PROCEEDINGS OF THE
AMERICAN MATHEMATICAL SOCIETY
Volume 59, Number 2, September 1976

A NEW FLAG TRANSITIVE AFFINE PLANE OF ORDER 27

M. L. NARAYANA RAO AND K. KUPPUSWAMY RAO

ABSTRACT. A flag transitive affine plane of order 27 is constructed. The stabilizer of the origin of this plane contains a cyclic subgroup which is transitive on the lines through the origin. It is also shown that this plane is not isomorphic to the flag transitive plane constructed by Hering.

1. Let π be a finite affine plane of order n. A collineation group G of π is defined to be flag transitive on π if G is transitive on incident point-line pairs or flags of π. A. Wagner [13] has shown that π is a translation plane so that $n = p^r$ for some prime p and for some integer $r > 0$. D. A. Foulser [4], [5] has determined all flag transitive groups of finite affine planes. While determining the flag transitive groups, Foulser remarks that the existence of non-Desarguesian flag transitive affine planes is still an open problem. However, Foulser constructs two flag transitive planes of order 25 [4] and shows that his two planes and the near field plane of order 9 have flag transitive collineation groups. C. Hering [7] has constructed a plane of order 27 which has a flag transitive collineation group. Recently, one of the authors [10], [11] has constructed a flag transitive plane of order 49 and a class of flag transitive planes of order q^2 where q is power of a prime $p > 3$. The aim of this paper is to construct a non-Desarguesian flag transitive plane of order 27 and establish that this is different from the plane constructed by Hering [7].

2. Let F be the set of all ordered triples (a, b, c) over $GF(3)$. Let C be a set of 3×3 matrices over $GF(3)$ satisfying:

(2.1) (i) C contains

$$\begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} \quad \text{and} \quad \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix},$$

(ii) C contains 27 matrices,

(iii) If $M, N \in C$ and $M \neq N$, then $|M - N| \neq 0$, where $|X|$ denotes determinant of matrix X.

The conditions (2.1) imply that corresponding to each ordered triple (a, b, c) in F, there is exactly one matrix of the form

Received by the editors October 17, 1974.
AMS (MOS) subject classifications (1970). Primary 50D30, 05B25; Secondary 15B25.
Key words and phrases. Projective planes, affine planes, flag transitivity, collineations.

$$\begin{pmatrix} a & b & c \\ p & q & r \\ x & y & z \end{pmatrix}$$

which will be denoted by $M(a,b,c)$. We now define addition "+" and multiplication "·" on F as follows.

$$(2.2)\qquad \begin{aligned} (a,b,c) + (d,e,f) &= (a+d, b+e, c+f), \\ (a,b,c) \cdot (d,e,f) &= (d,e,f)M(a,b,c) \end{aligned}$$

where $M(a,b,c)$ is the matrix in C corresponding to (a,b,c).

THEOREM 2.1. *The set F with operations + and · defined by* (2.2) *is a left Veblen-Wedderburn system.*

PROOF. See [3, §5] or [2, §5].

3. We now construct a set C of matrices over $GF(3)$ and show that C satisfies conditions (2.1).

Let $T = \left(\begin{smallmatrix} 0 & P \\ Q & R \end{smallmatrix}\right)$ be a 6×6 matrix over $GF(3)$ where 0 is 3×3 zero matrix and P,Q,R are 3×3 nonsingular matrices given by

$$P = \begin{pmatrix} 1 & 1 & 1 \\ 0 & 2 & 1 \\ 2 & 2 & 0 \end{pmatrix}, \quad Q = \begin{pmatrix} 0 & 2 & 1 \\ 0 & 0 & 1 \\ 1 & 2 & 0 \end{pmatrix}, \quad R = \begin{pmatrix} 1 & 2 & 1 \\ 0 & 0 & 2 \\ 2 & 0 & 2 \end{pmatrix}.$$

Let

$$M_0 = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}.$$

Matrices M_i, $2 \leqslant i \leqslant 27$, are inductively defined as

$$(3.1)\qquad M_2 = Q^{-1}R = \begin{pmatrix} 1 & 1 & 0 \\ 2 & 1 & 1 \\ 0 & 0 & 2 \end{pmatrix}, \quad M_{k+1} = Q^{-1}M_k^{-1}P + M_2, \qquad 2 \leqslant k \leqslant 26.$$

An inspection of Table 3.1 shows that for each k, $3 \leqslant k < 26$, M_{k-1} is nonsingular and, therefore, M_k is defined for $3 \leqslant k \leqslant 27$.

