PROCEEDINGS OF THE
AMERICAN MATHEMATICAL SOCIETY
Volume 69, Number 2, May 1978

A THEOREM ON C^*-EMBEDDING

F. K. DASHIELL, JR.

ABSTRACT. THEOREM. In a totally nonmeager and regular space, every countable intersection of open, normal, C^*-embedded subsets is normal and C^*-embedded.

A subspace G of a topological space S is called *C^*-embedded* if every bounded continuous f: $G \to \mathbf{R}$ extends continuously to S. This note gives a short and elementary proof of the theorem in the abstract. This extends a result of E. Aron ([**1**], otherwise unpublished) which asserts that in a compact space, every countable intersection of dense, C^*-embedded, open F_σ subsets is C^*-embedded. The original proof in [**1**] is rather long and nonelementary. The interest of this theorem and its relevance to certain problems in rings of continuous functions are discussed in [**4**, §6], [**5**], and [**6**, §5.8].

A space is called *totally nonmeager* (see [**2**, p. 252]) if every closed subspace is second category (i.e., nonmeager) relative to itself.

PROOF OF THE THEOREM. Suppose S is a totally nonmeager, regular space and $G_1, G_2, \ldots$ are open, normal, C^*-embedded subsets. It suffices to prove disjoint closures in S (for then, since G_1 is normal and $\overline{Z}_1 \cap \overline{Z}_2 \cap G_1 = \varnothing$, G would be normal and C^*-embedded in G_1 [**3**, Theorem 1.17]). Put

$$K = \overline{Z}_1 \cap \overline{Z}_2.$$

Clearly $G \cap K = \varnothing$. Suppose (toward a contradiction) that $K \neq \varnothing$. Fix n, pick any closed neighborhood F of a point in K, and set $A_i = G_n \cap \overline{Z}_i \cap F$, $i = 1, 2$. Then

$$\varnothing \neq \overline{Z_1 \cap F} \cap \overline{Z_2 \cap F} \subset \overline{A_1} \cap \overline{A_2},$$

so the relatively closed subsets A_1, A_2 of the normal, C^*-embedded set G_n do not have disjoint closures. Thus

$$\varnothing \neq A_1 \cap A_2 = G_n \cap K \cap F.$$

Since F was arbitrary and K is regular, $G_n \cap K$ is a dense relatively open subset of the nonmeager space K (for all n). Thus $\varnothing \neq \bigcap_{n=1}^{\infty}(G_n \cap K) = G \cap K$, a contradiction.

REFERENCES

1. E. Aron, *Embedding lattice-ordered algebras in uniformly closed algebras*, Thesis, U. Rochester, 1971.

Received by the editors August 22, 1977.
AMS (MOS) subject classifications (1970). Primary 54C45.

2. N. Bourbaki, *General topology*, Part 2, Hermann, Paris; Addison-Wesley, Reading, Mass., 1966.

3. L. Gillman and M. Jerison, *Rings of continuous functions*, Van Nostrand, Princeton, N. J.; Reinhold, New York, 1960.

4. A. W. Hager, *A class of function algebras (and compactifications, and uniform spaces)*, Symposia Mathematica, Vol. 17, Academic Press, New York, pp. 11–23.

5. A. W. Hager, P. Nanzetta and D. Plank, *Inversion in a class of lattice-ordered algebras*, Colloq. Math. **24** (1972), 225–234.

6. A. W. Hager and L. C. Robertson, *Representing and ringifying a Riesz space*, Symposia Mathematica, vol. 21, Academic Press, New York, 1977, pp. 411–431.

DEPARTMENT OF MATHEMATICS, CALIFORNIA INSTITUTE OF TECHNOLOGY, PASADENA, CALIFORNIA 91125

Current address: R and D Associates, P. O. Box 9695, Marina del Rey, California 90291