap \{A_i^*: i \in N\} \neq \emptyset$. Observe that $C'_X(A_i: i \in N)$ implies that $C_X(A_i: i \in N)$. We therefore have two associated Lodato proximities (M. S. Gagrut and S. A. Naimpally [5]): $A \delta_X B$ iff $C_X(A, B)$ and $A \delta'_X B$ iff $C'_X(A, B)$. Observe that if $\sigma \in X^*$, then $\{A_i: i \in N\} \subset \sigma$ implies $C'_X(A_i: i \in N)$.

R. A. Alo and H. L. Shapiro [1] say that X is \mathcal{L} -realcompact iff every

\mathcal{L} -ultrafilter with the countable intersection property has a nonempty intersection. It is easy to see that X is \mathcal{L} -realcompact iff for every $\sigma \in X^*$, there exists $x \in X$ such that $\{x\} \in \sigma$, and this condition is equivalent to X being homeomorphic to X^* by the map $e_X: X \rightarrow X^*$. R. A. Alo and H. L. Shapiro [1] proved (and their proof is valid in our more general setting) that X^* is \mathcal{L}^* -realcompact, and they call X^* the \mathcal{L} -realcompactification of X (by a slight abuse of language).

Our principal objective is to find a category for which $\eta(X, \mathcal{L}) = X^*$ becomes an epireflection functor. Thus we now turn to the study of maps $f: X \rightarrow Y$ where X and Y are endowed with countably productive separating bases \mathcal{L}_X and \mathcal{L}_Y respectively. We are interested in conditions which guarantee that f has a continuous extension $g: X^* \rightarrow Y^*$. By the phrase $g: X^* \rightarrow Y^*$ is an extension of $f: X \rightarrow Y$, we mean that $g \circ e_X = e_Y \circ f$.

THEOREM 1. *Let $f: X \rightarrow Y$ and $g: X^* \rightarrow Y^*$ be continuous. Then g extends f iff for each $\sigma \in X^*$, $f[\sigma] \subset g(\sigma)$.*

PROOF. Suppose that g extends f , that $\sigma \in X^*$, and that $L \in \sigma$. Then $\sigma \in L^*$ and $g(\sigma) \in g(L^*) = g(\text{Cl}_{X^*} L) \subset \text{Cl}_{Y^*}(gL) = \text{Cl}_{Y^*}(fL) = (fL)^*$. Hence $fL \in g(\sigma)$.

Conversely, suppose that $f[\sigma] \subset g(\sigma)$ for all $\sigma \in X^*$. Let $x \in X$. Then $\{f(x)\} = f[\{x\}] \in f[e_X(x)] \subset ge_X(x)$. If $A \in ge_X(x)$, then $f(x)\delta_Y A$ and so $A \in e_Y f(x)$. Thus $ge_X(x) \subset e_Y f(x)$ and because of the maximality of clusters, $ge_X(x) = e_Y f(x)$.

A function $f: X \rightarrow Y$ is called an ω -contiguity map iff $C'_X(A_i: i \in N)$ implies $C_Y(fA_i: i \in N)$, or equivalently iff $\bar{C}_Y(B_i: i \in N)$ implies $\bar{C}_X(f^{-1}B_i: i \in N)$.

If $f: X \rightarrow Y$ is an ω -contiguity map, then clearly $f: (X, \delta'_X) \rightarrow (Y, \delta_Y)$ is proximally continuous and therefore is continuous. Those maps which we are able to extend are defined as follows. The concept is a modification of D. Harris' [6] $\mathcal{W}\mathcal{O}$ -maps. If $A \subset X$ and $B \subset Y$, then we will write $A <^f B$ iff for all $H \subset X$, $H\delta_X A$ implies $(fH)\delta_Y B$. If $A_i \subset X$ and $B_j \subset Y$, then we will write $(A_i: i \in N) <^f (B_j: j \in N)$ iff for each $i \in N$ there exists $j \in N$ such that $A_i <^f B_j$. $f: X \rightarrow Y$ is called an ω -map iff whenever $\bar{C}_Y(B_j: j \in N)$ then there exists $(A_i: i \in N)$ with $\bar{C}_X(A_i: i \in N)$ and $(A_i: i \in N) <^f (B_j: j \in N)$. If $f: X \rightarrow Y$ is an ω -map, then we shall let, for each $\sigma \in X^*$, $f_\#(\sigma) = \{E \subset Y: \text{for all } H \in \sigma, (fH)\delta_Y E\}$.