LEMMA 3.1. *The set $C = \{M_i | i = 0, 2 \leqslant i \leqslant 27\}$ satisfies the conditions* (2.1) *and, hence, the corresponding $F(+, \cdot)$ is a left Veblen-Wedderburn system.*

PROOF. An inspection of Table 3.1 shows that C contains the zero and the unit matrices, the nonzero matrices of C are nonsingular and C consists of 27 matrices in all. Further, $Q^{-1}M_{27}^{-1}P + Q^{-1}R = M_0$. From (3.1) we obtain

$$(3.2)\qquad |M_{k+1} - M_2| = |Q^{-1}||M_k^{-1}||R| \neq 0$$

since Q, R and M_k are nonsingular for $2 \leqslant k \leqslant 26$. Since M_0 is the 3×3 zero matrix and M_k is nonsingular for $2 \leqslant k \leqslant 27$,

TABLE 3.1

i	M_i	i	M_i	i	M_i
0	$\begin{matrix}0&0&0\\0&0&0\\0&0&0\end{matrix}$	10	$\begin{matrix}2&0&0\\1&0&2\\1&2&1\end{matrix}$	19	$\begin{matrix}0&1&1\\1&1&0\\1&2&2\end{matrix}$
2	$\begin{matrix}1&1&0\\2&1&1\\0&0&2\end{matrix}$	11	$\begin{matrix}0&2&2\\2&2&2\\2&0&2\end{matrix}$	20	$\begin{matrix}2&2&0\\2&0&0\\0&2&2\end{matrix}$
3	$\begin{matrix}1&0&1\\1&2&2\\0&1&0\end{matrix}$	12	$\begin{matrix}2&1&2\\0&2&0\\2&0&1\end{matrix}$	21	$\begin{matrix}2&1&1\\1&1&2\\2&1&2\end{matrix}$
4	$\begin{matrix}1&2&2\\1&2&0\\2&2&2\end{matrix}$	13	$\begin{matrix}0&1&2\\2&0&1\\0&1&1\end{matrix}$	22	$\begin{matrix}2&2&1\\2&1&0\\2&0&0\end{matrix}$
5	$\begin{matrix}0&0&2\\2&2&0\\1&2&0\end{matrix}$	14	$\begin{matrix}1&0&0\\0&1&0\\0&0&1\end{matrix}$	23	$\begin{matrix}1&2&0\\2&0&2\\1&0&0\end{matrix}$
6	$\begin{matrix}1&1&2\\0&2&1\\2&1&0\end{matrix}$	15	$\begin{matrix}2&1&0\\1&2&1\\0&2&0\end{matrix}$	24	$\begin{matrix}1&2&1\\1&0&0\\1&1&1\end{matrix}$
7	$\begin{matrix}0&1&0\\1&0&1\\0&2&1\end{matrix}$	16	$\begin{matrix}0&2&0\\0&2&2\\1&1&2\end{matrix}$	25	$\begin{matrix}0&2&1\\1&1&1\\2&2&0\end{matrix}$
8	$\begin{matrix}2&0&2\\0&0&2\\1&1&0\end{matrix}$	17	$\begin{matrix}0&0&1\\0&1&2\\2&2&1\end{matrix}$	26	$\begin{matrix}1&1&1\\0&0&1\\1&0&2\end{matrix}$
9	$\begin{matrix}2&2&2\\0&1&1\\0&1&2\end{matrix}$	18	$\begin{matrix}2&0&1\\2&2&1\\1&0&1\end{matrix}$	27	$\begin{matrix}1&0&2\\2&1&2\\2&1&1\end{matrix}$

