PROCEEDINGS OF THE
AMERICAN MATHEMATICAL SOCIETY
Volume 50, July 1975

MALCEV ALGEBRAS WITH J_2-POTENT RADICAL

ERNEST L. STITZINGER

ABSTRACT. Let A be a Malcev algebra, B be an ideal of A and $J_2^1(B) = J(B, A, A)$ where $J(B, A, A)$ is the linear subspace of A spanned by all elements of the form $J(x, y, z) = (xy)z + (yz)x + (zx)y$, $x \in B$, $y, z \in A$. For $k \geq 1$, define $J_2^{k+1}(B) = J(J_2^k(B), A, A)$. Then B is called J_2-potent if there exists an integer $N \geq 1$ such that $J_2^N(B) = 0$. Now let A be a Malcev algebra over a field of characteristic 0 such that the radical R of A is J_2-potent. Then R is complemented by a semisimple subalgebra and all such complements are strictly conjugate in A. The proofs follow those in the Lie algebra case.

In recent years the theory of Malcev algebras has greatly advanced. However, the status of the Wedderburn principal theorem (Levi theorem) and accompanying Malcev-Harish-Chandra theorem does not appear to have been settled. The following special case, when the radical is J_2-potent, would seem to be of interest. In this situation the treatment is much like the Lie algebra case. All Malcev algebras and all modules are assumed finite dimensional over a field of characteristic 0.

We recall the following terminology. Let A be a Malcev algebra and define R_x to be right multiplication by x. For $x, y, z \in A$, let

$$J(x, y, z) = (xy)z + (yz)x + (zx)y = z(-R_{xy} + [R_x, R_y]).$$

For $x, y \in A$, let

$$\Delta(x, y) = [R_x, R_y] - R_{xy} \quad \text{and} \quad N(A) = \{z \in A;\ z\Delta(x, y) = 0\ \forall x, y \in A\}.$$

$N(A)$ is called the J-nucleus of A and is an ideal of A. Also let

$$D(x, y) = [R_x, R_y] + R_{xy}$$

and $D(x, y)$ is a derivation of A. If $B \trianglelefteq A$, define

$$J_2^1(B) = J(B, A, A) \quad \text{and} \quad J_2^{k+1}(B) = J(J_2^k(B), A, A), \qquad k \geq 1.$$

Then B is called J_2-potent if there exists an integer $N \geq 1$ such that

Received by the editors March 12, 1974 and, in revised form, April 29, 1974.
AMS (MOS) subject classifications (1970). Primary 17A30, 17B05.

$J_2^N(B) = 0$. Note that we have slightly altered the definition of J_2-potent (see [6, p. 444]). We prove the following.

Theorem 1. *Let A be a Malcev algebra over a field of characteristic 0. Suppose that the radical R of A is J_2-potent. Then there exists a semi-simple subalgebra S of A such that $A = R \oplus S$.*

The usual way of showing the Wedderburn principal theorem is to prove the case when the radical R is a minimal ideal of A such that $R^2 = 0$, and then the general case follows by a standard argument [1, p. 87] provided that one can obtain in the general case an ideal B of A such that B is properly contained in R and $(R/B)^2 = 0$. For Malcev algebras this approach is possible since $R^2 + J(R, R, A)$ is an ideal of A properly contained in R (provided $R \neq 0$) by [5, Theorem 1, p. 228]. Hence it suffices to prove our result holds in the case that R is a minimal ideal of A. Furthermore, if R is J_2-potent in A and is a minimal ideal in A, then $J(R, A, A)$ is an ideal of A [6, Theorem 3.5] and is properly contained in R, hence $J(R, A, A) = 0$ and $R \subseteq N(A)$. Then the natural representation of A on R is a homomorphism, i.e., $(b)[R_x, R_y] = (b)R_{xy}$ for all $b \in R$, $x, y \in A$. Since $R^2 = 0$, we consider R as an $\bar{A} = A/R$-module and the associated representation is still a homomorphism. Since $(\bar{A})^2 = \bar{A}$, A^2 is a supplement of R in A and if A^2 is properly contained in A, then A^2 is a complementary subalgebra of R and the result holds in this case. Hence we may assume that $A^2 = A$. Summarizing this paragraph we may consider the case when

(1) R is a minimal ideal of A,

(2) $(b)[R_x, R_y] = bR_{xy}$ for all $b \in R$, $x, y \in A$,

(3) $A^2 = A$.

