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GROSS' ABSTRACT WIENER MEASURE ON $C[0, \infty)$

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ABSTRACT. Classical Wiener measure on $C[0, \infty)$ is obtained by the construction of Gross' abstract Wiener measure on a suitable Banach subspace of $C[0, \infty)$.

In two other papers [2], [3] three classical Wiener measures were shown to be special cases of Gross' abstract measure. In each of those cases the space on which the measure was given was a Banach space with an obvious supremum norm. Since $C[0, \infty)$ is not a Banach space with the supremum norm, the problem of constructing the classical measure on it by Gross' method needs a slight modification.

Let C' be the subspace of absolutely continuous functions on $[0, \infty)$ with square integrable derivative. C' with the inner product

$$(x,y) = \int_0^\infty x'(t)y'(t)\,dt$$

is the Hilbert space to be used in the construction of abstract Wiener measure. It is easy to show for $x \in C'$ that $\|x\|$ exists, where

$$\|x\| = \sup_{t\in[0,\infty)} \sqrt{(2/\Pi)}\left|\int_0^t [x'(s)/\sqrt{(1+s^2)}]\,ds\right|,$$

and is a norm on C'. Also, by use of the C. O. N. set of functions

$$\left\{F_n(t) = \sqrt{(2/\Pi)}\int_0^t [1/\sqrt{(1+s^2)}]h_n[(2\arctan s)/\Pi]\,ds\right\}$$

where $h_n(s)$ is the nth Haar function on $[0, 1]$, [1, p. 16], it is easy to parallel the argument in [2] to show that $\| \ \|$ is a measurable norm. The completion of C' in this norm is the subspace B of $C[0, \infty)$ for which $\int_0^\infty [1/\sqrt{(1+s^2)}]dx(s)$ converges, and it is on B that the abstract measure is given.

Now from the law of the iterated logarithm in classical Wiener space [4] (which implies $x(t) = O\sqrt{(t \ln t)}$ a.e.), and from the definition of B above there follows that B has measure one in classical Wiener space. Finally, a consideration of linear functionals on B which are integrals of step functions shows that the abstract measure assigned to $\{x \in B: x(t_i) \in [a_i, b_i): i = 1, 2, \ldots, n\}$ is the same as the classical measure assigned to the same set. Thus

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the classical measure is obtained from the abstract measure by the enlargement of B to $C[0, \infty)$ and the assignment of measure zero to subsets of $C[0, \infty) \backslash B$.

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