(3.3) $$|M_k - M_0| \neq 0.$$

Let $1 \leqslant k \leqslant 26$, $1 \leqslant j \leqslant 26$, $k \neq j$. There is no loss of generality if we assume $k > j$. From (3.1) we obtain

$$M_{k+1} - M_{j+1} = Q^{-1} M_k^{-1} (M_j - M_k) M_j^{-1} R$$

which implies that

(3.4) $$|M_{k+1} - M_{j+1}| = \mu(j,k) |M_j - M_k|$$

where $\mu(j,k) = |Q^{-1}||M_k^{-1}||M_j^{-1}||R| \in GF(3)$ and $\mu(j,k) \neq 0$. Applying (3.4) repeatedly we obtain

(3.5) $$|M_{k+1} - M_{j+1}| = \mu |M_{k-j+2} - M_2|$$

where μ is a nonzero element of $GF(3)$ which depends upon k and j. In view of (3.2) we get

$$|M_{k+1} - M_{j+1}| \neq 0. \tag{3.6}$$

Now the lemma follows from (3.2), (3.3), (3.6) and Theorem 2.1.

4. Let Π_N be the projective plane coordinatized by $F(+, \cdot)$ [4, p. 353], [6, p. 45], [10, p. 31]. That is Π_N has points (c), (a,d), (∞) and lines $[k]$, $[m,b]$, $[\infty] = L_\infty$ for $a, b, c, d, k, m \in F$ and $\infty \notin F$. Incidence in Π_N is defined by $(x,y) \in [m,b]$ if $y = m \cdot x + b$ and $(x,y) \in [k]$ if $x = k$. The plane Π_N may also be considered as a six dimensional right vector space $V(6,3)$ with three dimensional subspaces of $V(6,3)$ as lines and vectors of $V(6,3)$ as points. The line corresponding to the equation $y = m \cdot x$, $m \in F$, is given by the subspace

$$V(m) = \{(a,b,c,d,e,f) \mid (a,b,c) \in F,\ (d,e,f) = (a,b,c)M(m)\}$$

where $M(m)$ is the unique matrix in C corresponding to $m \in F$. The line $x = 0$ corresponds to the subspace

$$V(\infty) = \{(0,0,0,a,b,c) \mid (a,b,c) \in F\}.$$

The lines $y = m \cdot x + b$ correspond to appropriate translates of $V(m)$ for $m \in F$ or $m = \infty$. The group G_0 of all collineations fixing (0,0,0) of Π_N consists of all nonsingular linear transformations of $V(6,3)$ which permute the subspaces $V(m)$ for $m \in F$ or $m = \infty$ among themselves [1, Satz 19], [12, p. 208].

We now show that T given in §3 is a collineation of Π_N fixing the point corresponding to zero vector in $V(6,3)$. For convenience let the subspace $V(m)$ be denoted by V_k, where M_k is the unique matrix in C corresponding to $m \in F$, and $V(\infty)$ be denoted by V_1. It is clear that T is nonsingular. Since P is nonsingular it is easy to see that $V_0 T = V_1$. Now

$$\begin{aligned} V_1 T &= \{(0,0,0,a,b,c) \mid (a,b,c) \in GF(3)\}T \\ &= \{(x,y,z,p,q,r) \mid (x,y,z) = (a,b,c)Q \\ &\qquad\qquad \text{and } (p,q,r) = (a,b,c)R,\ (a,b,c) \in GF(3)\} \\ &= \{(x,y,z,p,q,r) \mid (p,q,r) = (x,y,z)Q^{-1}R \\ &\qquad\qquad\qquad = (x,y,z)M_2,\ (x,y,z) \in GF(3)\} \\ &= V_2. \end{aligned}$$

Similarly, in view of relations

$$M_{i+1} = Q^{-1}M_i^{-1}P + Q^{-1}R \quad \text{and} \quad M_0 = Q^{-1}M_{27}^{-1}P + Q^{-1}R,$$

we obtain that $V_i T = V_k$ where $k \equiv i + 1 \pmod{28}$ for $2 \leqslant i \leqslant 27$. Applying T repeatedly we obtain that

$$V_i T^j = V_k \quad \text{where } k \equiv i + j \pmod{28}.$$

Thus $\langle T \rangle$ is a collineation group of Π_N which permutes the lines through the origin transitively. The action of T restricted to the lines through the origin of Π_N may be given by T: $(0, 1, 2, 3, \ldots, 27)$ where x stands for the line V_x. Let π_N be the affine plane obtained from Π_N by deleting the line containing the ideal points.

