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EMBEDDINGS OF COMPACTA WITH SHAPE DIMENSION IN THE TRIVIAL RANGE

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ABSTRACT. In this paper a loop condition is defined which generalizes the cellularity criterion and applies to compacta with nontrivial shape. It is shown that if X, $Y \subset E^n$, $n \geqslant 5$, are compacta which satisfy this loop condition and whose shape classes include a space having dimension in the trivial range with respect to n, then $\mathrm{Sh}(X) = \mathrm{Sh}(Y)$ is equivalent to $E^n - X \approx E^n - Y$. An application is given to compacta with the shape of a compact connected abelian topological group.

1. **Introduction and statements of main results.** Recently several individuals have studied special cases of the following general problem: if X and Y are compacta in Euclidean n-space E^n, under what conditions is $E^n - X \approx E^n - Y$ equivalent to $\mathrm{Sh}(X) = \mathrm{Sh}(Y)$? The results of this paper concern compacta whose shape classes include a space having dimension in the trivial range with respect to n. We give a global homotopy condition under which the equivalence holds for such compacta. Before stating our main result we make some definitions.

DEFINITION. Let X be a compact subset of the manifold M. X is said to satisfy the *inessential loops condition* (ILC) if for every neighborhood U of X in M there exists a neighborhood V of X in U such that each loop in $V - X$ which is null-homotopic in V is also null-homotopic in $U - X$. (See §2 for the definitions of other loop conditions and a discussion of some of the relationships among them.) For any compactum X, the *shape dimension of* X ($\mathrm{Sd}(X)$) is defined by $\mathrm{Sd}(X) = \min\{\dim Y\colon \mathrm{Sh}(X) = \mathrm{Sh}(Y)\}$. We say that k is in the *trivial range* with respect to n if $2k + 2 \leqslant n$.

THEOREM 1. *Let X and Y be compacta in E^n, $n \geqslant 5$, satisfying ILC and having shape dimension in the trivial range with respect to n. Then $E^n - X \approx E^n - Y$ if any only if* $\mathrm{Sh}(X) = \mathrm{Sh}(Y)$.

As a consequence of Theorem 1 we prove the following theorem about compacta with the shape of a topological group. For example, A and B in Theorem 2 could be solenoids. Recall that every finite dimensional compact connected abelian topological group is metrizable [16].

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THEOREM 2. *Let X, $Y \subset E^n$, $n \geqslant 5$, be globally* 1-*alg compacta and let A, B be compact connected abelian topological groups with* $2 \dim A + 2 \leqslant n$. *If* $\mathrm{Sh}(X) = \mathrm{Sh}(A)$ *and* $\mathrm{Sh}(Y) = \mathrm{Sh}(B)$ *then the following are equivalent*:
(i) $E^n - X \approx E^n - Y$,
(ii) $\mathrm{Sh}(X) = \mathrm{Sh}(Y)$, *and*
(iii) *A and B are topologically isomorphic.*

Theorem 1 is related to several other recent results. Chapman [3] proved that if $\dim X$, $\dim Y \leqslant k$ and $3k + 3 \leqslant n$, then there are copies X' and Y' of X and Y, respectively, in E^n so that $\mathrm{Sh}(X) = \mathrm{Sh}(Y)$ if and only if $E^n - X' \approx E^n - Y'$. Geoghegan and Summerhill [5] refined Chapman's theorem by reducing the unnecessary condition $3k + 3 \leqslant n$ to the trivial range and by making more explicit which copies of X and Y are acceptable. Specifically, they required that the copies of X and Y be 1-ULC. Hollingsworth and Rushing [7] improved this result by replacing 1-ULC (which is a local condition) with the small loops condition (which is global). The global condition is more desirable for a weak flatness theorem of this type–see [7] for more details.

The work of Hollingsworth and Rushing is generalized in Theorem 1 since the same conclusion is drawn for compacta which themselves do not necessarily have dimension in the trivial range but merely have the shape of such. Coram, Daverman and Duvall [4] have previously proved Theorem 1 in the special case that $\dim X \leqslant n - 3$ and Y is a finite polyhedron with dimension in the trivial range.

Theorem 2 answers a question raised by J. Keesling.

