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# THE $Q$-TOPOLOGY, WHYBURN TYPE FILTERS AND THE CLUSTER SET MAP

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ABSTRACT. We use nonstandard topology and the $Q$-topology to characterize normal, almost-normal, regular, almost-regular, semiregular spaces. The cluster [resp. $\theta$-cluster] set relation is used to characterize regular, almost-regular [resp. strongly-regular] spaces. The Whyburn [resp. Dickman] filter bases are characterized and it is shown that the cluster [resp. $\theta$-cluster] set relation restricted to the domain of the Whyburn [resp. Dickman] filter bases is an essentially continuous [resp. strongly $\theta$-continuous] map iff the space is Hausdorff [resp. Urysohn].

1. **Introduction.** This paper has three major purposes. First, we investigate the $Q$-topology on an enlargement $^*X$ of a topological space $X$ as introduced by Robinson [9] and show, among other results, that the $Q$-closure of a point or set monad is the $\theta$-monad [6]. Moreover, using the $Q$-topology and the point or set monad, we characterize regular, semiregular, almost-regular [10], normal and almost-normal [11] spaces by means of a collection of highly analogous statements.

Fuller [4] defines a topology on the set of all clustering filters on $X$ and using the lower semifinite topology shows that the cluster set map is continuous iff $X$ is locally compact. Employing a different topology on the set of all converging filters, Wyler [13] shows that the convergence of a filter on a Hausdorff space $X$ is a continuous map iff $X$ is regular. We use that standard part [resp. $\theta$-standard part] relation, which can be considered the cluster [resp. $\theta$-cluster] set map, and show that, from the nonstandard viewpoint, regular [resp. almost-regular, strongly-regular] spaces are characterizable by similar statements involving the inverse of this relation. Further, by considering the near-standard [resp. $\theta$-near-standard] points and employing the induced $Q$-topology, we show that the cluster [resp. $\theta$-cluster] set relation is a continuous map iff $X$ is Hausdorff [resp. Urysohn].

In [12], Whyburn introduces the concept of a filter base being directed toward $A \subset X$ and uses this concept to characterize perfect (not necessarily continuous) maps. Dickman [2], [3] modifies Whyburn's definition and introduces the concept of a filter base almost-converging to $A \subset X$. Among our final results, we show that a filter base is directed toward [resp. almost-converges to] $A \subset X$ iff its nucleus satisfies a nonstandard condition analo-

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gous to the criterion for compactness [resp. quasi-$H$-closedness].

Throughout this paper, we let $\mathfrak{M} = (\mathfrak{U}, \in, \text{pr}, \text{ap})$ be the standard set-theoretic structure constructed by Machover and Hirschfeld [8] and, as usual, assume that all standard objects are elements of $\mathfrak{U}$. Even though some of the results only require $^*\mathfrak{M} = (^*\mathfrak{U}, {}^*\!\in, {}^*\text{pr}, {}^*\text{ap})$ to be an enlargement, it is convenient to assume that the extension $^*\mathfrak{M}$ is $\kappa$-saturated, where $\kappa$ is any cardinal larger than the cardinality of $\mathfrak{M}$. In the usual manner [7], [8], [9] we let $\mathfrak{L}$ be a first order language with equality and the usual assortment of abbreviations which formally describes $\mathfrak{M}$. Also we do not distinguish between the formal constant, relation and operator symbols in $\mathfrak{L}$ and the corresponding objects in $\mathfrak{M}$. We assume that the reader is familiar with the concepts and methods associated with nonstandard topology [7], [8], [9]. We use much of the notation found in [8].

