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UNIFORMITIES INDUCED BY COZERO AND BAIRE SETS[1]

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ABSTRACT. This paper treats the cozero- and Baire-fine uniform spaces, those X such that each cozero (resp., Baire) function on X is uniformly continuous. The emphasis is on the general method, with the results about coz and Ba as corollaries. Some of these, stated just for coz: The coz functor out of Unif has no left adjoint, but its restrictions to precompact, and to separable, spaces do. A space is coz-fine iff it is proximally fine and each finite coz-cover is uniform. A cozero field $\mathcal{Q}$ has a compatible coz-fine uniform space iff the meet of two completely additive $\mathcal{Q}$-covers is another.

1. **Φ-fine spaces.** Let U be the category of uniform spaces, and let Φ be a class of functions between objects of U such that $U(X, Y) \subset \Phi(X, Y)$ always, and $\Phi \circ \Phi \subset \Phi$. The simplest examples are $\Phi =$ continuous functions, or all functions. We are concerned mainly with $\Phi =$ cozero functions, or Baire functions: $\operatorname{coz}(X, Y) = \{f | f^{-1}(\operatorname{coz} Y) \subset \operatorname{coz} X\}$, where $\operatorname{coz} X = \{\operatorname{coz} f | f \in U(X, R)\}$; $\operatorname{Ba}(X, Y) = \{f | f^{-1}(\operatorname{Ba} Y) \subset \operatorname{Ba} X\}$, where $\operatorname{Ba} X$ is the σ-field generated by $\operatorname{coz} X$. Secondarily, we deal with functions generated from a reflective modification (reflection maps are one-to-one and onto) $U \to^r R$ as $r(X, Y) = \{f | f \in U(rX, rY)\}$. The relevant examples are the precompact and separable reflections: pX (resp., eX) has base of finite (resp., countable) X-covers [7]. Subsuming all these, let τ: $U \to T$ be a functor to a concrete category which preserves underlying sets, and set $\Phi(X, Y) = T(\tau X, \tau Y)$.

Suppose given Φ and a reflective modification $U \to^r R$. Then: Φ-fine$|R$ is the (full) subcategory of R (and U) with objects those $X \in R$ with $\Phi(X, Y) = R(X, Y)$ $(= U(X, Y))$, for each $Y \in R$. We permit $R = U$ with r the identity, and then just write Φ-fine. E.g., coz-fine, Ba-fine; or p-fine, and e-fine. (The p-fine spaces are the proximally fine spaces of [1], [10].) Or, taking $r = p$, coz-fine$|P$, etc.

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[1]This paper is a continuation of [4a], [4b], and is excerpted from [4e]. Some of the results were announced in [4c], [4d], and some were contained in a course of lectures delivered by the author at the Charles University, Prague, in 1973. Most of the work was done in Fall, 1972, with sabbatical support of Wesleyan University. The first version of [4e] was completed with support of the Academies of Sciences of Czechoslovakia and the United States. I am pleased to thank these Universities and Academies for their support.

1.1 Proposition. *Φ-fine$|R$ is coreflective in R; the coreflections $^r\varphi X$ are modifications.*

Proof. By §37 of [5], it will suffice that (1) R have sums and (monic $\circ$ quotient)-factorizations: these are $r\Sigma$ and $r(m \circ q)$ where Σ is the sum in U, and $m \circ q$ a (monic $\circ$ quotient)-factorization in U; and that (2) Φ-fine $|R$ be closed under R-sums and R-quotients. These verifications are routine. Further, any coreflection in R is a modification; this is an easy extension of Kennison's theorem [9].

2. **Types.** Let Φ and R be as before, and let $X \in R$. The r-relative Φ-type of X, denoted $^r\langle X\rangle_\Phi$ or $^r\langle X\rangle$, consists of all X' on the same set as X with $X' \in R$ and with the identity $1 \in \Phi(X', X) \cap \Phi(X, X')$; equivalently, with $\Phi(X', Y) = \Phi(X, Y)$ for each $Y \in R$. The type category $T(\Phi, R)$ has objects $^r\langle X\rangle$ and morphisms $T(^r\langle X\rangle, {}^r\langle Y\rangle) = \Phi(X, Y)$ $(X, Y \in R)$, and $^rt(X) = {}^r\langle X\rangle$ is a functor $R \to T(\Phi, R)$. When $R = U$, we suppress left superscripts and write $\langle X\rangle$, t (and φ, from 1.1).

