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SHORTER NOTES

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PIECEWISE LINEAR EMBEDDINGS OF BALLS INTO CONTRACTIBLE MANIFOLDS

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ABSTRACT. The simplicial space of proper piecewise linear embeddings of k-balls into n-dimensional contractible manifolds which are equal to a fixed embedding along the boundary is contractible if $n - k = 3$.

As a consequence of the Alexander isotopy theorem the simplicial space of flat (which equals locally flat unless $n - p = 2$) piecewise linear embeddings of D^p into D^n which are standard on the boundary, $E(D^p, D^n; \text{rel } \partial D^p)$, is contractible. The purpose of this note is to publicize the fact that this is often the case when D^n is replaced by any contractible p.l. manifold, N. Since ∂N need not be simply connected this result does not follow from Morlet's disjunction lemma [**1**], [**3**].

PROPOSITION. *If N^n is a contractible p.l. manifold and $n - p \geqslant 3$, then $\Pi_i(E(D^p, N; \text{rel } \partial D^p)) = 0$ for all i.*

A direct proof of this fact employs Theorem 1.11 of my AMS Memoir, *Piecewise linear concordances and isotopies* [**2**], to show that $\Pi_i(E(D^p, N; \text{rel } \partial D^p))$ is isomorphic to $\Pi_0(E(D^p \times I^i, N \times I^i; \text{rel } \partial(D^p \times I^i)))$, where one has a fixed standard embedding of $D^p \subset N, f$, and takes the associated standard embedding $f \times 1$: $D^p \times I^i \to N \times I^i$. One then analyzes the isotopy classes of embeddings of $D^p \times I^i$ in $N \times I^i$ as follows: Suppose f_{-1} and f_{+1} are two such embeddings–there is an associated embedding

$$f\colon \partial\left((D^p \times I^i) \times I\right) \to \partial\left((N \times I^i) \times I\right)$$

given by

$$f_{-1}|D^p \times I^i \times \{-1\}, \quad f_{+1}|D^p \times I^i \times \{+1\},$$

and

$$f_{-1} = f_{+1}|\partial(D^p \times I^i) \times I.$$

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We note that, if $n + i \geqslant 5$, $N \times I^i \times I$ is PL equivalent to an $n + i + 1$ ball, by the h-cobodism theorem, so that the Zeeman unknotting theorem implies that f extends to

$$f\colon (D^p \times I^i) \times I \to (N \times I^i) \times I$$

defining a concordance between f_{-1} and f_{+1}. Hudson's concordance implies isotopy theorem for $n - p \geqslant 3$ implies that f_{-1} and f_{1+1} are isotopic. Thus we need only consider the cases where $n + i \leqslant 4$. The case $p = 0$ is trivial since N is connected. The remaining case is $n = 4$, $p = 1$, $i = 0$. In this case $f|\partial(D^1 \times I)$ is constructed as above and extended as a map to $D^1 \times I$ since $N \times I$ is contractible. By general position, this extension can be made an embedding, and Hudson's theorem applied to the result completes the proof.

References

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