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SHORTER NOTES

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SPHERICALLY SYMMETRIC ENTIRE SOLUTIONS OF $\Delta^p u = f(u)$

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ABSTRACT. A sufficient condition is given for existence of spherically symmetric entire solutions of the equation $\Delta^p u = f(u)$, $p \geqslant 2$.

Some results on spherically symmetric entire solutions of $\Delta u = f(u)$ are given in [1], [2]. In particular, it is known that $\Delta u = e^u$ has no entire solutions for all $n \geqslant 2$. However, W. Walter [3] obtained the startling result that when $n \geqslant 3$, $p \geqslant 2$, there do exist spherically symmetric entire solutions of $\Delta^p u = e^u$. The purpose of this note is to prove the following theorem. For simplicity, we shall state and prove the theorem only for the equation $\Delta^2 u = f(u)$. The theorem holds true for the equation $\Delta^p u = f(u)$, $p \geqslant 2$.

THEOREM. *Suppose that $f(t)$ is a positive continuously differentiable function with $f'(t) \geqslant 0$ for all t. Suppose f satisfies the condition*

$$\int^{\infty}\left(\int_0^t f(s)\, ds\right)^{-1/2} dt = \infty. \tag{A}$$

Then the equation $\Delta^2 u = f(u)$ has a spherically symmetric entire solution, where Δ is the n-dimensional Laplacian.

PROOF. Let $r^2 = x_1^2 + x_2^2 + \cdots + x_n^2$. Then

$$\Delta u(r) = r^{1-n} \frac{d}{dr}\left(r^{n-1} \frac{du}{dr}\right).$$

Employing standard iteration procedure, one can show that there exists a solution $u(r)$ satisfying $\Delta^2 u = f(u)$ in a maximal interval $[0, R)$, with $u(0) = u'(0) = \Delta u(0) = (\Delta u)'(0) = 0$. It is easily seen that $u(r) \to \infty$ as $r \to R$. Since $(\Delta u)' > 0$, from $\Delta^2 u = f(u)$ it follows that $(\Delta u)'' < f(u)$. Integrating this inequality twice, we obtain $\Delta u < 2^{-1} r^2 f(u)$. Since $u' > 0$, we have $u'u'' < 2^{-1} r^2 u' f(u)$. Integrating twice, we obtain

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$$\int^{\infty}\left(\int_0^u f(s)\,ds\right)^{-1/2} du < R^2.$$

R cannot be finite because of the condition (A).

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