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REPRESENTATIONS OF ALMOST CONNECTED GROUPS

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ABSTRACT. A simple proof is given that an irreducible representation of an almost connected group on a quasi-complete locally convex space is essentially a Lie group representation.

Let G be a locally compact group and G_0 the connected component of the identity in G. The group G is called almost connected if G/G_0 is compact.

R. L. Lipsman has proved in [2] that an irreducible continuous unitary representation of an almost connected group is essentially a Lie group representation. In the present note we give a simple proof of this result holding also in a more general situation.

Theorem. *Let U be a continuous topologically irreducible representation of an almost connected group G on a quasi-complete locally convex space E. Then $G/\ker U$ is a Lie group.*

Proof. Let $\mathfrak{L}(G)$ be the set of all compact normal subgroups H of G such that G/H is a Lie group. Then $\mathfrak{L}(G)$ is directed by inclusion [3, p. 177], and for every neighborhood V of the identity in G there exists an $H \in \mathfrak{L}(G)$ such that $H \subset V$ [3, p. 175].

For every $H \in \mathfrak{L}(G)$ we denote by μ_H the normalised Haar measure on H. By an elementary result [1, Chapter VIII, §2, Corollary 2 of Lemma 4], $\lim U(\mu_H)x = x$ holds true for every $x \in E$. Hence there exists an $H \in \mathfrak{L}(G)$ such that $U(\mu_H) \neq 0$. Obviously, the subgroup H being compact and normal, $U(\mu_H)\,U(a) = U(a)\,U(\mu_H)$ for every $a \in G$; by the irreducibility of U the continuous projection $U(\mu_H)$ is equal to the identity. Now $U(a) = U(a)U(\mu_H) = U(\mu_H) = 1_E$ for every $a \in H$; hence, H is contained in $\ker U$ and this suffices to prove the Theorem.

The same reasoning yields the result also for factorial representations of G on a Hilbert space.

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REFERENCES

1. N. Bourbaki, *Éléments de mathématique*. Livre VI: *Intégration*. Chaps. I–IX. Actualités Sci. Indust., Nos. 1175, 1343, Hermann, Paris, 1952, 1969. MR **14**, 960; 43 #2183.

2. R. Lipsman, *Representation theory of almost connected groups*, Pacific J. Math. **42** (1972), 453–467.

3. D. Montgomery and L. Zippin, *Topological transformation groups*, Interscience, New York, 1955. MR **17**, 383.

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