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2. **Definitions and notation.** Let X be a compactum in the manifold M^n. X is said to satisfy the *cellularity criterion (small loops condition)* if given a neighborhood U of X there exists a neighborhood V of X in U (and a number $\varepsilon > 0$) such that any loop in $V - X$ (any ε-loop in $V - X$) is null-homotopic in $U - X$. X is said to be *globally 1-alg* in M if given a neighborhood U of X there exists a neighborhood V of X in U such that any loop in $V - X$ which is null-homologous in $U - X$ is null-homotopic in $U - X$.

These loop conditions are closely related. For example if $\dim X \leqslant n - 2$, then ILC is equivalent to the small loops condition. On the other hand, if X has the shape of a point, then ILC is equivalent to the cellularity criterion. In case $\mathrm{Sd}(X) \leqslant n - 3$, Alexander duality shows that the inclusion induced homomorphism $H_1(V - X) \to H_1(V)$ is an isomorphism. Hence if $\mathrm{Sd}(X) \leqslant n - 3$, X globally 1-alg implies X satisfies ILC. Finally, it can easily be seen that if X has the shape of the inverse limit of a sequence of ANR's where each of these ANR's has abelian fundamental group, then X is globally 1-alg whenever X satisfies ILC.

Throughout this paper the symbols $\approx$ and $\simeq$ will have the following meanings: $\approx$ means "is homeomorphic to," "is isomorphic to," or "is topologically isomorphic to," depending on the context; while $\simeq$ means "is homotopic to." H_* denotes reduced singular homology, $\check{H}^*$ Čech cohomology and H_c^* Alexander cohomology with compact supports [13] all with integer coefficients.

All spaces are assumed to be metric.

For definitions of concepts related to shape theory, the reader is referred to [1] and [10]. For all other definitions consult [12].

3. **Compacta in standard position**. We begin by making a definition [3] which is basic to the entire proof of Theorem 1.

DEFINITION. Let $X \subset E^n$ be a compactum and $k = \mathrm{Sd}(X)$. X is in *standard position* if there exist sequences $\{P_i\}_{i=1}^{\infty}$ and $\{N_i\}_{i=1}^{\infty}$ such that

(i) each P_i is a compact polyhedron in E^n, $\dim P_i \leqslant k$,

(ii) each N_i is a regular neighborhood of P_i in E^n,

(iii) each $N_{i+1} \subset \operatorname{int} N_i$, and

(iv) $X = \bigcap_{i=1}^{\infty} N_i$.

If $\dim X = k$ and $2k + 1 \leqslant n$, then the set of embeddings $f\colon X \to E^n$ such that $f(X)$ is in standard position is a dense G_δ-subset of the set of maps of X into E^n [5, Theorem 3.3]. In this section we show that compacta in E^n which satisfy ILC and have shape dimension less than $n - 2$ are in standard position.

For any pair (A, B), the notation $\pi_i(A, B) = 0$ means that every map f: $(\Delta^i \times 0, \partial\Delta^i \times 0) \to (A, B)$ extends to a map $\bar{f}$: $(\Delta^i \times [0, 1], \partial\Delta^i \times [0, 1] \cup \Delta^i \times 1) \to (A, B)$. ($\Delta^i$ denotes the standard i-simplex.)

LEMMA 1. *Let $X \subset E^n$ be a compactum satisfying ILC and let $k = \mathrm{Sd}(X)$. Then $\pi_i(U, U - X) = 0$, $0 \leqslant i \leqslant n - k - 1$, for every compact neighborhood U of X in E^n.*

PROOF. It may be assumed that U is connected because otherwise the following proof can be applied to each component of U. Since $\mathrm{Sd}(X) = k$, there exists a compactum Y with $\dim Y = k$ and $\mathrm{Sh}(X) = \mathrm{Sh}(Y)$. Embed Y in E^{2k+1} in standard position; say $Y = \bigcap_{i=1}^{\infty} N_i$ where each N_i is a regular neighborhood of P_i in E^{2k+1}, $\dim P_i \leqslant k$ and $N_{i+1} \subset \operatorname{int} N_i$.