2.1. Proposition. *Let Φ come from a functor τ: $U \to T$, as in §1. Then $X' \in \langle X\rangle$ iff $\tau X' = \tau X$; $T(\Phi, U)$ is isomorphic to the range $\tau(U)$ (as a full subcategory of T), and $t = \tau$ (up to the isomorphism).*

2.2 Proposition. *Suppose that $rX \in \langle X\rangle$ for each $X \in U$. Then*:
(a) *$\Phi(X, Y) = \Phi(rX, rY)$ for each X, $Y \in U$.*
(b) *$^r\langle rX\rangle = \langle X\rangle \cap R$ for each $X \in U$.*
(c) *$T(\Phi, R) = T(\Phi, U)$ and $t = {}^rt \circ r$ (up to isomorphism).*
(d) *Φ-fine $\subset$ r-fine.*

The reader should interpret 2.1 and 2.2 for $\Phi = p$, e, coz, Ba with $R = U$, and for $\Phi =$ coz, Ba with $R = P$, E. The proofs are trivial, and omitted.

For $X \in R$, let $^r\overline{X}$ be the space weakly generated by all $\Phi(X, Y)$ $(Y \in R)$. Clearly, $X \in \Phi$-fine$|R$ iff $X = {}^r\overline{X}$, and $^r\varphi X$ is obtained by transfinitely iterating $^r\overline{X}$.

2.3 Proposition. *These conditions on $^r\langle X\rangle$ $(X \in R)$ are equivalent.*
(a) *$^r\langle X\rangle \cap (\Phi$-fine$|R) \neq \varnothing$ (or, $^r\varphi X \in {}^r\langle X\rangle$).*
(b) *$^r\overline{X} \in {}^r\langle X\rangle$ (and $^r\overline{X} = {}^r\varphi X$).*
(c) *$\Phi(X, Y) = R(^r\varphi X, Y)$ for each $Y \in R$.*

Proof. The equivalence of the two parts of (a) is clear. Assume (c). Then $1_X \in R(^r\varphi X, X) \subset R(^r\varphi X, {}^r\overline{X}) = \Phi(X, {}^r\overline{X})$; this suffices for (b). Assuming (b), $\Phi(^r\overline{X}, Y) = \Phi(X, Y) \subset R(^r\overline{X}, Y)$ (by definition of $^r\overline{X}$); hence $^r\overline{X} \in \Phi$-fine$|R$, so (a) holds. (a) $\Rightarrow$ (c). $\Phi(X, Y) \subset \Phi(^r\varphi X, Y) = R(^r\varphi X, Y)$ always holds. With (a), $R(^r\varphi X, Y) = \Phi(^r\varphi X, Y) = \Phi(X, Y)$.

3. **Adjoints.** 2.3(c) is an adjunction condition, namely, $^rs(^r\langle X\rangle) \equiv {}^r\varphi X$ defines a functor ^rs: $T(\Phi, R) \to R$, and

3.1 THEOREM. *These conditions on* Φ *and* R *are equivalent.*

(a) *Each type* ${}^r\langle X\rangle$ *has a member* (${}^r\overline{X} = {}^r\varphi X$) *in* Φ-*fine*$|R$.

(b) ${}^rs$ *is left adjoint to* ${}^rt$.