Our fundamental result is the following.

THEOREM 2. *Let $f: X \rightarrow Y$ be an ω -map. Then*

- (1) *f is an ω -contiguity map.*
- (2) *$f_\#: X^* \rightarrow Y^*$ is an ω -map.*
- (3) *If $\sigma \in X^*$ and if $f[\sigma] \subset \sigma' \in Y^*$, then $f_\#(\sigma) = \sigma'$.*
- (4) *$f_\#$ extends f .*

PROOF OF (1). This result follows easily from the fact that if $A <' B$, then $f^{-1}B \subset \text{Cl}_X A$.

PROOF OF (2). First observe that if $H, L \in \sigma \in X^*$, then $\sigma \in H^* \cap L^*$ and so $H\delta'_X L$ and $(fH)\delta_Y(fL)$. This implies that for each $\sigma \in X^*$, $f[\sigma] \subset f_\#(\sigma)$.

In order to show that $f_\#$ sends X^* into Y^* , let $\sigma \in X^*$. Let $\{B_j: j \in N\} \subset f_\#(\sigma)$ and suppose that $\bar{C}_Y(B_j: j \in N)$. Then there exists $(A_i: i \in N)$ with $\bar{C}'_X(A_i: i \in N)$ and $(A_i: i \in N) <' (B_j: j \in N)$. There exists $i \in N$ with $\sigma \notin A_i^*$. $A_i \notin \sigma$ so there exists $H \in \sigma$ with $H\delta'_X A_i$. Let $j \in N$ with $A_i <' B_j$. Then $(fH)\delta_Y B_j$ which contradicts that $B_j \in f_\#(\sigma)$. To show that $f_\#(\sigma)$ is maximal, let $A \subset Y$ such that $A \notin f_\#(\sigma)$. Then for some $H \in \sigma$, $(fH)\delta_Y A$. Since $f[\sigma] \subset f_\#(\sigma)$, then $fH \in f_\#(\sigma)$ and so $f_\#(\sigma)$ is maximal and $f_\#(\sigma) \in Y^*$.

Now, to show that $f_\#: X^* \rightarrow Y^*$ is an ω -map, let $\bar{C}_Y(B_j: j \in N)$. Then there exists $\{L_j: j \in N\} \subset \mathcal{L}_Y$ with $B_j \subset L_j^*$ and $\bigcap \{L_j^*: j \in N\} = \emptyset$. So $\bar{C}_Y(L_j: j \in N)$ and since $f: X \rightarrow Y$ is an ω -map, for some $(A_i: i \in N)$, $\bar{C}'_X(A_i: i \in N)$ and $(A_i: i \in N) <' (L_j: j \in N)$. Then $\bigcap \{A_i^*: i \in N\} = \emptyset$ and since $e_{X^*}: X^* \rightarrow X^{**}$ is a homeomorphism, then $A_i^{**} = \text{Cl}_{X^{**}}(e_{X^*} A_i^*) = e_{X^*} A_i^*$ and $\bigcap \{A_i^{**}: i \in N\} = \emptyset$. Therefore $\bar{C}'_{X^*}(A_i^*: i \in N)$ and it suffices to show that $(A_i^*: i \in N) <_{f_\#} (B_j: j \in N)$. Let $A_i <' L_j$. To show that $A_i^* <_{f_\#} B_j$, let $P\delta'_X A_i^*$. For some $H, K \in \mathcal{L}_X$, $P \subset H^*$, $A_i^* \subset K^*$ and $H \cap K = \emptyset$. Then $H\delta'_X A_i$ and so $(fH)\delta_Y L_j$. There exists $J \in \mathcal{L}_Y$ with $fH \subset J$ and $J \cap L_j = \emptyset$. Then $A_i \subset L_j^*$ and $f_\# P \subset J^*$ and so $(f_\# P)\delta_Y A_i$ as required.

PROOF OF (3). Let $\sigma \in X^*$ and $f[\sigma] \subset \sigma' \in Y^*$. Let $E \in f_\#(\sigma)$ and suppose that $E \notin \sigma'$. Then there exists $B \in \sigma'$ with $B\delta'_Y E$. Thus, $B \notin f_\#(\sigma)$ and so there exists $H \in \sigma$ with $B\delta'_Y(fH)$. This contradicts $fH \in f[\sigma] \subset \sigma'$.