We first construct a convenient sequence of neighborhoods whose intersection is X. Let $\{f_i, X, Y\}_{E^n, E^{2k+1}}$ and $\{g_i, Y, X\}_{E^{2k+1}, E^n}$ be fundamental sequences which show that X and Y have the same shape [2, Theorem 2.4]. Choose an integer j such that $g_i(N_j) \subset U$ for almost all i. Now choose a neighborhood V of X such that $f_i(V) \subset N_j$ and $g_i f_i | V \simeq 1_V$ in U for almost all i. It may be assumed that $g_i | P_j$ is piecewise linear. Thus the inclusion map β: $V \hookrightarrow U$ is homotopic in U to a map of V into $g_i(P_j)$. Inductively then we can construct a sequence of neighborhoods V_i in U and polyhedra K_i such that $\dim K_i \leqslant k$, $V_i \cup K_i \subset V_{i-1}$, $\bigcap_{i=1}^{\infty} V_i = X$ and the inclusion map $V_i \to V_{i-1}$ is homotopic in V_{i-1} to a map of V_i into K_i.

Consider the universal cover p: $\tilde{U} \to U$. Denote $p^{-1}(V_i)$ by $\overline{V}_i$ and $p^{-1}(X)$ by $\overline{X}$. We show that $H_c^q(\overline{X}) = 0$ for $q > k$. Let f: $V_i \times [0, 1] \to V_{i-1}$ be a homotopy such that $f_0 = 1_{V_i}$ and $f_1(V_i) \subset K_i$. The diagram

$$\begin{tikzcd} & \tilde{U} \arrow[d, "p"] \\ \overline{V}_i \arrow[ur, hook] \arrow[r, "f_0 p|\overline{V}_i"'] & U \end{tikzcd}$$

commutes, so $f_t\, p|\overline{V}_i$ can be lifted to a homotopy g_t. Since $g_1(\overline{V}_i) \subset p^{-1}(K_i)$ and p is a local homeomorphism, $\dim g_1(\overline{V}_i) \leqslant k$.

Let $\overline{V}_i^+$ and $\overline{X}^+$ denote the one-point compactifications of $\overline{V}_i$ and $\overline{X}$ respectively. Since g_t is a proper map, g_t can be extended to $\bar{g}_t\colon \overline{V}_i^+ \to \overline{V}_{i-1}^+$. Hence the inclusion $\overline{V}_i^+ \hookrightarrow \overline{V}_{i-1}^+$ factors up to homotopy through a map into a space of dimension at most k. Thus the continuity axiom implies that $H^q(\overline{X}^+) = 0$ for $q > k$. Finally [13, Corollary 6.6.12] shows that $H_c^q(\overline{X}) = 0$, $q > k$.

Now Alexander duality [13, Theorem 6.9.10] gives $H_q(\tilde{U}, \tilde{U} - \overline{X}) \approx H_c^{n-q}(\overline{X}) \approx 0$ for $n - q \geqslant k + 1$. We look at the homology sequence of the pair $(\tilde{U}, \tilde{U} - \overline{X})$ and see that $H_0(\tilde{U} - \overline{X}) = 0$. So $\tilde{U} - \overline{X}$ is connected.

$\pi_1(U, U - X) = 0$ since if $f\colon (\Delta^1 \times 0, \partial\Delta^1 \times 0) \to (U, U - X)$, f can be lifted to $f'\colon (\Delta^1 \times 0, \partial\Delta^1 \times 0) \to (\tilde{U}, \tilde{U} - \overline{X})$. $\tilde{U} - \overline{X}$ is connected, so $f'(0)$ and $f'(1)$ can be joined by an arc in $\tilde{U} - \overline{X}$. The resulting loop is null-homotopic in $\tilde{U}$. The projection down of this homotopy is the desired homotopy in U.