(c) ${}^rt$ *has a left adjoint.*

PROOF. We sketch (b), "concretized", says that $R({}^rsA, Y) = T(A, {}^rtY)$ for each A and Y. Using the definitions, this says 2.3(c) holds for each X, Y, i.e., (a). Of course, (b) $\Rightarrow$ (c). Now (c) asserts a functor l: $T(\Phi, R) \to R$ for which, concretizing again, $R(l\,{}^r\langle X\rangle, Y) = T({}^r\langle X\rangle, {}^r\langle Y\rangle) = \Phi(X, Y)$ for each X, Y. If $X = l\,{}^r\langle X\rangle$, this yields $\Phi(X, Y) = R(X, Y)$ for each Y; i.e., $X \in \Phi$-fine$|R$. And if $X \in \Phi$-fine$|R$, then $R(X, Y) = \Phi(X, Y) = R(l\,{}^r\langle X\rangle, Y)$ for each Y; hence $X = l\,{}^r\langle X\rangle$. Since $l \circ {}^rt$ is a coreflection with the same range as ${}^rs \circ {}^rt$, l and ${}^rs$ are naturally equivalent.

We first consider $R = U$: in [8], Katětov exhibits a p-type with no p-fine member. It is easy to see that the same kind of construction works for e, coz, Ba. From 3.1, then: *There is no left adjoint to* p, e, coz, *or* Ba.

We next consider the four pairs from $R = P$ or E, and $\Phi =$ coz or Ba. For now, denote $P_0 = P$ and $p_0 = p$, $P_1 = E$ and $p_1 = e$, and (referring to one of coz or Ba) ${}^r\overline{X}$, ${}^r\varphi$, ${}^rs$ become ${}^i\overline{X}$, ${}^i\varphi$, ${}^is$ ($i = 0, 1$); note that ${}^i\langle X\rangle = \langle X\rangle$ by 2.2.

3.2 THEOREM. *Let* $i = 0$ *or* 1.

(a) *For* $X \in P_i$, *the collection of* coz X-*covers* (*resp*., Ba X-*partitions*) *of power* $< \omega_i$ *is the base for a space* X_i^* *with* coz $X_i^* =$ coz X (*resp*., Ba $X_i^* =$ Ba X) *and* $U(X_i^*, Y) = \text{coz}(X, Y)$ (*resp*., $\text{Ba}(X, Y)$) *for each* $Y \in P_i$.

(b) *The functors* coz $|P_i$ *and* Ba $|\,P_i$ *have left adjoints* ${}^is(\langle X\rangle) = {}^i\varphi X = X_i^*$.

PROOF. (a) The case $i = 1$ for coz is in §2 of [4a]. The same technique handles the other three cases. For Ba, it is easier because partitions can be used; $i = 1$ for Ba is in 2.2 and 6.5 of [4b].

(b) By (a), $X_i^* \in \langle X\rangle$ and X_i^* is weakly generated by coz(X, Y) (resp., Ba(X, Y)) for $Y \in P_i$. Thus, $X_i^* = {}^i\overline{X}$. Now (b) follows from 3.1.

The above references to [4a], [4b] are to the construction for spaces in $P_1 = E$ of the metric-fine and measurable coreflections. In particular, 3.2 and [4a], [4b] show that coz-fine $|E$ is the class of separable metric-fine spaces, and Ba-fine $|E$ the separable measurable spaces. See [4a], [4b] for greater detail.

The nonseparable theory of metric-fine and measurable spaces is in [3b], [3d] and [11]. In particular, X is measurable iff X is metric-fine and coz $X =$ Ba X (extending the result 4.4 of [4a] for separable spaces). Analogously, it is very easy to prove, using the space $(p_0X)_0^*$ (with respect to Ba) that X is Ba-fine iff X is coz-fine and coz $X =$ Ba X.

4. **Fine types.** For suitable Φ, we can refine 2.3(b) to describe those Φ-types (relative to $R = U$) which contain a Φ-fine member. For $\Phi =$ coz and Ba, the result is

4.1 THEOREM. *Let* $\mathcal{Q}$ *be a cozero* (*resp., Baire*) *field. The* coz (*resp.* Ba)-*type of* $\mathcal{Q}$ *contains a* coz (*resp.*, Ba)-*fine member iff* $\mathfrak{U} \wedge \mathfrak{V}$ *is a completely additive* $\mathcal{Q}$-*cover* (*resp., partition*) *wherever* $\mathfrak{U}$ *and* $\mathfrak{V}$ *are.*

