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ERRATUM TO "ON THE ORDER OF SOME ERROR FUNCTIONS RELATED TO k-FREE INTEGERS"

BY

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1. **Introduction.** Let k be a fixed integer ≥ 2. A positive integer is said to be k-free, if it is not divisible by the kth power of any prime. $Q_k(x)$ and $Q'_k(x)$ denote, respectively, the number and the sum of k-free integers not exceeding x. Let the error terms be denoted by $\Delta_k(x)$ and $\Delta'_k(x)$, i.e.,

$$\Delta_k(x) = Q_k(x) - x/\zeta(k) \quad \text{and} \quad \Delta'_k(x) = Q'_k(x) - x^2/2\zeta(k).$$

It has been proved by the author [1] that on assumption of Riemann hypothesis,

$$\Delta'_k(x) - x\Delta_k(x) = O(x^{1+1/2k+\epsilon}) \tag{1.1}$$

and

$$\frac{1}{x}\int_1^x \Delta_k(t)\,dt = O(x^{1/2k+\epsilon}). \tag{1.2}$$

The details, other notations, and related references can be found in [1].

In this note, we first point out a false argument (which has been pointed out by Dr. D. Suryanarayana in personal correspondence to the author), and in the last section we give a correct argument (which has been found independently by Dr. Suryanarayana and the author).

2. **False argument.** On p. 330 of [1], it is concluded that $I_{22} = O(1)$ as follows:

$$\begin{aligned} I_{22} &= \int_{-1}^{+1} \frac{\zeta\left(\frac{1}{2k} + \epsilon + it\right)}{\zeta\left(\frac{1}{2} + k\epsilon + ikt\right)\left(\frac{1}{2k} + \epsilon + it\right)\cdots\left(\frac{1}{2k} + m + \epsilon + it\right)}\,dt \\ &= O\left(\frac{1}{m!}\int_0^1 t^{1/2-1/4k+\epsilon}\,dt\right) \quad \text{by Theorems 2 and 3} \\ &= O(1). \end{aligned} \tag{2.1}$$

The false argument takes place at the second step of the above argument, for it is derived using the results which are true if $|t| \longrightarrow \infty$. But since here the range of integration is finite, these results cannot be used.

3. **Correct argument.** Assuming the Riemann hypothesis to be true, it would be clear from the details of [1] that the integrand in (2.1) is analytic, hence continuous and bounded for $-1 \leq t \leq 1$. Therefore, $I_{22} = O(1)$.

We note that actually, $I_{22} = \int_C F(s)\,ds$ where C is the line segment joining $1/2k + \epsilon - i$ and $1/2k + \epsilon + i$ and

$$F(s) \equiv \zeta(s)/\zeta(ks)s(s+1)\cdots(s+m),$$

and $s = \sigma + it$ is a complex variable. On the R.H.S. $F(s)$ is analytic in the half-plane $\sigma > 1/2k$ except at $s = 1$.

The author wishes to express his sincere thanks to Dr. D. Suryanarayana for pointing out the mistake and, then, suggesting a correct proof.

REFERENCE

1. V. S. Joshi, *On the order of some error functions related to k-free integers*, Proc. Amer. Math. Soc. 35 (1972), 325–332.

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