We next prove that $\pi_1(\tilde{U} - \overline{X}) = 0$. Let $f\colon S^1 \to \tilde{U} - \overline{X}$ be a loop. f extends to $\bar{f}\colon \Delta^2 \to \tilde{U}$. Consider $p\bar{f}\colon (\Delta^2, S^1) \to (U, U - X)$. Choose $V \subset U$ to be a neighborhood of X satisfying the inessential loops condition relative to U. Now triangulate Δ^2 so that each simplex whose image intersects X is mapped into V. Using the fact that $\pi_1(V, V - X) = 0$ (by the same argument as was used to show that $\pi_1(U, U - X) = 0$), we can push the image of the 1-skeleton of this triangulation off X. If σ is a 2-simplex in Δ^2, $p\bar{f}|\partial\sigma\colon \partial\sigma \to V - X$ and $p\bar{f}|\partial\sigma \simeq 0$ in V, so $p\bar{f}|\sigma$ may be replaced by a map of σ into $U - X$ which agrees with $p\bar{f}$ on $\partial\sigma$. Thus $p\bar{f}|S^1 \simeq 0$ in $U - X$. Lifting this homotopy, we see that $f \simeq 0$ in $\tilde{U} - \overline{X}$.

Finally, we apply the relative Hurewicz Theorem [13, Theorem 7.5.4] and conclude that $\pi_i(\tilde{U}, \tilde{U} - \overline{X}) \approx H_i(\tilde{U}, \tilde{U} - \overline{X}) = 0$, $2 \leqslant i \leqslant n - k - 1$. The homotopy lifting property now can be used to show that $\pi_i(U, U - X) = 0$, $i \leqslant n - k - 1$.

Lemma 2. *Let $X \subset E^n$ be a compactum such that* $\mathrm{Sd}(X) \leqslant n - 3$. *Given a neighborhood U of X there exists a neighborhood V of X such that for any compact polyhedron $K \subset V$ with* $\dim K \leqslant n - 3$ *there is a polyhedron P and a regular neighborhood N of P such that $K \subset \operatorname{int} N \subset N \subset U$ and* $\dim P \leqslant \mathrm{Sd}(X)$.

Proof. The proof is by induction on $k = \dim K$. If $k \leqslant \mathrm{Sd}(X)$, $P = K$ and $V = U$ will work; so it may be assumed that $k > \mathrm{Sd}(X)$ and that the lemma is true for polyhedra of dimension less than k. Choose $V' \subset U$ using this inductive hypothesis. As before choose a neighborhood V of X in V' and a polyhedron $P' \subset V'$ such that $\dim P' \leqslant \mathrm{Sd}(X)$ and the inclusion $A \hookrightarrow V'$ is homotopic in V' to a map of V into P'.

Let $f\colon K \times [0, 1] \to V'$ be a homotopy such that $f_0 = 1_K$ and $f_1(K) \subset P'$. By Zeeman's Piping Lemma [15, Lemma 48], we may assume that there exists a polyhedron $J \subset K \times I$ such that

(1) $S(f) \subset J$,

(2) $\dim J \leqslant 2k - n + 2 \leqslant k - 1$,

(3) $\dim[J \cap (K^{k-1} \times [0, 1])] \leqslant 2k - n + 1 \leqslant k - 2$, and

(4) $K \times [0, 1] \to J \cup (K^{k-1} \times [0, 1]) \cup K \times 1$.(Here K^i denotes the i-skeleton of K.)

Let L be a $(k-2)$-dimensional subpolyhedron of K such that $L \supset K^{k-2}$ and $L \times [0, 1] \supset J \cap (K^{k-1} \times [0, 1])$. By induction it may be assumed that $f(L \times [0, 1] \cup J) \cup P' \subset N \subset U$ where N is a regular neighborhood of some polyhedron P with dimension $\leqslant \mathrm{Sd}(X)$. It remains only to engulf $(k-1)$- and k-simplexes of K.

$$K^{k-1} \times [0, 1] \searrow K^{k-2} \times [0, 1] \cup K^{k-1} \times 1 \cup (L \cap K^{k-1}) \times [0, 1].$$

The image of the latter set is already contained in N and contains $S(f|K^{k-1} \times [0, 1])$. Following the image of this collapse, K^{k-1} can be engulfed with N. Similarly $K \times [0, 1] \searrow J \cup K^{k-1} \times [0, 1] \cup K \times 1$, so N can be pushed out to cover all of K.