PROOF OF (4). By (1) and (2) both f and $f_\#$ are continuous. Therefore by Theorem 1 and the observation at the beginning of the proof of (2) above, $f_\#$ extends f .

An immediate consequence of (3) of Theorem 2 is the following result.

THEOREM 3. *If an ω -map $f: X \rightarrow Y$ has a continuous extension $g: X^* \rightarrow Y^*$, then $g = f_\#$.*

We do not know whether the composition of two ω -maps is always an ω -map. In order to get a category, we modify the concept as follows. $f: X \rightarrow Y$ is called a *reduced ω -map* iff whenever $\bar{C}_Y(B_j: j \in N)$ then there exists $(A_i: i \in N)$ with $\bar{C}'_X(A_i: i \in N)$ and $(A_i: i \in N) <' (B_j: j \in N)$. Obviously each reduced ω -map is also an ω -map. Clearly the composition of reduced ω -maps is again a reduced ω -map. Thus, T_1 -spaces endowed with countably productive separating bases together with reduced ω -maps form a category \mathcal{A} . It is obvious that the embedding $e_X: X \rightarrow X^*$ is a reduced ω -map.

THEOREM 4. *The embedding $e_X: X \rightarrow X^*$ is an epimorphism in the category \mathcal{A} .*

PROOF. Let Y be a T_1 -space with a countably productive separating base \mathcal{L}_Y and let $g, h: X^* \rightarrow Y$ be reduced ω -maps such that $g \circ e_X = h \circ e_X$. We must show that $g = h$. Let $f = g \circ e_X$. Then $f: X \rightarrow Y$ is a reduced ω -map and since $(e_Y \circ h) \circ e_X = e_Y \circ f = (e_Y \circ g) \circ e_X$, then both of $e_Y \circ h$ and $e_Y \circ g$ are continuous extensions of f . By Theorem 3, $e_Y \circ h = f_{\#} = e_Y \circ g$ and since e_Y is injective then $h = g$.

The following theorem follows directly from Theorems 2, 3, and 4. Extend the function η to all of the category \mathcal{A} by defining $\eta(f) = f_{\#}$ for each reduced ω -map f .

THEOREM 5. *$\eta: \mathcal{A} \rightarrow \mathcal{A}$ is an epireflection functor with the maps $e_X: X \rightarrow X^*$ as universal arrows.*

We now turn to the case where Y is Tychonoff and is endowed with a nest generated intersection ring \mathcal{L}_Y . In this case we have a fairly simple characterization of ω -maps and at the same time, several necessary and sufficient conditions for extendibility.

THEOREM 6. *Let \mathcal{L}_X be a countably productive separating base on X and let \mathcal{L}_Y be a nest generated intersection ring on Y . Let $f: X \rightarrow Y$ be a function. Then the following are equivalent:*

- (1) $f: X \rightarrow Y$ is an ω -map.
- (2) $f: X \rightarrow Y$ is an ω -contiguity map.
- (3) $f: (X, \delta'_X) \rightarrow (Y, \delta_Y)$ is proximally continuous.
- (4) $f: (X, \delta'_X) \rightarrow (Y, \delta'_Y)$ is proximally continuous.
- (5) f has a continuous extension $g: X^* \rightarrow Y^*$.
- (6) f has a continuous extension $g: X^* \rightarrow Y^*$ and $g: X^* \rightarrow Y^*$ is an ω -map.

PROOF. To show that (3) implies (2), assume (3) and let $\bar{C}_Y(B_j: j \in N)$. Then there exist $L_j \in \mathcal{L}_Y$ with $B_j \subset L_j$ and $\bigcap \{L_j: j \in N\} = \emptyset$. There exist $E_{j_n}, F_{j_n} \in \mathcal{L}_Y$ with $F_{j_{n+1}} \subset Y - E_{j_n} \subset F_{j_n}$ and $L_j = \bigcap \{F_{j_n}: n \in N\}$. Let $P_{j_n} = f^{-1}F_{j_{n+1}}$ and $Q_{j_n} = X - f^{-1}F_{j_n}$. Since $F_{j_{n+1}} \delta_Y (Y - f^{-1}F_{j_n})$, then $P_{j_n} \delta'_X Q_{j_n}$. We now proceed by contradiction. Suppose $C_X(f^{-1}B_j: j \in N)$ and let $\sigma \in \bigcap \{(f^{-1}B_j)^*: j \in N\}$. Since $X = \bigcup \{Q_{j_n}: j \in N \text{ and } n \in N\}$ and σ is maximal, there exist $j, n \in N$ with $Q_{j_n} \in \sigma$. Then $P_{j_n} \notin \sigma$ which contradicts that $f^{-1}B_j \in \sigma$ and that $f^{-1}B_j \subset f^{-1}L_j \subset P_{j_n}$.