We first consider the critical case when $\bar{A}$ is Lie. Then $A\Delta(x, y) \subseteq R$ and $R\Delta(x, y) = 0$. Consequently

$$\Delta(x, y)\Delta(u, v) = 0 = \Delta(x\Delta(u, v), y) \tag{4}$$

holds in A.

Following the ideas of [6], one obtains the following identity for any Malcev algebra

$$[\Delta(x, y), \Delta(u, v)] = \Delta(x\Delta(u, v), y) + \Delta(x, y\Delta(u, v)) + 6R_{J(x,y,uv)} - 6\Delta(uv, xy) \tag{5}$$

as follows:

$$\begin{aligned}[\Delta(x, y), \Delta(u, v)] &= [\Delta(x, y), D(u, v)] - 2[\Delta(x, y), R_{uv}] \\ &= \Delta(xD(u, v), y) + \Delta(x, yD(u, v)) - 3\Delta(uv, xy) \\ &\quad + \Delta(x, y(uv)) + \Delta(y, (uv)x)\end{aligned}$$

by the proof of [6, Proposition 8.14, p. 454] and [6, 2.35, p. 432]. Then

$$\begin{aligned}\Delta(xD(u, v), y) + \Delta(x, yD(u, v)) &= \Delta(x\Delta(u, v), y) + 2\Delta(x(uv), y) \\ &\quad + \Delta(x, y\Delta(u, v)) + 2\Delta(x, y(uv)).\end{aligned}$$

Substitution then gives

$$\begin{aligned}[\Delta(x, y), \Delta(u, v)] &= \Delta(x\Delta(u, v), y) + \Delta(x, y\Delta(u, v)) \\ &\quad - 3\Delta(uv, xy) + 3\Delta(x, y(uv)) + 3\Delta(y, (uv)x).\end{aligned}$$

Using [6, 2.32, p. 432] on the last two terms gives (5). Now for the algebra under consideration, from (4) and (5) we obtain the identity

$$R_{J(x,y,uv)} = \Delta(uv, xy). \tag{6}$$

Since $A^2 = A$, (6) yields the identity

$$tJ(x, y, z) = t\Delta(z, xy) = J(t, z, xy) \tag{7}$$

which holds in A.

We have the usual criterion for R to have a complementary subalgebra in A [1, pp. 86–89]. That is, let σ be a linear map from $\overline{A}$ into A such that $\overline{a^{\sigma}} = \overline{a}$ for all $a \in A$ and define

$$g(\overline{b}, \overline{c}) = \overline{b}^{\sigma}\overline{c}^{\sigma} - (\overline{b}\overline{c})^{\sigma} \in R \tag{8}$$

for all $\overline{b}, \overline{c} \in \overline{A}$.

Since $R^2 = 0$, R is an A-module under the product $r\overline{a} = r\overline{a}^{\sigma}$ and because of (2), the associated representation is a homomorphism. Then R has a complementary subalgebra if and only if there exists a linear mapping ρ of $\overline{A}$ into R such that

$$g(\overline{b}, \overline{c}) = \overline{b}^{\rho}\overline{c} - \overline{c}^{\rho}\overline{b} - (\overline{b}\overline{c})^{\rho}. \tag{9}$$

We collect some properties of g. Since A is antisymmetric,

$$g(\overline{b}, \overline{b}) = 0 \tag{10}$$

which yields $g(\overline{b}, \overline{c}) = -g(\overline{c}, \overline{b})$. Next write

$$\overline{b}_1^{\sigma}\overline{b}_2^{\sigma} = (\overline{b}_1\overline{b}_2)^{\sigma} + g(\overline{b}_1, \overline{b}_2)$$

and compute

$$\begin{aligned}(\overline{b}_1^{\sigma}\overline{b}_2^{\sigma})\overline{b}_3^{\sigma} &= (\overline{b}_1\overline{b}_2)^{\sigma}\overline{b}_3^{\sigma} + g(\overline{b}_1, \overline{b}_2)\overline{b}_3^{\sigma} \\ &= ((\overline{b}_1\overline{b}_2)\overline{b}_3)^{\sigma} + g(\overline{b}_1\overline{b}_2, \overline{b}_3) + g(\overline{b}_1, \overline{b}_2)\overline{b}_3^{\sigma}.\end{aligned}$$