2. **The $Q$-topology.** For a topological space $(X, \tau)$, the $Q$-topology on $^*X$, denoted by $\mathfrak{T}$, is the topology generated by $\{{}^*A \mid A \in {}^*\tau\}$ as a base. Recall that if $A \in {}^*\mathfrak{U}$, then $^*A = \{p \| [p \in {}^*\mathfrak{U}] \wedge [p \in {}^*A]\}$. If $A \in {}^*\tau$, then $^*A$ is said to be *-open. If $B \in \mathfrak{T}$, then $B$ is said to be $Q$-open, etc. We let $\mu(p)$ and $\mu(A)$ be the point and set monad [9] and define

$$\mu_\alpha(p) = \bigcap\{{}^*(\text{int}_X \, \text{cl}_X \, G) \mid p \in G \in \tau\},$$

$$\mu_\alpha(A) = \bigcap\{{}^*(\text{int}_X \, \text{cl}_X \, G) \mid A \subset G \in \tau\},$$

$$\mu_\theta(p) = \bigcap\{{}^*(\text{cl}_X \, G) \mid p \in G \in \tau\},$$

$$\mu_\theta(A) = \bigcap\{{}^*(\text{cl}_X \, G) \mid A \subset G \in \tau\}$$

to be the $\alpha$ and $\theta$ point and set monads respectively.

For many properties of the $Q$-topology not mentioned in this paper, we refer the reader to [1], [9]. In particular, Button [1] has shown that the $Q$-topology preserves much of the structure of $\tau$ and, indeed, $({}^*X, \mathfrak{T})$ is discrete iff $(X, \tau)$ is discrete.

THEOREM 2.1. *If nonempty* $\mathcal{G} \subset \tau$, *then* $\text{Nuc}\,\mathcal{G}$ *is $Q$-open.*

PROOF. If $\mathcal{G}$ does not have the finite intersection property, then $\text{Nuc}\,\mathcal{G} = \varnothing$. Assume that $\mathcal{G}$ has the finite intersection property and let $\mathcal{F}$ be the open filter generated by $\mathcal{G}$. Luxemburg's Theorem 2.1.6 [7] holds for any filter on any meet-semilattice of sets [5]. Hence $\text{Nuc}\,\mathcal{G} = \bigcup\{{}^*E \| [E \in {}^*\mathcal{F}] \wedge [{}^*E \subset \text{Nuc}\,\mathcal{G}]\}$.

COROLLARY 2.1.1. *For each $p \in X$ and $A \subset X$, the monads $\mu(p)$, $\mu(A)$, $\mu_\alpha(p)$, $\mu_\alpha(A)$ are $Q$-open.*

REMARK. In [1], Button obtains 2.1 by using a considerably more elaborate technique.

Clearly, if $\mathcal{G}$ is an open filter on $X$, then every infinitesimal *element in $\mathcal{G}$ is *-open. Indeed, we have a converse to this assertion.

THEOREM 2.2. *Let $\mathcal{F}$ be a filter base on $X$. If each infinitesimal *element in $\mathcal{F}$ is *-open, then* $\text{Nuc}\,\mathcal{F} = \text{Nuc}\,\mathcal{G}$, *where* $\mathcal{G} = \{G \| [G \in \tau] \wedge [G \in \mathcal{F}]\}$.

PROOF. Since $\mathfrak{F}$ is a filter base, then there exists an infinitesimal *element in $\mathfrak{F}$. Thus, it follows by transfer that $\mathcal{G} = \{G \| [G \in \tau] \wedge [G \in \mathfrak{F}]\} \neq \varnothing$. Clearly, $\operatorname{Nuc} \mathfrak{F} \subset \operatorname{Nuc} \mathcal{G}$. Now let $F \in \mathfrak{F}$ and $\mathcal{E} = \{E \| [E \in {}^*\tau] \wedge [E \in {}^*\mathfrak{F}] \wedge [{}^*E \subset {}^*F]\}$. Using saturation and Luxemburg's Theorem 2.7.3(c) [7], which also holds for filter bases, we have that there exists an open $G \in \mathfrak{F}$ such that $G \subset F$. Consequently, $\operatorname{Nuc} \mathcal{G} \subset \operatorname{Nuc} \mathfrak{F}$ and the result follows.

Clearly, for $A \subset X$, ${}^*(\mathrm{cl}_X A)$ is *-closed. Hence $\mu_\theta(p)$ and $\mu_\theta(A)$ are $Q$-closed. Of course, $\mathrm{ns}\,({}^*X) = \bigcup\{\mu(p) \mid p \in X\}$ is $Q$-open.

THEOREM 2.3. *For each* $p \in X$ [*resp.* $A \subset X$], *the monad* $\mu_\theta(p) = \mathrm{cl}_{{}^*X}\,(\mu(p))$ [*resp.* $\mu_\theta(A) = \mathrm{cl}_{{}^*X}\,(\mu(A))$].