To show that (2) implies (1), assume (2) and let $\bar{C}_Y(B_j: j \in N)$. Then there exist $L_j \in \mathcal{L}_Y$ with $B_j \subset L_j$ and $\bigcap \{L_j: j \in N\} = \emptyset$. Next, there exist $E_{j_n}, F_{j_n} \in \mathcal{L}_Y$ with $F_{j_{n+1}} \subset Y - E_{j_n} \subset F_{j_n}$ and $L_j = \bigcap \{F_{j_n}: n \in N\}$. Then $\bigcap \{F_{j_n}: j \in N \text{ and } n \in N\} = \emptyset$ so $\bar{C}_Y(F_{j_n}: j \in N \text{ and } n \in N)$ and therefore $\bar{C}'_X(f^{-1}F_{j_n}: j \in N \text{ and } n \in N)$. Let $j, n \in N$. Claim that $f^{-1}F_{j_n} \subset' B_j$.

Let $H\delta_X(f^{-1}F_{jn})$. Then $H \cap f^{-1}F_{jn} = \emptyset$ and $(fH) \cap F_{jn} = \emptyset$. Therefore $fH \subset E_{jn}$, $B_j \subset L_j \subset F_{j,n+1}$ and $E_{jn} \cap F_{j,n+1} = \emptyset$. This shows that $(fH)\delta_Y B_j$.

That (1) implies (6) is the content of Theorem 2. That (6) implies (5) is a triviality. That (5) implies (4) is well known and easy to prove. That (4) implies (3) is obvious.

The preceding theorem (with Theorem 2) implies that Tychonoff spaces endowed with nest generated intersection rings together with ω -maps form a category \mathbf{B} and $\eta: \mathbf{B} \rightarrow \mathbf{B}$ is an epireflection functor.

We remark that a sufficient but not necessary condition for the existence of a continuous extension $g: \eta(X, \mathcal{L}_X) \rightarrow \eta(Y, \mathcal{L}_Y)$ of a continuous map $f: X \rightarrow Y$ has been discovered by D'Aristotle [3]. Theorem 6 above thus represents an improvement.

We will now illustrate the utility of Theorem 6 with a simple example. Let X be a T_1 -space and let \mathcal{L} be the collection of all closed subsets of X . Then \mathcal{L} is obviously a countably productive separating base on X , and so we have an extension $rX = \eta(X, \mathcal{L})$ which is a subspace of the Wallman compactification wX of X . By an easy application of Theorem 6, rX has the property that every real valued continuous map $f: X \rightarrow \mathbf{R}$ has a unique continuous extension $f_\#: rX \rightarrow \mathbf{R}$. Since $r\mathbf{R} = v\mathbf{R}$, the Hewitt realcompactification of \mathbf{R} , and $v\mathbf{R} = \mathbf{R}$, then every continuous map $f: X \rightarrow \mathbf{R}$ has a continuous extension $g: rX \rightarrow \mathbf{R}$, i.e. X is C -embedded in rX . We have been unable to answer the following question: Is rX the largest subspace of wX in which X is C -embedded? If X is a normal space, then $wX = \beta X$ and so it follows from the definitions (as an easy exercise) that in this case $rX = vX$, the Hewitt realcompactification of X .

The following theorem is an exact analogue of D. Harris' theorem on the Wallman compactification being a functor [6]; it is a corollary to Theorem 2 above.

THEOREM 7. *The correspondence $X \rightarrow rX, f \rightarrow f_\#$ is an epireflection functor on the category of T_1 -spaces and reduced ω -maps (with respect to the collections $\mathcal{L}_X, \mathcal{L}_Y$ of all closed subsets of X, Y respectively).*

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