THEOREM 4.1. *The plane π_N is a non-Desarguesian flag transitive affine plane.*

PROOF. An examination of Table 3.1 reveals that the set $C = \{M_i | i = 0, 2 \leqslant i \leqslant 27\}$ of matrices does not form a Galois field under matrix addition and multiplication implying Π_N is non-Desarguesian [3, p. 230]. Thus π_N is non-Desarguesian. The rest of the theorem follows from the action of T restricted to the lines of π_N through the origin.

LEMMA 4.2. *Any collineation of π_N fixing V_0, V_1, V_{14}, V_{15} and permuting the other lines through the origin among themselves fixes V_7 also.*

PROOF. Let $S = \left(\begin{smallmatrix} U & V \\ W & X \end{smallmatrix}\right)$, where U, V, W, X are 3×3 matrices over $GF(3)$, be a matrix inducing a collineation of π_N fixing V_0, V_1, V_{14}, V_{15} and permuting the other lines of π_N through the origin. Since S fixes V_0 and V_1, S reduces to the form $\left(\begin{smallmatrix} U & 0 \\ 0 & X \end{smallmatrix}\right)$ where 0 is 3×3 zero matrix, and U and X are 3×3 nonsingular matrices [9, Lemma 1.2]. Since S permutes the lines V_i, $2 \leqslant i \leqslant 27$, among themselves, U and X must be such that for each $M_i \in C$ there must be a $M_j \in C$ satisfying $UM_i = M_jX$, $i \neq 0, 1, J \neq 0, 1$. Obviously, S fixes V_i if and only if $UM_i = M_iX$. Since S fixes M_{14} and $M_{14} = I$, the 3×3 unit matrix, we get that $U = X$. But S fixes V_{15} also so that $UM_{15}U^{-1} = M_{15}$. An examination of Table 3.1 shows that

$$M_7 = M_{14} + M_{15} = I + M_{15}.$$

Then

$$UM_7U^{-1} = UIU^{-1} + UM_{15}U^{-1} = I + M_{15} = M_7.$$

Thus S fixes V_7 also. Hence, the lemma.

5. We give a brief description of the construction of Hering's affine plane π_H [7] and deduce some of its collineations in order to compare it with the plane π_N. Let 0 be 3×3 zero matrix. Let

$$s = \begin{pmatrix} A_1 & 0 \\ 0 & A_2 \end{pmatrix} \quad \text{where } A_1 = \begin{pmatrix} 0 & 2 & 0 \\ 2 & 0 & 2 \\ 1 & 0 & 0 \end{pmatrix}, \quad A_2 = \begin{pmatrix} 2 & 0 & 1 \\ 2 & 0 & 0 \\ 0 & 2 & 0 \end{pmatrix},$$

$$r = \begin{pmatrix} B_1 & B_2 \\ B_3 & B_4 \end{pmatrix} \quad \text{where } B_1 = \begin{pmatrix} 2 & 0 & 0 \\ 2 & 1 & 0 \\ 1 & 2 & 2 \end{pmatrix}, \quad B_2 = \begin{pmatrix} 2 & 0 & 0 \\ 2 & 1 & 0 \\ 1 & 2 & 2 \end{pmatrix},$$

$$B_3 = \begin{pmatrix} 2 & 0 & 0 \\ 2 & 1 & 0 \\ 2 & 2 & 2 \end{pmatrix}, \quad B_4 = \begin{pmatrix} 1 & 0 & 0 \\ 1 & 2 & 0 \\ 2 & 1 & 1 \end{pmatrix},$$

$$h = \begin{pmatrix} 0 & C_1 \\ C_2 & 0 \end{pmatrix} \quad \text{where } C_1 = \begin{pmatrix} 2 & 0 & 0 \\ 2 & 2 & 0 \\ 0 & 1 & 2 \end{pmatrix}, \quad C_2 = \begin{pmatrix} 1 & 0 & 0 \\ 1 & 1 & 0 \\ 2 & 2 & 1 \end{pmatrix}.$$