PROOF. We only show the first assertion, the second being similar. Let $p \in X$. Since $\mu(p) \subset \mu_\theta(p)$, then $\mathrm{cl}_{{}^*X}\,(\mu(p)) \subset \mu_\theta(p)$. Assume that there exists $q \in \mu_\theta(p)$ and $q \notin \mathrm{cl}_{{}^*X}\,(\mu(p))$. Now there exists $E \in {}^*\tau$ such that $q \in {}^*E$ and ${}^*E \cap \mu(p) = \varnothing$. Saturation implies that there exists $G \in \tau$ such that $p \in G$ and ${}^*E \cap {}^*G = \varnothing$. Hence ${}^*E \cap {}^*(\mathrm{cl}_X G) = \varnothing$ by transfer. However, $q \in \mu_\theta(p)$ implies ${}^*E \cap {}^*(\mathrm{cl}_X G) \neq \varnothing$ and the result follows.

Since $X$ is regular [resp. almost-regular [10]] iff $\mu(p) = \mu_\theta(p)$ [resp. $\mu_\alpha(p) = \mu_\theta(p)$] for each $p \in X$ [6], then it follows that a space $X$ is regular [resp. almost-regular] iff $\mu(p)$ [resp. $\mu_\alpha(p)$] is $Q$-closed for each $p \in X$. Also, it is easy to show that a space $X$ is normal [resp. almost-normal [11]] iff $\mu(A) = \mu_\theta(A)$ [resp. $\mu_\alpha(A) = \mu_\theta(A)$] for each closed $A \subset X$. Hence a space $X$ is normal [resp. almost-normal] iff $\mu(A)$ [resp. $\mu_\alpha(A)$] is $Q$-closed for each closed $A \subset X$.

REMARK. Button [1], using a different technique, also gives the $Q$-open and $Q$-closed characterizations for regular and normal spaces.

In [6], we give some nonstandard characterizations for semiregular spaces. Using the $Q$-topology, we obtain another characterization. Let $\tau_s$ be the topology generated by the set of all regular-open subsets in $X$ and $\mathfrak{T}_s$ its associated $Q$-topology.

THEOREM 2.4. *A space* $(X, \tau)$ *is semiregular iff* $\mu(p) \in \mathfrak{T}_s$ *for each* $p \in X$.

PROOF. For the necessity, let $(X, \tau)$ be semiregular. Then $\mathfrak{T}_s = \mathfrak{T}$. Thus applying 2.1.1, we have that $\mu(p) \in \mathfrak{T}_s$ for each $p \in X$.

For the sufficiency, let $\mu(p) \in \mathfrak{T}_s$. Since ${}^*\tau_s$ is a base for $\mathfrak{T}_s$ and $p \in \mu(p)$, then it follows that there exists $E \in {}^*\tau_s$ such that $p \in {}^*E \subset \mu(p) \subset \mu_\alpha(p)$. Let $G$ be any open set such that $p \in G$. Then the sentence in $\mathfrak{L}$,

$$\exists x[[x \in \tau_s] \wedge [p \in X] \wedge [x \subset G]],$$

holds in $\mathfrak{M}$ by transfer. Consequently, since we are dealing with filter bases, we have that $\mu_\alpha(p) \subset \mu(p)$. Thus $\mu(p) = \mu_\alpha(p)$. This implies that $(X, \tau)$ is semiregular [6].

3. **The cluster set map.** As is well known if $\mathfrak{F}$ is a filter base on $X$, then $\mathrm{St}\,[\operatorname{Nuc} \mathfrak{F}]$ is the cluster set for $\mathfrak{F}$, where for $W \subset {}^*X$, $\mathrm{St}\,[W] = \{p \| [p \in X] \wedge [\mu(p) \cap W \neq \varnothing]\}$. Recall that a set $W \subset {}^*X$ is *nuclear* if there exists $\mathfrak{F} \subset \mathcal{P}(X)$ such that $W = \operatorname{Nuc} \mathfrak{F}$. Hence "St" restricted to $\mathrm{ns}\,({}^*X)$ is essentially the cluster set map for filter bases on $X$. Of course, in this case "St" may be considered a map from $\mathrm{ns}\,({}^*X)$ into $X$ iff $X$ is Hausdorff. A space $(X, \tau)$ is

called *strongly-regular* if for closed $F \subset X$ and $p \in X - F$ there exist $G, H \in \tau$ such that $p \in G$, $F \subset H$ and $\mathrm{cl}_X G \cap \mathrm{cl}_X H = \varnothing$. Observe that completely regular implies strongly-regular implies regular.