LEMMA 3. *Let $X \subset E^n$, $n \geqslant 5$, be a compactum with* $\mathrm{Sd}(X) \leqslant n-3$. *Then X satisfies ILC if and only if X is in standard position.*

PROOF. It suffices to show that given a neighborhood U of X there exists a polyhedron P in U with $\dim P \leqslant \mathrm{Sd}(X)$ and a regular neighborhood N of P such that $X \subset \operatorname{int} N \subset N \subset U$. Let $V \subset U$ be given by Lemma 2 and let M be a compact PL manifold neighborhood of X in V. Denote the $(n-3)$-skeleton of M by M^{n-3} and the dual 2-skeleton by M^2_*. By Lemma 2 there exists a polyhedron P with $\dim P \leqslant \mathrm{Sd}(X)$ and a regular neighborhood N of P such that $M^{n-3} \subset \operatorname{int} N \subset N \subset U$.

By Lemma 1 and Stallings' engulfing theorem [14], there exists a PL homeomorphism h_1: $\operatorname{int} M \to \operatorname{int} M$ with compact support such that $h_1(\operatorname{int} M - X) \supset M^2_* \cap \operatorname{int} M$. Extend h_1 via the identity to U. Let h_2 be a homeomorphism of U which pushes N across the join structure between M^{n-3} and M^2_* until $M \subset h_1(U - X) \cup h_2(\operatorname{int} N)$. Then $h_1^{-1}h_2(N)$ is the regular neighborhood we are looking for.

4. **Proofs of Theorems 1 and 2**. In this section we complete the proof of Theorem 1 and prove Theorem 2. The following lemma is used to keep an inductive argument going in the proof of Theorem 1.

LEMMA 4. *Let X, $Y \subset E^n$, $n \geqslant 5$, be compacta in standard position with shape dimensions in the trivial range with respect to n and let $\{f_i, X, Y\}$ and $\{f_i', Y, X\}$ be fundamental sequences which are homotopy inverse to one another. Let U_0 be a neighborhood of X and h be a PL homeomorphism of E^n such that $Y \subset h(U_0)$ and such that there exists a neighborhood W_0 of Y with $h^{-1}|W_0 \simeq f_i'|W_0$ in U_0 for almost all i. Then for every open set V_0, $Y \subset V_0 \subset h(U_0)$, there exists a PL homeomorphism q of E^n such that $q|E^n - U_0 = h|E^n - U_0$, $X \subset q^{-1}(V_0)$, and $q|U_1 \simeq f_i|U_1$ in V_0 for almost all i where U_1 is some neighborhood of X.*

PROOF. Assume that $X = \bigcap_{j=1}^\infty M_j$ where each M_j is a regular neighborhood of a compact polyhedron L_j and that $\dim L_j \leqslant \mathrm{Sd}(X)$. Choose neighborhoods $V \subset V_0$ of Y and $U \subset U_0$ of X and an integer i_0 such that $h^{-1}|V \simeq f_i'|V$ in U_0, $f_i|U \simeq f_{i+1}|U$ in V, and $f_i'f_i|U \simeq 1_U$ in U_0 for $i \geqslant i_0$. Let j be an integer large enough so that $M_j \subset U$. $f_{i_0}|L_j$ can be approximated by a PL embedding $\hat{f}$.

Notice that $h^{-1} \circ \hat{f} \simeq f_{i_0}' \circ \hat{f} \simeq f_{i_0}' \circ f_{i_0}|L_j \simeq 1_{L_j}$ in U_0.
So $\hat{f} \simeq h|L_j$ in $h(U_0)$. Hence there exists a PL homeomorphism r of E^n which is the identity outside $h(U_0)$ and such that $rh|L_j = \hat{f}$ [6]. It may be assumed

that $rh(M_j) \subset V$. Taking $q = rh$ and $U_1 = M_j$ gives the desired conclusion.

PROOF OF THEOREM 1. Suppose that $\mathrm{Sh}(X) = \mathrm{Sh}(Y)$. A homeomorphism of $E^n - X$ onto $E^n - Y$ can be constructed using the technique of [5, Lemma 4.2]. Lemma 4 above replaces Lemma 4.1 of [5]. Now suppose that $E^n - X \approx E^n - Y$. There exist X' and Y' with dimension in the trivial range satisfying $\mathrm{Sh}(X) = \mathrm{Sh}(X')$ and $\mathrm{Sh}(Y) = \mathrm{Sh}(Y')$. It may be assumed that X' and Y' are embedded in E^n as 1-ULC subsets [7, Lemmas 2 or 5, §3]. The first part of the theorem implies that $E^n - X' \approx E^n - Y'$, so $\mathrm{Sh}(X) = \mathrm{Sh}(Y)$ by [5] again.