Let L_{26} and L_{27} be three dimensional subspaces of $V(6,3)$ defined by basis vectors as

$L_{26} = \langle (1,0,0,0,0,0), (0,1,0,0,0,0), (0,0,1,0,0,0) \rangle$,
$L_{27} = \langle (0,0,0,1,0,0), (0,0,0,0,1,0), (0,0,0,0,0,1) \rangle$.

Let

$$L_i = L_{26} rs^i, \qquad 0 \leqslant i \leqslant 12, \quad L_{13+i} = L_{27} rs^i, \qquad 0 \leqslant i \leqslant 12.$$

The incidence structure π_H with L_i $(0 \leqslant i \leqslant 27)$ and their cosets in the additive group of $V(6,3)$ as lines and vectors of $V(6,3)$ as points with inclusion as incidence relation is the flag transitive plane of order 27 constructed by Hering [7].

We will now show that s,h and r induce collineations on π_H. It is easy to verify that

$L_{26} s = L_{26}$ and $L_{27} s = L_{27}$ and
$L_i s^j = L_k$ where $k \equiv i + j \pmod{13}$, $0 \leqslant i \leqslant 12$, $0 \leqslant j \leqslant 12$, and
$L_{13+i} s^j = L_{13+k}$ where $k \equiv i + j \pmod{13}$.

Thus s induces a collineation on π_H the action of which restricted to lines through the origin is given by s: $(26)(27)(0,1,2,\ldots,12)(13,14,\ldots,25)$. Here (x) stands for line L_x.

In order to consider actions of h and r on π_H we need the following relations between generators r,h and s of the group G considered by Hering [7].

$$s^{13} = I, \quad h^6 = r^2 = 2I, \quad r^{-1}hr = h^{-1},$$

where I is the 6×6 unit matrix.

$$h^{-1}sh = s^4, \quad r^{-1}sr = s^{-1}rs^{-1}.$$

For $1 < i \leqslant 12$, $r^{-1}s^i r$ can be computed according to the relations given by Hering [7, p. 205].

It can be checked that

$$L_{26} h = L_{27}, \quad L_{27} h = L_{26} \quad \text{and} \quad L_0 h = L_{26} rh = L_{26} h^{-1} r = L_{27} r = L_{13}.$$

Similarly, $L_{13} h = L_0$.

Let $1 < i \leqslant 12$. Then

$$\begin{aligned} L_i h &= L_{26} rs^i h = L_{26} rhh^{-1} s^i h = L_{26} rhs^{4i} \\ &= L_{26} h^{-1} rs^{4i} = L_{27} rs^{4i} = L_{13} s^{4i} = L_{13+(4i \bmod 13)}. \end{aligned}$$

Similarly, $L_{13+i} h = L_{4i \pmod{13}}$.

Thus it follows that h induces a collineation and its action restricted to the lines of π_H through origin is

$$\begin{gathered} h\colon (26,27), (0,13), (1,17,3,25,9,23), (2,21,6,24,5,20), \\ (4,16,12,22,10,14), (7,15,8,19,11,18). \end{gathered}$$

The action of h^2 restricted to the lines π_H is given by

$$h^2\colon (26)(27)(0)(13)(1,3,9)(2,6,5)(4,12,10)(7,8,11)(14,16,22)$$
$$(15,19,18)(17,25,23)(20,21,24).$$