DEFINITION 3.1. For each $W \subset {}^*X$, let $\mathrm{St}_\theta[W] = \{p \| [p \in X] \wedge [\mu_\theta(p) \cap W \neq \varnothing]\}$ and $\mathrm{ns}_\theta({}^*X) = \bigcup\{\mu_\theta(p) | p \in X\}$. Notice that if $\mathfrak{F}$ is a filter base, then $\mathrm{St}_\theta[\mathrm{Nuc}\,\mathfrak{F}]$ is the set of all $\theta$-cluster points [3] for $\mathfrak{F}$. Also, "$\mathrm{St}_\theta$" is a map from $\mathrm{ns}_\theta({}^*X)$ into $X$ iff $X$ is Urysohn [6] (i.e. for distinct $p, q \in X$ there exist neighborhoods $N_p$, $N_q$ such that $\mathrm{cl}_X N_p \cap \mathrm{cl}_X N_q = \varnothing$).

THEOREM 3.1. *Let* $(X, \tau)$ *be Hausdorff and* $\mathrm{St} : \mathrm{ns}({}^*X) \to X$. *Then*:

(i) $X$ *is regular iff* $\mathrm{St}^{-1}[F] = \mu(F) \cap \mathrm{ns}({}^*X)$ *for each closed* $F \subset X$.

(ii) $X$ *is almost-regular iff* $\mathrm{St}^{-1}[F] = \mu_\alpha(F) \cap \mathrm{ns}({}^*X)$ *for each regular-closed* $F \subset X$.

PROOF. (i) For the necessity, let closed $F \subset X$ and $q \in \mathrm{St}^{-1}[F]$. Then $\mathrm{St}(q) = p$ implies that $q \in \mu(p)$ and $\mu(p) \cap {}^*F \neq \varnothing$. Hence $p \in F$. Thus $\mu(p) \subset {}^*G$ for each open $G \supset F$. Consequently, $q \in \mu(F) \cap \mathrm{ns}({}^*X)$ implies that $\mathrm{St}^{-1}[F] \subset \mu(F) \cap \mathrm{ns}({}^*X)$. Now assume that $X$ is regular and $q \in \mu(F) \cap \mathrm{ns}({}^*X)$. Then $q \in \mu(p)$ for some $p \in X$. Assume that $p \notin F$. Then there exist disjoint $G, H \in \tau$ such that $p \in G$ and $F \subset H$. Thus $\mu(p) \cap {}^*H = \varnothing$. However, this implies the contradiction that $q \notin \mu(F)$. Consequently, $\mathrm{St}[\mu(p)] = p \in F$ and the necessity follows.

For the sufficiency, let closed $F \subset X$ and $p \notin F$. Then $\mathrm{St}^{-1}(p) = \mu(p) \subset \mathrm{ns}({}^*X)$ and $\mathrm{St}^{-1}[F] = \mu(F) \cap \mathrm{ns}({}^*X)$. Observe that $\mu(p) \cap {}^*F = \varnothing$. Hence

$$\begin{aligned} \varnothing &= \mathrm{St}^{-1}[F \cap \{p\}] = \mathrm{St}^{-1}[F] \cap \mathrm{St}^{-1}(p) \\ &= \mu(F) \cap \mathrm{ns}({}^*X) \cap \mu(p) = \mu(F) \cap \mu(p). \end{aligned}$$

Thus there exist disjoint $G, H \in \tau$ such that $p \in G$ and $F \subset H$.

(ii) Observe that if $F \subset X$ is regular-closed in $X$, then

$$\mathrm{St}^{-1}[F] \subset \mu(F) \cap \mathrm{ns}({}^*X) \subset \mu_\alpha(F) \cap \mathrm{ns}({}^*X).$$

The result follows in the same manner as in (i) since the operator "$\mathrm{int}_X\, \mathrm{cl}_X$" preserves disjointness for open sets.