Here, a completely additive (ca) $\mathcal{Q}$-cover is a cover $\mathfrak{U}$ such that: $\mathfrak{U}' \subset \mathfrak{U}$ implies $\bigcup \mathfrak{U}' \in \mathcal{Q}$; $\mathfrak{U}$ initiates a normal sequence of such covers (automatically fulfilled if $\mathfrak{U}$ is a partition). These covers arise by analyzing the subbase for $\overline{X}$:

For general Φ, $\overline{X}$ has subbase of covers $f^{-1}(\mathfrak{U})$, where $Y \in U$, $\mathfrak{U} \in Y$ (i.e., $\mathfrak{U}$ is a uniform cover of Y) and $f \in \Phi(X, Y)$. A cover refined by one of these will be called a Φ-cover of X, and, Φ will be said to be *determined by covers* if $f \in \Phi(X, Y)$ if (and only if) $f^{-1}(\mathfrak{U})$ is a Φ-cover for each $\mathfrak{U} \in Y$. (It suffices here to consider just metric Y's, by I.14 of [7].)

4.2 PROPOSITION. *Of these conditions on* $\langle X \rangle$:
(a) $\langle X \rangle$ *has a* Φ-*fine member* (*namely*, $\overline{X}$);
(b) *If* $f \in \Phi(X, Y)$, $g \in \Phi(X, Z)$, *then* $(f, g) \in \Phi(X, Y \times Z)$;
(c) *If* $\mathfrak{U}$ *and* $\mathfrak{V}$ *are* Φ-*covers of* X, *then so is* $\mathfrak{U} \wedge \mathfrak{V}$;
(a) $\Rightarrow$ (b) $\Rightarrow$ (c); *and* (c) $\Rightarrow$ (a) *if* Φ *is determined by covers.*

PROOF. (a) $\Rightarrow$ (b). Let f, g be as in (b). Then $(f, g) \in U(X, Y \times Z) \subset \Phi(\overline{X}, Y \times Z)$; with (a), the last is $\Phi(X, Y \times Z)$. (b) $\Rightarrow$ (c). Consider $\mathfrak{U} \wedge \mathfrak{V} > f^{-1}(\mathfrak{U}_1) \wedge g^{-1}(\mathfrak{V}_1) = (f, g)^{-1}(\mathfrak{U}_1 \times \mathfrak{V}_1)$. (c) $\Rightarrow$ (a). Clearly, (c) implies that each uniform cover of $\overline{X}$ is a Φ-cover of X. Then, if Φ is determined by covers, $1: X \to \overline{X}$ is in Φ and $\overline{X} \in \langle X \rangle$ follows. By 2.3, we have (a).

Routine arguments show that $\Phi = p, e$, coz, Ba are determined by covers. Thus, 4.2 is applicable; the characterization 4.2(c) for p is in [1]. For coz and Ba, 4.2 and the following prove 4.1.

4.3 PROPOSITION. (a) *Each* coz (*resp.*, Ba)-*cover of* X *is refined by a* ca coz X-*cover* (*resp.*, ca Ba X-*partition*). (b) *Each* ca coz X-*cover* (*resp.*, ca Ba-*partition*) *is a* coz (*resp.*, Ba)-*cover of* X.

PROOF. (b). If $\{U_\alpha : \alpha \in A\}$ is a ca Ba-partition, then give A the discrete uniformity, whence Ba A = all subsets of A, and write the obvious map $X \to^f A$. By ca, $f \in \mathrm{Ba}(X, A)$, and $f^{-1}\{\{\alpha\}: \alpha \in A\} = \{U_\alpha\}$. Next, let $\mathfrak{U} >^* \mathfrak{U}_1 >^* \cdots >^* \mathfrak{U}_n >^* \ldots$ where each $\mathfrak{U}_n$ is a ca coz-cover. By the usual construction [7], let ρ be the pseudometric associated with $\{\mathfrak{U}_n\}$ and let M be the metric identification of $\{X, \rho\}$. We have a map $X \to^f M$ such that $f^{-1}(\mathfrak{S}(\varepsilon)) < \mathfrak{U}$ for suitable ε. Since each $\mathfrak{U}_n$ is a ca coz-cover, each open ρ-ball in X is in coz X, and hence $f \in \mathrm{coz}(X, M)$.