From the definition of L_0 and L_{13} we get $L_{26}r = L_0$ and $L_{27}r = L_{13}$. Since $r^2 = 2I$, $L_0 r = L_{26}r^2 = L_{26}$. Similarly $L_{13}r = L_{27}$. From $r^{-1}sr = s^{12}rs^{12}$ it follows that

$$L_1 r = L_{26}rsr = L_{26}r^2r^{-1}sr = L_{26}s^{12}rs^{12} = L_{12}.$$

Here we have used the facts that $L_{26}r^2 = L_{26}$ and $L_{26}s^i = L_{26}$ for any i. Similarly, using conjugates of s^i by r and noting that h interchanges L_{26} and L_{27}, we may conclude that r induces a collineation and its action restricted to the lines L_i is

$$r\colon (0,26)(13,27)(1,12)(2,19)(3,4)(5,18)(6,15)(7,24)(8,21)(9,10)$$
$$(11,20)(14,25)(16,17)(22,23).$$

THEOREM 5.1. *Any collineation of π_H that fixes L_{26} also fixes L_{27}.*

PROOF. Let β be any collineation of π_H fixing L_{26} and moving L_{27} to L_x, $0 \leqslant x \leqslant 12$. Since s fixes L_{26} and is transitive on the lines L_i, $0 \leqslant i \leqslant 12$, we may as well take β such that β fixes L_{26} and maps L_{27} onto L_0. Then the action of

$$\beta^{-1}s\beta = (26)(0)(x_0, x_1, \ldots, x_{12})(y_{13}, \ldots, y_{25}),$$

where $L_{x_i} = L_i\beta$; $L_{y_{13+i}} = L_{13+i}\beta$. Now two cases need to be considered.

Case (i). Suppose at least one number i, $0 \leqslant i \leqslant 12$, and at least one number j, $13 \leqslant j \leqslant 25$, are contained in the set $\{x_0, \ldots, x_{12}\}$. Then obviously $G_1 = \langle s, \beta^{-1}s\beta\rangle$ fixes L_{26} and is transitive on the remaining lines through the origin. Then $\langle G, G_1\rangle$ is doubly transitive on the lines of π_H through the origin.

Case (ii). Suppose $\{x_0, \ldots, x_{12}\}$ consists of the numbers $\{13, 14, \ldots, 25\}$. Then the action of the restriction of $r^{-1}\beta^{-1}s\beta r$ to the lines L_i of π_H through the origin is given by

$$r^{-1}\beta^{-1}s\beta r\colon (26)(0)(z_0, z_1, \ldots, z_{12})(a_0, a_1, \ldots, a_{12})$$

where $L_{z_i} = L_{13+j}r$ for some j, $0 \leqslant j \leqslant 12$.

Since $L_{19}r = L_2$, $L_{17}r = L_{16}$, $L_{13}r = L_{27}$ we find that $\{z_0, z_1, \ldots, z_{12}\}$ contains 2,16 and 27. This implies that $G_2 = \langle s, r^{-1}\beta^{-1}s\beta r\rangle$ fixes L_{26} and is transitive on the remaining lines of π_H through the origin. Here again, the group $\langle G, G_2\rangle$ is doubly transitive on the lines of π_H through the origin.

Similarly we may conclude that the collineation group of π_H is doubly transitive on the lines of π_H through the origin if β fixes L_{26} and maps L_{27} onto L_{13}.

Since the order 27 of π_H is odd and $27 \not\equiv 1 \pmod 8$, the double transitivity implies that π_H is Desarguesian [3, p. 217]. This is a contradiction since π_H is non-Desarguesian. From this contradiction the truth of lemma follows.

DEFINITION 5.2. If every collineation of π_H fixing a line L_x also fixes L_y, then L_y is called a companion of L_x.

LEMMA 5.3. *Every line L_x of π_H through the origin has a unique companion.*

PROOF. By Theorem 5.1 any collineation that fixes L_{26} also fixes L_{27} and L_{27} is, therefore, a companion of L_{26}. Since s fixes L_{26} and L_{27} and moves the remaining lines through the origin, none of the lines through the origin other than L_{27} can be a companion of L_{26}. Thus L_{27} is the unique companion of L_{26}. From the transitivity of G on the lines L_i, we get that every line L_x has a unique companion L_y. We may also remark that if L_y is a companion of L_x then L_x is a companion of L_y.