Clearly, a strongly-regular $T_1$ space is Urysohn. Of course, since a strongly-regular space is regular, then in a strongly-regular space $X$, $F \subset X$ is closed iff $\mathrm{St}_\theta[{}^*F] = F$. The following result is obtained in the same manner as is Theorem 3.1.

THEOREM 3.2. *Let* $X$ *be Urysohn. Then* $X$ *is strongly-regular iff* $\mathrm{St}_\theta^{-1}[F] = \mu_\theta(F) \cap \mathrm{ns}_\theta({}^*X)$ *for each closed* $F \subset X$.

4. **Whyburn and Dickman filter bases.** In [12], Whyburn says that a filter base $\mathfrak{F}$ on $X$ is *directed toward* $A \subset X$ if every filter base $\mathfrak{G}$ stronger than $\mathfrak{F}$ has a cluster point in $A$. Dickman [3] modifies Whyburn's definition and says that a filter base $\mathfrak{F}$ on $X$ is *almost-convergent to* $A \subset X$ if every filter base $\mathfrak{G}$ stronger than $\mathfrak{F}$ has an almost-cluster point in $A$ (i.e. $\mathrm{St}_\theta[\mathrm{Nuc}\,\mathfrak{G}] \cap A \neq \varnothing$). We call a filter base $\mathfrak{F}$ a *Whyburn* [resp. *Dickman*] filter base if $\mathfrak{F}$ is directed toward [resp. almost-convergent to] some $A \subset X$.

DEFINITION 4.1. A set $W \subset {}^*X$ is $A$-compact [resp. $\theta A$-compact] for $A \subset X$ if $W \subset \bigcup\{\mu(p) \mid p \in A\}$ [resp. $\{\mu_\theta(p) \mid p \in A\}$].

THEOREM 4.1. *Let* $\mathfrak{F}$ *be a filter base on* $X$. *Then the following statements are equivalent.*

(i) *For each open cover* $\mathcal{C}$ *of* $A$, *we have that* $\operatorname{Nuc}\mathfrak{F} \subset \bigcup\{{}^*G \mid G \in \mathcal{C}\}$ [*resp.* $\{{}^*(\mathrm{cl}_X\, G) \mid G \in \mathcal{C}\}$].

(ii) $\operatorname{Nuc}\mathfrak{F}$ *is* $A$-*compact* [*resp.* $\theta A$-*compact*].

(iii) *For each open cover* $\mathcal{C}$ *of* $A$ *there exists a finite subcover* $\mathfrak{D}$, *such that* $\operatorname{Nuc}\mathfrak{F} \subset \bigcup\{{}^*D \mid D \in \mathfrak{D}\}$ [*resp.* $\{{}^*(\mathrm{cl}_X\, D) \mid D \in \mathfrak{D}\}$].

(iv) *For each open cover* $\mathcal{C}$ *of* $A$ *there exists a finite subcover* $\mathfrak{D}$ *and an* $F \in \mathfrak{F}$, *such that* $F \subset \bigcup\{D \mid D \in \mathfrak{D}\}$ [*resp.* $\{\mathrm{cl}_X\, D \mid D \in \mathfrak{D}\}$].

PROOF. We only prove the first conclusions since the second follow in a similar manner.

(i) $\to$ (ii). Assume that $q \in \operatorname{Nuc}\mathfrak{F}$ and $q \notin \bigcup\{\mu_\theta(p) \mid p \in A\}$. Then for each $p \in A$ there exists some open neighborhood $G$ such that $q \notin {}^*G$. Thus $\mathcal{C} = \{G \mid [G \in \tau] \wedge [q \notin {}^*G]\}$ is an open cover of $A$ such that $\operatorname{Nuc}\mathfrak{F} \not\subset \bigcup\{{}^*G \mid G \in \mathcal{C}\}$.