(a). Since a coz (resp., Ba)-cover looks like $f^{-1}(\mathfrak{U})$, where $f \in \mathrm{coz}(X, Y)$ (resp., $\mathrm{Ba}(X, Y)$) and $\mathfrak{U} \in Y$, and since we may assume Y metric and $\mathfrak{U}$ open, it will suffice to prove this: In a metric space Y, an open cover $\mathfrak{U}$ is a ca coz Y-cover (which is obvious) and is refined by a ca Ba Y-partition $\mathfrak{B}$. In producing $\mathfrak{B}$, we may assume $\mathfrak{U}$ σ-discrete, by A. H. Stone's theorem [2]. Now, index $\mathfrak{U}$ by an initial segment of ordinals, $\mathfrak{U} = \{U_\alpha\}$, and define inductively: $B_0 = U_0, \ldots, B_\alpha = U_\alpha - \bigcup\{B_\beta: \beta < \alpha\}, \ldots$. Evidently, $\{B_\alpha\}$ is a partition shrinking $\{U_\alpha\}$. Since $B_\alpha = U_\alpha \cap \bigcap\{X - B_\beta: \beta < \alpha\}$, B_α is the intersection of an open set and a closed set, and hence is a G_δ-set. Now, if $\mathfrak{B}' \subset \{B_\alpha\}$, then $\mathfrak{B}'$ shrinks a subfamily $\mathfrak{U}'$ of U. Since $\mathfrak{U}$ is σ-discrete, so is $\mathfrak{U}'$, and so is $\mathfrak{B}'$. The union of a discrete collection of G_δ-sets is Baire, and thus $\bigcup \mathfrak{B}'$ is Baire.

5. r-**simplicity.** Given Φ and R, as before, extend the definition of $'\overline{X}$ to all $X \in U$: $'\overline{X}$ is weakly generated by all $\Phi(X, Y)$, $Y \in R$. Φ will be called *r-simple* if for each $X \in U$, $\langle X \rangle$ contains both rX and $'\overline{X}$. Note that, then, 2.2 and 3.1 obtain.

5.1 PROPOSITION. *Let Φ be r-simple.*

(a) *$\langle X \rangle$ has a Φ-fine member (namely, $X = \varphi X$) iff $r\overline{X} = {}'\overline{X}$.*

(b) *$X \in \Phi$-fine iff $X \in r$-fine and $1_X \in U(X, {}'\overline{X})$.*

PROOF. (a). If $r\overline{X} = {}'\overline{X}$, then $r\overline{X} \in \langle {}'\overline{X} \rangle = {}'\langle X \rangle = \langle X \rangle \cap \underline{R} \subset \underline{\langle X \rangle}$. We assume that $r\overline{X} \in \langle \overline{X} \rangle$. So $\langle \overline{X} \rangle = \langle X \rangle$. Conversely, $1 \in U(\overline{X}, {}'\overline{X})$, hence $1 \in U(r\overline{X}, {}'\overline{X})$ (since $'\overline{X} \in R$); and if $\overline{X} \in \langle X \rangle$, then $r\overline{X} \in \langle X \rangle \cap \mathfrak{R} = {}'\langle X \rangle$ and $1 \in U({}'\overline{X}, r\overline{X})$ since $'\overline{X}$ is finest in $'\langle X \rangle$.

(b). If $X \in \Phi$-fine, then $X \in r$-fine, by 2.2. Since $\overline{X} = X$, $r\overline{X} = rX$. By (a), $r\overline{X} = {}'\overline{X}$, so $1 \in U(X, {}'\overline{X})$. Conversely, if $1 \in U(X, {}'\overline{X})$, and if $f \in \Phi(X, Y)$, then $f \in U({}'\overline{X}, {}'\overline{Y})$ and the composition

$$X \to^1 {}'\overline{X} \to^f {}'\overline{Y} \to r\,{}'\overline{Y} = rY$$

is uniform. If X is r-fine, $f \in U(X, Y)$ follows.