PROOF OF THEOREM 2. The implication (iii) $\Rightarrow$ (ii) is obvious and (ii) $\Rightarrow$ (iii) is exactly Corollary 1.3 of [9]. Theorem 1 gives (iii) $\Rightarrow$ (i). Our proof that (i) $\Rightarrow$ (iii) actually establishes a stronger result which we state as Theorem 3.

THEOREM 3. *Let $X, Y \subset E^n$ be compacta and let A, B be compact connected abelian topological groups with* $\mathrm{Sh}(X) = \mathrm{Sh}(A)$ *and* $\mathrm{Sh}(Y) = \mathrm{Sh}(B)$. *Then $E^n - X \approx E^n - Y$ implies $A \approx B$.*

PROOF. Suppose $E^n - X \approx E^n - Y$. Then by Alexander duality [13, Theorem 6.2.16] $H^1(X) \approx H^1(Y)$; hence $H^1(A) \approx H^1(B)$ [10, Theorem 16]. Therefore char $A \approx$ char B [8, Theorem 1.4] and so Pontryagin duality [11, Theorem 52] shows that $A \approx B$.

REFERENCES

1. K. Borsuk, *Concerning homotopy properties of compacta*, Fund. Math. **62** (1968), 223–254. MR **37** #4811.

2. ———, *Remark on a theorem of S. Mardešić*, Bull. Acad. Polon. Sci. Sér. Sci. Math. Astronom. Phys. **19** (1971), 475–483. MR **48** #2986.

3. T. A. Chapman, *Shapes of finite dimensional compacta*, Fund. Math. **76** (1972), no. 3, 261–276. MR **47** #9531.

4. D. Coram, R. Daverman and P. Duvall, Jr., *A loop condition for embedded compacta*, Proc. Amer. Math. Soc. **53** (1975), 205–212.

5. R. Geoghegan and R. R. Summerhill, *Concerning the shapes of finite dimensional compacta*, Trans. Amer. Math. Soc. **179** (1973), 281–292.

6. H. Gluck, *Embeddings in the trivial range*, Ann. of Math. (2) **81** (1965), 195–210. MR **30** #3456.

7. J. G. Hollingsworth and T. B. Rushing, *Embeddings of shape classes of compacta in the trivial range* (to appear).

8. J. Keesling, *An algebraic property of the Čech cohomology groups which prevents local connectivity and movability*, Trans. Amer. Math. Soc. **190** (1974), 151–162.

9. ———, *Shape theory and compact connected abelian topological groups*, Trans. Amer. Math. Soc. **194** (1974), 349–358. MR **49** #9803.

10. S. Mardešić and J. Segal, *Shapes of compacta and ANR-systems*, Fund. Math. **72** (1971), no. 1, 41–59. MR **45** #7686.

11. L. S. Pontrjagin, *Continuous groups*, 2nd ed., GITTL, Moscow, 1954; English transl., *Topological groups*, Gordon and Breach, New York, 1966. MR **17**, 171; **34** #1439.

12. T. B. Rushing, *Topological embeddings*, Pure and Appl. Math., vol. 52, Academic Press, New York and London, 1973. MR **50** #1247.

13. E. H. Spanier, *Algebraic topology*, McGraw-Hill, New York, 1966. MR **35** #1007.

14. J. R. Stallings, *The piecewise-linear structure of Euclidean space*, Proc. Cambridge Philos. Soc. **58** (1962), 481–488. MR **26** #6945.

15. E. C. Zeeman, *Seminar on combinatorial topology*, Mimeographed notes, Inst. Hautes Études Sci., Paris, 1963.

16. J. D. Newburgh, *Metrization of finite dimensional groups*, Duke Math. J. **20** (1953), 287–293. MR **14**, 949.

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