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ANR'S ADMITTING AN INTERVAL FACTOR ARE Q-MANIFOLDS

TERRY L. LAY AND JOHN J. WALSH

In this note we show that if an ANR admits an interval factor, then it must be a Q-manifold. The proof uses a characterization of Q-manifolds due to H. Toruńczyk [T]. As a corollary we get a partial affirmative answer to the question (see the problem list in [Ch]): Is Q the only compact, homogeneous, metric space homeomorphic to its own cone? The question is reduced to showing that such a space is an AR.

MAIN THEOREM. *If X is a locally compact ANR and $X \simeq X \times I$ ($\simeq$ = homeomorphic), then X is a Q-manifold.*

Since $X \simeq \operatorname{cone}(X)$ implies $X \simeq X \times I$ [S] and since Q is the only compact, contractible Q-manifold [Ch], we obtain the following corollary.

COROLLARY. *If X is a compact AR and $X \simeq \operatorname{cone}(X)$, then $X \simeq Q$.*

PROOF OF MAIN THEOREM. Using the theorem stated below, it suffices to show that for each $n > 0$ and pair of maps $f, g\colon I^n \to X$ there are arbitrarily close approximations f' and g' with $f'(I^n) \cap g'(I^n) = \varnothing$. A consequence of $X \simeq X \times I$ is that $X \simeq X \times I^{2n+1}$. Using the latter homeomorphism, f' and g' are obtained by general positioning in the second factor.

The following theorem is a widely known variation of the characterization given in [T]; for completeness, a reduction to the characterization stated in [T] is included.

THEOREM (TORUŃCZYK). *A locally compact ANR X is a Q-manifold if for each $n > 0$ and pair of maps $f, g\colon I^n \to X$ there are arbitrarily close approximations f' and g' to f and g such that $f'(I^n) \cap g'(I^n) = \varnothing$.*

PROOF. It must be shown that for each $n > 0$ and map $h\colon I^n \to X$, we can approximate h by a map h' with $h'(I^n)$ a z-set in X (see Theorem 1 of [T]). (A closed subset A of X is said to be a z-set if each mapping of Q into X can be approximated by a mapping of Q into $X \setminus A$.) First observe that for $f\colon I^n \to X$ and $u\colon Q \to X$ there are approximations f' and u' so that $f'(I^n) \cap u'(Q) = \varnothing$. Let $\{u_1, u_2, \dots\}$ be a countable dense set of maps from Q to X.

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Successively adjust h and the u_j's to obtain maps h_j and u'_j so that $h_j(I^n) \cap u'_i(Q) = \varnothing$ for $i < j$, $h' = \lim h_j$ is close to h, $h'(I^n) \cap u'_j(Q) = \varnothing$ for all j, and the set $\{u'_1, u'_2, \ldots\}$ is dense.

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DEPARTMENT OF MATHEMATICS, UNIVERSITY OF TENNESSEE, KNOXVILLE, TENNESSEE 37916