LEMMA 5.4. *π_H has a collineation the restriction of which to the lines L_i fixes each line of any two pairs of companions and moves the remaining lines through the origin.*

PROOF. Follows from Lemma 5.3, structure of h^2 and s and the transitivity of G on the lines L_i.

LEMMA 5.5. *If π_N is Hering's plane, then the line V_k is the unique companion of V_x where $k \equiv x + 14 \pmod{28}$.*

PROOF. Suppose π_N is Hering's plane. Then there is a collineation γ whose action restricted to the lines V_i is given by

$$\gamma\colon (x)(y)(a_0, a_1, \ldots, a_{12})(b_0, b_1, \ldots, b_{12})$$

where V_y is the unique companion of V_x. Then $T^{y-x}\gamma T^{x-y}$ fixes V_x and V_j where $j \equiv 2x - y \pmod{28}$, moving the remaining lines. In view of Lemma 5.3, V_j must be V_y. This is possible only when $y \equiv 2x - y \pmod{28}$ which gives $2x \equiv 2y \pmod{28}$, since $x \neq y$. This implies $y = x + 14 \pmod{28}$.

THEOREM 5.6. *π_N is not isomorphic to π_H.*

PROOF. If π_N is isomorphic to π_H, then in view of Lemmas 5.4 and 5.5, π_N has a collineation fixing V_0, V_{14}, V_1 and V_{15} and moving the remaining lines through the origin. But this is not possible by Lemma 4.2. Hence π_N is not isomorphic to π_H.

REFERENCES

1. J. André, *Über nicht-Desarguessche Ebenen mit transitiver Translationsgruppe*, Math. Z. **60** (1954), 156—186. MR **16**, 64.

2. R. H. Bruck and R. C. Bose, *The construction of translation planes from projective spaces*, J. Algebra **1** (1964), 85—102. MR **28** #4414.

3. P. Dembowski, *Finite geometries*, Ergebnisse der Mathematik und ihrer Grenzgebiete, Band 44, Springer-Verlag, Berlin and New York, 1968. MR **38** #1597.

4. D. A. Foulser, *The flag-transitive collineation groups of the finite Desarguesian affine planes*, Canad. J. Math. **16** (1964), 443—472. MR **29** #3549.

5. ———, *Solvable flag transitive affine groups*, Math. Z. **86** (1964), 191—204. MR **30** #1190.

6. M. Hall, Jr., *The theory of groups*, Macmillan, New York, 1959. MR **21** #1996.

7. C. H. Hering, *Eine nicht-desarguessche zweifach transitive affine Ebene der Ordnung* 27, Abh. Math. Sem. Univ. Hamburg **34** (1969/70), 203—208. MR **42** #8390.

8. D. R. Hughes, *Review of some results in collineation groups*, Proc. Sympos. Pure Math., vol. 1, Amer. Math. Soc., Providence, R.I., 1959, pp. 42—55. MR **22** #7053.

9. T. G. Ostrom, *Homologies in translation planes*, Proc. London Math. Soc. (3) **26** (1973), 605—629. MR **48** #2893.

10. M. L. Narayana Rao, *A flag transitive plane of order* 49, Proc. Amer. Math. Soc. **32** (1972), 256—262. MR **44** #7428.

11. ———, *A class of flag transitive planes*, Proc. Amer. Math. Soc. **39** (1973), 51—56. MR **47** #4132.

12. G. Pickert, *Projecktive Ebenen*, Die Grundlehren der math. Wissenschaften, Band 80, Springer-Verlag, Berlin, 1955. MR **17**, 399.

13. A. Wagner, *On finite affine line transitive planes*, Math. Z. **87** (1965), 1—11. MR **30** #2391.

DEPARTMENT OF MATHEMATICS, OSMANIA UNIVERSITY, HYDERABAD-500007, INDIA