Permute b_1, b_2, b_3 cyclically, add and make use of the Jacobi identity in $\bar{A}$ to obtain

$$(11)\qquad \begin{aligned} J(\bar{b}_1^\sigma, \bar{b}_2^\sigma, \bar{b}_3^\sigma) &= g(\bar{b}_1\bar{b}_2, \bar{b}_3) + g(\bar{b}_1, \bar{b}_2)\bar{b}_3^\sigma + g(\bar{b}_2\bar{b}_3, \bar{b}_1) \\ &\quad + g(\bar{b}_2, \bar{b}_3)\bar{b}_1^\sigma + g(\bar{b}_3\bar{b}_1, \bar{b}_2) + g(\bar{b}_3, \bar{b}_1)\bar{b}_2^\sigma. \end{aligned}$$

Since $\bar{b}^\sigma\bar{c}^\sigma - (\bar{b}\bar{c})^\sigma \in R$ and $J(R, A, A) = 0$, (7) yields

$$(12)\qquad J(\bar{c}^\sigma, \bar{d}^\sigma, \bar{b}^\sigma)\bar{a} = J(\bar{c}^\sigma, \bar{d}^\sigma, \bar{b}^\sigma)\bar{a}^\sigma = -J(\bar{a}^\sigma, \bar{b}^\sigma, (\bar{c}\bar{d})^\sigma).$$

Also [6, 2.14, p. 429] yields

$$(13)\qquad \begin{aligned} -2J(\bar{a}^\sigma, \bar{b}^\sigma, \bar{c}^\sigma)\bar{d} &= J(\bar{d}^\sigma, \bar{a}^\sigma, (\bar{b}\bar{c})^\sigma) \\ &\quad + J(\bar{d}^\sigma, \bar{b}^\sigma, (\bar{c}\bar{a})^\sigma) + J(\bar{d}^\sigma, \bar{c}^\sigma, (\bar{a}\bar{b})^\sigma). \end{aligned}$$

Now J can be used to define a trilinear mapping, which we also denote by J, from $\bar{A}$ into R by $J(\bar{a}, \bar{b}, \bar{c}) = J(\bar{a}^\sigma, \bar{b}^\sigma, \bar{c}^\sigma)$ and (10), (11), (12) and (13) all hold. To complete the proof of this case we show a slight extension of the second Whitehead lemma.

Lemma 1. *Let L be a semisimple Lie algebra over a field of characteristic 0 and let M be a finite dimensional (Lie) L-module. Let $(x_1, x_2) \to g(x_1, x_2)$ be a bilinear mapping of $L \times L \to M$ such that*

$$(14)\qquad g(x, x) = 0,$$

$$(15)\qquad \begin{aligned} g(x_1x_2, x_3) + g(x_1, x_2)x_3 + g(x_2x_3, x_1) + g(x_2, x_3)x_1 \\ + g(x_3x_1, x_2) + g(x_3, x_1)x_2 = J(x_1, x_2, x_3) \end{aligned}$$

where

$$(16)\qquad -2J(x_1, x_2, x_3)x_4 = J(x_4, x_1, x_2x_3) + J(x_4, x_2, x_3x_1) + J(x_4, x_3, x_1x_2)$$

and

$$(17)\qquad J(x_1, x_2, x_3x_4) = -J(x_3, x_4, x_2)x_1.$$

Then there exists a linear mapping $x \to x^\rho$ of L into M such that

$$(18)\qquad g(x_1, x_2) = x_1^\rho x_2 - x_2^\rho x_1 - (x_1x_2)^\rho.$$

Proof. We use the machinery developed in the proofs of the Whitehead lemmas [1]. Let K be the kernel of the induced representation S and let L_1 be a complementary ideal of K in L. Then the restriction of S to L_1 is one-to-one and the induced trace form is nondegenerate on L_1. Let (u_i) and (u^i) be complementary bases of L_1 with respect to the trace form. Then if $u_i a = \Sigma_j \alpha_{ij} u_j$ and $u^k a = \Sigma_m \beta_{km} u^m$, it follows that $\alpha_{ik} = -\beta_{ki}$ since the trace form is invariant. Let Γ be the Casimir operator on M; that is, $\Gamma = \Sigma S_{u_i} S_{u^i}$, and recall that Γ commutes with each S_a, $a \in L$, and that $\operatorname{tr} \Gamma = \Sigma \operatorname{tr} S_{u_i} S_{u^i} = \dim L_1$ [1, p. 78]. As in the proof in [1, p. 89] set $x_3 = u_i$ in (15), take the module product with respect to u^i and add on i. This gives

$$\sum J(x_1, x_2, u_i)u^i = \sum g(x_1x_2, u_i)u^i + g(x_1, x_2)\Gamma + \sum g(x_2u_i, x_1)u^i$$
$$+ \sum (g(x_2, u_i)x_1)u^i + \sum g(u_ix_1, x_2)u^i + \sum (g(u_i, x_1)x_2)u^i.$$

