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THE STRONG DUAL OF A STRONGLY NUCLEAR SPACE NEED NOT BE NUCLEAR

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ABSTRACT. A strongly nuclear sequence space is exhibited whose strong dual is normed.

Let l^0 denote the intersection of all the l^p spaces with $p > 0$. Then $l^{0\times}$, the Köthe dual of l^0, is easily seen to be l^∞. The normal topology on l^∞ induced by l^0 is the topology generated by the seminorms $(x_n) \mapsto \Sigma|a_n x_n|$, for (x_n) in l^∞ and (a_n) in l^0.

Köthe [2, (5)] has established that a sequence space λ with the normal topology induced by a sequence space μ in the above sense is strongly nuclear if and only if $l^0 \cdot \mu \supseteq \mu$. Since l^0 is closed under the operation $(x_n) \mapsto (|x_n|^{1/2})$, it follows that the normal topology induced on l^∞ by l^0 is strongly nuclear. But as is well known [1, (5)], the bounded subsets of any perfect ($\lambda = \lambda^{\times\times}$) sequence space λ with a nuclear normal topology are precisely the subsets of solid hulls of points; that is, for every bounded subset B of λ, there is an element (b_n) of λ such that $|x_n| \leqslant |b_n|$ for all n, and for all (x_n) in B. Hence the bounded subsets of l^∞ under the normal topology induced by l^0 are the same as the bounded subsets of l^∞ under its usual norm topology. Therefore l^∞ under this strongly nuclear topology has a strong dual which is normed.

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