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COMPACT MULTIPLIERS ON BANACH ALGEBRAS

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ABSTRACT. An elementary proof is provided to show that for a large class of Banach algebras, the compact multipliers are trivial.

In [1] Dutta and Tewari prove the above result for Segal algebras on noncompact lca groups (i.e., those lca groups with nondiscrete dual groups). We give a much simpler proof in a more general setting.

Let A be a commutative semisimple Banach algebra considered as a subalgebra of $C(\Delta)$ where Δ denotes the maximal ideal space of A. We assume the following regularity condition: For each x in Δ and neighborhood V of x, there exists f in A with $\|f\| \equiv \|f\|_A \leqslant 1$ (or any constant independent of x and V), $f(x) = 1$, and whose support $\operatorname{supp} f \subset V$.

THEOREM. *Let A be a Banach algebra satisfying the above conditions and let T be a compact multiplier on A. Then T is trivial if Δ contains no isolated points.*

PROOF. Since T is a multiplier there exists a continuous function φ on Δ such that $T(f) = \varphi f$ for all f in A. We must show that $\varphi = 0$. Suppose that there exists x_0 in Δ such that $\varphi(x_0) \neq 0$. Choose $a > 0$ and a neighborhood V of x_0 such that $|\varphi(x)| \geqslant a$ for all x in V. Since x_0 is not an isolated point, we may choose a sequence $\{x_n\}$ in V with disjoint neighborhoods $\{V_n\}$ such that $x_n \in V_n \subset V$ for all n. For each n choose f_n in A with $f_n(x_n) = 1$, $\|f_n\| \leqslant 1$, and $\operatorname{supp} f \subset V_n$. Then for all n

$$\|T\| \geqslant \|\varphi f_n\| \geqslant \|\varphi f_n\|_\infty \geqslant a. \tag{1}$$

Since T is compact we have (passing to a subsequence) that $\varphi f_n \to h$ in A. So $h(x) = \lim(\varphi f_n)(x) = 0$ for all x. But by (1) $\|h\| \geqslant a$, which is a contradiction.

REFERENCES

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