Then, since S is Lie,

$$\sum J(x_1, x_2, u_i)u^i = g(x_1, x_2)\Gamma + \sum g(x_1x_2, u_i)u^i + \sum g(x_2u_i, x_1)u^i$$
$$+ \sum g(x_2, u_i)(x_1u^i) + \sum (g(x_2, u_i)u^i)x_1$$
$$+ \sum g(u_ix_1, x_2)u^i + \sum g(u_i, x_1)(x_2u^i) + \sum (g(u_i, x_1)u^i)x_2.$$

Using the above relation between $u_i a$ and $u^k a$ we compute that

$$\sum g(x_2, u_i)(x_1u^i) = \sum g(x_2, u_ix_1)u^i, \qquad \sum g(u_i, x_1)(x_2u^i) = \sum g(u_ix_2, x_1)u^i,$$

$$\sum J(u^i, x_1, x_2u_i) = -\sum J(x_2u^i, x_1, u_i),$$

and

$$\sum J(u^i, x_2, u_ix_1) = -\sum J(u^ix_1, x_2, u_i).$$

Then

$$-g(x_1, x_2)\Gamma = \sum g(x_1x_2, u_i)u^i + \sum (g(x_2, u_i)u^i)x_1$$
$$+ \sum (g(u_i, x_1)u^i)x_2 - \sum J(x_1, x_2, u_i)u^i$$

and

$$-2\sum J(x_1, x_2, u_i)u^i$$
$$= \sum J(u^i, x_1, x_2u_i) + \sum J(u^i, x_2, u_ix_1) + \sum J(u^i, u_i, x_1x_2)$$
$$= -\sum J(x_2u^i, x_1, u_i) - \sum J(u^ix_1, x_2, u_i) + \sum J(u^i, u_i, x_1x_2)$$
$$= -\sum J(x_1, u_i, x_2u^i) - \sum J(x_2, u_i, u^ix_1) + \sum J(u^i, u_i, x_1x_2)$$
$$= \sum J(x_2, u^i, u_i)x_1 + \sum J(u^i, x_1, u_i)x_2 + \sum J(u^i, u_i, x_1x_2)$$
$$= \sum J(u^i, u_i, x_2)x_1 - \sum J(u^i, u_i, x_1)x_2 + \sum J(u^i, u_i, x_1x_2).$$

Hence

$$-g(x_1, x_2)\Gamma = \sum g(x_1x_2, u_i)u^i + \sum (g(x_2, u_i)u^i)x_1 + \sum (g(u_i, x_1)u^i)x_2$$
$$+ \frac{1}{2}\sum J(u^i, u_i, x_1x_2) + \frac{1}{2}\sum J(u^i, u_i, x_2)x_1 - \frac{1}{2}\sum J(u^i, u_i, x_1)x_2.$$

If Γ is nonsingular, define

$$x^{\rho} = \sum (g(x, u_i)u^i + \tfrac{1}{2} J(u^i, u_i, x))\Gamma^{-1}.$$

Then ρ is the desired mapping in (18).

Suppose Γ is nilpotent. Then $0 = \operatorname{tr} \Gamma = \dim L_1$ and $\Gamma = 0$. Hence $ML = 0$. By (17) $J(x, y, z) = 0$ and we have the classical case of the second Whitehead lemma, hence the result holds in this case. In the general case, decompose M as the direct sum of the Fitting null and one components of M with respect to Γ. These spaces are submodules and the conditions carry over to them. The conclusion holds on each of the submodules and hence on M by adding the linear transformations obtained on the submodules.