For $R = P$ or E, and $\Phi = \text{coz}$ or Ba, we have $rX \in \langle X \rangle$; and $'\overline{X}$ is the appropriate $(p_i X)_i^*$ of 3.2(a), so $'\overline{X} \in \langle X \rangle$. That is, coz *and* Ba *are p- and e-simple.*

X is called metric-fine [4a] if $U(X, M) = U(X, \alpha M)$ for each metric M, α being the fine coreflector [7]; X is measurable iff X is metric-fine and $\text{coz}\, X = \text{Ba}\, X$. (See §3.) These classes are coreflective; m and b denote the functors. With $i = p$ or e, X will be called i(metric)-fine (resp., i(metric)-measurable) if $U(X, iM) = U(X, mM)$ (resp., $U(X, bM)$) for each metric M. (Note that $mM = \alpha M$.) We need just: If X is metric-fine, so is eX. eX is metric-fine iff each countable coz X-cover is in X. ([4a]; the second is referred to in §3, of course.) These follow: If X is measurable, so is eX, and eX is measurable iff each countable Ba X-cover is in X (cf. §3).

5.2 COROLLARY. *The following conditions on X are equivalent.*

(a) *X is* coz*-fine (resp.,* Ba*-fine).*

(b) *X is e-fine and each countable* coz X *(resp.,* Ba X*)-cover is in X.*

(c) *X is p-fine and each finite* coz X *(resp.,* Ba X*)-cover is in X.*

(d) *X is metric-fine (resp., measurable) and i-fine for $i = e$, or p.*

(e) *X is i(metric)-fine (resp., i(metric)-measurable), for $i = e$, or p.*

PROOF. The equivalence of (a), (b), (c) follows from 5.1. We complete the proof for coz; the proof for Ba is similar. M stands for a metric space.

(a) $\Rightarrow$ (d). $U(X, M) \subset \text{coz}(X, \alpha M)$ (because coz $M = \text{coz } \alpha M$). So coz-fine $\subset$ metric-fine. And clearly, coz-fine $\subset$ p-fine $\subset$ e-fine.

(d) $\Rightarrow$ (e). If (d) holds, $U(X, iM) = U(X, M) = U(X, \alpha M)$.

(e) $\Rightarrow$ (b). If X is metric-fine then the second condition in (b) holds: This condition says that X is finer than the coz-fine$|E$ coreflection of eX; because X and eX are metric-fine, this is exactly eX (which is coarser than X). So, if X is also e-fine (the weakest i-fine condition), then (b) holds.

One more application of 5.1: X is said to be finest in $\langle X \rangle_\Phi$ if $1 \in U(X, X')$ for each $X' \in \langle X \rangle_\Phi$. It is known, and not completely trivial, that a space finest in $\langle X \rangle_p$ is p-fine [1], [7], [10]. This implies that *a space finest in its* coz (*resp.*, Ba)-*type is* coz (*resp.*, Ba)-*fine*, via the general proposition: if Φ is r-simple, and if finest in r-type implies r-fine, then finest in Φ-type implies Φ-fine. (*Proof.* Apply 5.1(b) and 2.2(d).) In 1972, I conjectured that a space finest in e-type need *not* be e-fine, and that an example is in the e-type of $N \times eD$, N countable discrete and D uncountable discrete. This is still open.

6. **Remarks.** Something akin to the Φ-fine scheme is discussed in [6], and some of the ideas of the present paper are treated independently in [3c]. (In [3c] is asserted a construction of the coz-fine coreflection, which if true would mean that it is $(\bar{\bar{X}})$ (see §2, here); but the proof seems inconclusive).

4.1 here, for Ba, answers the question raised on p. 143 of [3a] (where a partial result appears).

That coz-fine = (p-fine) $\cap$ (metric-fine) (part of 5.2 here) occurs independently in [3b].

In late 1975 appeared the report of a Prague seminar directed by Z. Frolík, "Seminar Uniform Spaces, 1973–74" M. U. ČSAV v Praze. The papers by Frolík considerably overlap with this paper, and the papers by V. Rödl and V. Kůrková-Pohlová treat interestingly (but do not settle) the conjecture at the end of §5.

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