(ii) $\to$ (iii). Assume that there exists some open cover $\mathcal{C}$ of $A$ such that for no finite $\mathfrak{D} \subset \mathcal{C}$ do we have that $\operatorname{Nuc}\mathfrak{F} \subset \bigcup\{{}^*D \mid D \in \mathfrak{D}\}$. Now there exists internal $E^* \in \mathfrak{F}$ such that ${}^*E \subset \operatorname{Nuc}\mathfrak{F}$ and

$${}^*E - \bigcup\{{}^*D \mid D \in \mathfrak{D}\} \neq \varnothing$$

for any finite $\mathfrak{D} \subset \mathcal{C}$. For if we assume that

$${}^*E - \bigcup\{{}^*D \mid D \in \mathfrak{D}\} = \varnothing$$

for some nonempty finite $\mathfrak{D} \subset \mathcal{C}$, where $|\mathfrak{D}| = n$, then the sentence in $\mathfrak{L}$,

$$\exists x[[x \in \mathfrak{F}] \wedge [x \subset D_1 \cup \cdots \cup D_n]],$$

holds in ${}^*\mathfrak{M}$. Thus by transfer there would exist $F \in \mathfrak{F}$ such that ${}^*F \subset {}^*(D_1 \cup \cdots \cup D_n) = {}^*D_1 \cup \cdots \cup {}^*D_n$. This would imply the contradiction that $\operatorname{Nuc}\mathfrak{F} \subset {}^*F \subset {}^*D_1 \cup \cdots \cup {}^*D_n$. Consequently, using saturation, ${}^*E - \bigcup\{{}^*C \mid C \in \mathcal{C}\} \neq \varnothing$ implies that there exists $q \in {}^*E \subset \operatorname{Nuc}\mathfrak{F}$ such that $a \notin {}^*C$ for any $C \in \mathcal{C}$ and the result follows.

(iii) $\to$ (iv). Simply let $E$ be the infinitesimal *element which is contained in ${}^*\mathfrak{F}$. Then the sentence in $\mathfrak{L}$,

$$\exists x[[x \in \mathfrak{F}] \wedge [x \subset \bigcup\{D \mid D \in \mathfrak{D}\}]],$$

holds in ${}^*\mathfrak{M}$; hence in $\mathfrak{M}$ by transfer.

(iv) $\to$ (i) is obvious.

COROLLARY 4.1.1. *A filter base* $\mathfrak{F}$ *on* $X$ *is Whyburn* [*resp. Dickman*] *iff* $\operatorname{Nuc}\mathfrak{F} \subset \mathrm{ns}\,({}^*X)$ [*resp.* $\operatorname{Nuc}\mathfrak{F} \subset \mathrm{ns}_\theta\,({}^*X)$].

COROLLARY 4.1.2. *A filter base* $\mathfrak{F}$ *on* $X$ *is directed toward* [*resp. almost-converges to*] $A \subset X$ *iff* $\operatorname{Nuc}\mathfrak{F}$ *is* $A$-*compact* [*resp.* $\theta A$-*compact*].

REMARK. The reader may wish to compare Theorem 4.1 with the known results that a set $A \subset X$ is compact [resp. quasi-$H$-closed relative to $X$] iff ${}^*A$ is $A$-compact [9] [resp. $\theta A$-compact [6]].

Recall that a map $f\colon X \to Y$ is strongly $\theta$-continuous at $p \in X$ if for every open neighborhood $N$ of $f(p)$ there exists some open neighborhood $G$ of $p$ such that $f[\mathrm{cl}_X\, G] \subset N$. Since in the $Q$-topology $\mu(p)$ is open and $\mathrm{cl}_{*X}\,(\mu(p)) = \mu_\theta(p)$ for each $p \in X$, then the next result follows easily and compares nicely with the results of Fuller [4] and Wyler [13].

THEOREM 4.2. *Let* ns $({}^*X)$ [*resp.* $\mathrm{ns}_\theta\,({}^*X)$] *carry the topology induced by the $Q$-topology on ${}^*X$. Then* $\mathrm{St}\colon \mathrm{ns}\,({}^*X) \to X$ [*resp.* $\mathrm{St}_\theta\colon \mathrm{ns}_\theta\,({}^*X) \to X$] *is a continuous* [*resp. strongly $\theta$-continuous*] *map iff $X$ is Hausdorff* [*resp. Urysohn*].

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