Now we have the general result in the case $\overline{A} \cong A/R$ is Lie. Suppose now that $\overline{A}$ is not necessarily Lie and that R is again a minimal ideal in A. Then $\overline{A} \cong N(\overline{A}) \oplus J(\overline{A}, \overline{A}, \overline{A})$. Let E be the ideal of A containing R such that $N(\overline{A}) \cong E/R$. R is the radical of E and E/R is Lie. Then R is complemented in E by a subalgebra T. Then

$$J(E, E, E) \subseteq J(T, T, T) + J(T, T, R) + J(T, R, R) + J(R, R, R) = 0.$$

Hence E is Lie and $A = J(A, A, A) + N(A)$. Either $R \cap J(A, A, A) = 0$ or $R \subseteq J(A, A, A)$. In the first case $J(A, A, A) \oplus T$ is a complementary subalgebra of R since $J(A, A, A)T \subseteq J(A, A, A)N(A) = 0$. In the second case $R \subseteq J(A, A, A) \cap N(A)$ and $J(A, A, A)R = 0$. Then $J(A, A, A)$ may be considered as a $J(A, A, A)/R$-module and since $J(A, A, A)/R$ is semisimple, $J(A, A, A)$ is completely reducible under $J(A, A, A)/R$ by [2, Corollary 8, p. 244]. Hence there exists a complementary subalgebra D of R in $J(A, A, A)$. Since $DT \subseteq J(A, A, A)N(A) = 0$, $D + T$ is a complementary subalgebra to R in A. This completes the proof when R is a minimal ideal of A and the result follows as in the paragraph after the statement of Theorem 1.

We turn to the conjugacy of the subalgebras complementary to the radical. Suppose that A is a semisimple Malcev algebra of characteristic 0 and M is an A-module whose associated representation S is Lie; that is, $S_{xy} = [S_x, S_y]$ for all $x, y \in A$. Form the Malcev algebra $A + M = B$ with the natural product. Then M is the radical of B. Form the Lie triple system T_B associated with B by defining the composition

(19) $$[x, y, z] = 2(xy)z - (yz)x - (zx)y = z(-2R_{xy} - [R_x, R_y])$$

in the vector space B (see [4, p. 554]) and let

(20) $$R(x, y) = -2R_{xy} - [R_x, R_y].$$

Then the radical of T_B coincides with the radical of B [4, Satz 2, p. 557]. Let D be a derivation of A into M and then D is a derivation of T_A into T_B. By [3, Theorem 2.18, p. 225] there exist $x_i, y_i \in B$ such that $\Sigma R(x_i, y_i)$ is a derivation of T_B which extends D. Since D: $T_A \to T_M$, when restricted to A, $D = \Sigma R(x_i, y_i)$ where $x_i \in M$, $y_i \in A$. Since the representation S is Lie, $R_{xy} = [R_x, R_y]$ for each $x \in M$, $y \in A$. Hence

$$R(x, y) = -R_{xy} - D(x, y) = -(3/2)D(x, y)$$

and D is the sum of derivations $D(x, y)$, $x \in M$, $y \in A$. Formally this shows the following.

Lemma 2. *Let A be a semisimple Malcev algebra over a field of characteristic 0 and let M be an A-module whose associated representation is Lie. Let D be a derivation of A into M. Then there exist $x_i \in M$, $y_i \in A$ such that $D = \Sigma D(x_i, y_i)$.*

Remark. In the lemma we are considering $D(x_i, y_i)$ as defined in the second paragraph of this paper and considering M as a 2-sided A-module.

As usual we say that two subalgebras of A are strictly conjugate if one is mapped onto the other by an automorphism of the form $G_1 \cdots G_k$ where $G_i = \exp D_i$ and D_i is a nilpotent derivation. Recall that $\exp D \exp D'$ is of the form $\exp D''$ where D'' is a nilpotent derivation by the Campbell-Hausdorff formula. Finally we note if $y \in A$ and $x \in$ nilradical of A, then $D(x, y)$ is in the radical of the multiplication algebra of A [2, Theorem 2, p. 233].

Theorem 2. *Let A be a Malcev algebra over a field of characteristic 0 whose radical R is J_2-potent. Let L_1 be a semisimple subalgebra of A and let L be a semisimple subalgebra of A such that $A = L \oplus R$. Then L_1 is strictly conjugate to a subalgebra of L by an automorphism $G = \exp D$ where D is in the radical of the multiplication algebra of A.*

