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THE COMPACT 3-MANIFOLDS COVERED BY $S^2 \times R^1$

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ABSTRACT. The classification of all free actions by a finite group on $S^2 \times S^1$ follows from the observation that there exist only four compact 3-manifolds which have $S^2 \times R^1$ for a universal covering space.

Theorem. *If $S^2 \times R^1$ is a covering space of the compact 3-manifold M then M is homeomorphic to $S^2 \times S^1$, N, $P^2 \times S^1$, or $P^3 \# P^3$ (N denotes the nonorientable 2-sphere bundle over S^1 and P^n denotes real projective n-space).*

Proof. We assume all spaces, subspaces and maps are PL. Let p: $S^2 \times R^1 \to M$ be a covering space projection onto the compact space M. Since $\pi_2(M) \neq 0$, M contains either a noncontractible 2-sphere or a two-sided projective plane [1], say $F \subset M$. Let U denote a regular neighborhood of F in M. Each component of $S^2 \times R^1 - p^{-1}(\mathrm{Int}(U))$ is homeomorphic to $S^2 \times [0, 1]$ and covers a component of $M - \mathrm{Int}(U)$. Thus, each component of $M - \mathrm{Int}(U)$ is either homeomorphic to $S^2 \times [0, 1]$ or is double-covered by $S^2 \times [0, 1]$. In the latter case, [2] and [3] can be applied (by capping the boundaries of $S^2 \times [0, 1]$ with 3-cells to obtain S^3) to see that $S^2 \times [0, 1]$ double-covers only $P^2 \times [0, 1]$ and $P^3 - \{\text{open 3-cell}\}$. It is now easily seen that M must be homeomorphic to $S^2 \times S^1$, N, $P^2 \times S^1$ or $P^3 \# P^3$.

Corollary 1. *$S^2 \times S^1$ is a covering space of only $S^2 \times S^1$, N, $P^2 \times S^1$, and $P^3 \# P^3$.*

Corollary 2. *Suppose G is a finite group acting freely on $S^2 \times S^1$. Let M denote the orbit space of G. Then*

(i) *$G \cong Z_p$ and $M \approx S^2 \times S^1$ (for p odd), $M \approx S^2 \times S^1$, N, or $P^2 \times S^1$ (for p even); or*

(ii) *$G \cong Z_p \times Z_2$ (p even) and $M \approx P^2 \times S^1$; or*

(iii) *$G \cong D_n$, the dihedral group of order $2n$ ($n \geq 1$), and $M \approx P^3 \# P^3$.*

Corollary 3. *The 3-manifolds $P^2 \times S^1$ and $P^3 \# P^3$ may cover only them-*

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selves and N may cover only itself and $P^2 \times S^1$.

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