Proof. The proof follows along the usual lines for the Malcev-Harish-

Chandra theorem. Let $x \in L_1$. Then x can be written uniquely as $x = x^\lambda + x^\sigma$ where $x^\lambda \in L$ and $x^\sigma \in R$. Hence λ and σ are linear maps of L_1 into L and R, respectively, and λ is one-to-one. If $y \in L_1$, then

$$(xy)^\lambda = x^\lambda y^\lambda \tag{21}$$

and

$$(xy)^\sigma = x^\sigma y - y^\sigma x + x^\sigma y^\sigma. \tag{22}$$

Hence $(xy)^\sigma \in RA \subseteq N$, where N is the nilradical of A, by [2, Corollary 3, p. 235]. Since $L_1 L_1 = L_1$, $x^\sigma \in N$ for each $x \in L_1$ and $L_1 \subseteq L \oplus N$. Note that any minimal ideal D of A contained in N satisfies $D^2 + J(D, A, A) = 0$ since D is J_2-potent and since any minimal ideal of A contained in R is abelian [5, Theorem 1, p. 228]. Now construct a chain of ideals of A,

$$N = A_1 \supset A_2 \supset \cdots \supset A_k = 0,$$

such that A_i/A_{i+1} is a minimal ideal in A/A_{i+1} and $A_i A_i + J(A_i, A, A) \subseteq A_{i+1}$. Then the natural representation of A on A_i/A_{i+1} is Lie. We prove by induction that there exists an automorphism E_i such that $L_1 E_i \subseteq L + A_i$ for $i = 1, \cdots, k$. For $i = 1$, let E_1 = identity and it suffices to prove the inductive step. We may assume that $L_1 \subseteq L \oplus A_i$ and show the existence of E_{i+1}. The definitions of λ and σ above are modified according to this assumption. Now A_i/A_{i+1} is an L_1-module with product defined by $\overline{a}x = \overline{ax^\lambda}$, $a \in A_i$, $x \in L_1$. Since $x^\sigma y^\sigma \in A_{i+1}$, (22) becomes

$$\overline{(xy)^\sigma} = \overline{x^\sigma y^\lambda} - \overline{y^\sigma x^\lambda} = \overline{x^\sigma}\, y - \overline{y^\sigma}\, x. \tag{23}$$

If we set $f(x) = \overline{x^\sigma}$, then $x \to f(x)$ is a linear mapping of L_1 into A_i/A_{i+1} and (23) becomes

$$f(xy) = f(x)y - f(y)x. \tag{24}$$

By Lemma 2, there exist $y_i \in L_1$, $\overline{x_i} \in A_i/A_{i+1}$ such that

$$\overline{z^\sigma} = f(z) = z \sum D(\overline{x_i}, y_i) = \overline{z^\lambda \sum D(x_i, y_i^\lambda)}$$

or

$$z^\sigma = z^\lambda \sum D(x_i, y_i) \mod A_{i+1}$$

where $D(x_i, y_i^\lambda)$ is a derivation of A contained in the radical of the multiplication algebra of A. Let $D = \Sigma D(x_i, y_i^\lambda)$ and $E_{i+1} = \exp(-D)$. Then

$$x^{E_{i+1}} = (x - xD) \bmod A_{i+1} = (x^\lambda + x^\sigma - x^\lambda D - x^\sigma D) \bmod A_{i+1}$$

$$= (x^\lambda - x^\sigma D) \bmod A_{i+1} = x^\lambda \bmod A_{i+1}.$$

Hence $L_1^{E_{i+1}} \subseteq L \oplus A_{i+1}$ and $E_1 \cdots E_k$ is an automorphism of the desired type.

Corollary. *Let A and R satisfy the conditions of Theorem* 2. *Let A_1 and A_2 be subalgebras of A which complement R. Then there exists an automorphism $G = \exp D$, D a derivation in the radical of the multiplication algebra of A, such that $A_1^G = A_2$.*

REFERENCES

1. N. Jacobson, *Lie algebras*, Interscience Tracts in Pure and Appl. Math., no. 10, Interscience, New York, 1962. MR **26** #1345.

2. E. N. Kuz'min, *Mal'cev algebras and their representations*, Algebra i Logika 7 (1968), no. 4, 48–69 = Algebra and Logic 7 (1968), 233–244. MR **40** #5688.

3. W. G. Lister, *A structure theory of Lie triple systems*, Trans. Amer. Math. Soc. **72** (1952), 217–242. MR **13**, 619.

4. O. Loos, *Über eine Beziehung zwischen Malcev-Algebren und Lie-Tripelsystemen*, Pacific J. Math. **18** (1966), 553–562. MR **33** #7385.

5. T. S. Ravisankar, *On Malcev algebras*, Pacific J. Math. **42** (1972), 227–234. MR **47** #1905.

6. A. A. Sagle, *Malcev algebras*, Trans. Amer. Math. Soc. **101** (1961), 426–458. MR **26** #1343.

DEPARTMENT OF MATHEMATICS, NORTH CAROLINA STATE UNIVERSITY, RALEIGH, NORTH CAROLINA 27607