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SEQUENTIAL CONDITIONS AND FREE TOPOLOGICAL GROUPS

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ABSTRACT. Most of the results in this paper concern relationships between sequential properties of a pointed topological space (X, p) and sequential properties of the Graev free topological group on X. In particular, it is shown that the free group over a sequential k_ω-space is sequential, and that a nondiscrete sequential free group has sequential order equal to ω_1 (the first uncountable ordinal). The free topological group on a space X which includes a convergent sequence contains a closed subspace homeomorphic to S_ω, a previously studied homogeneous, zero-dimensional sequential space. Finally, it is shown that there is no topological group homeomorphic to S_ω.

0. Introduction. In this paper we discuss relationships between sequential properties of a pointed topological space (X, p) and sequential properties of its Graev free topological group $F_G(X, p)$. Sequential spaces have been discussed in **[D]**, **[Fr$_1$]**, **[Fr$_2$]**, **[A-Fr]**, **[R]**; we make heavy use of the space S_ω of sequential order ω_1 constructed in **[A-Fr]**. The Graev free topological group $F_G(X, p)$ on a pointed Tychonoff space (X, p) was introduced in **[Gr$_1$]**; its topology has proved to be rather intractable, but in the last few years good results have been obtained in the case when X is a k_ω-space, that is, a weak union of countably many compact subsets **[O$_1$]**, **[H-M]**, **[MMO]**.

Definitions and preliminary results about sequential spaces appear in §1; §2 contains preliminaries about free topological groups and about k_ω-spaces. §3 contains results about sequential properties of free topological groups and their consequences. The result that S_ω supports no group structure (answering in the negative a question of S. P. Franklin) appears in §4.

We thank S. P. Franklin for posing to us the question just mentioned, and for several helpful conversations and suggestions.

1. Sequential spaces. Our definition of sequential spaces, sequential order, and the particular example S_ω, are based on **[A-Fr]**.

A subset U of a topological space X is *sequentially open* if each sequence converging to a point in U is eventually in U. The space X is *sequential* if each sequentially open subset of X is open. For each subset A of X, let $s(A)$ denote the set of all limits of sequences of points of A. X is of *sequential order* 1 (X is also called a *Fréchet space*) if $s(A)$ is the closure of A for every A.

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We define higher sequential orders by induction. Let $s_0(A) = A$, and for each ordinal $\alpha = \beta + 1$, let $s_\alpha(A) = s(s_\beta(A))$. If α is a limit ordinal, let $s_\alpha(A) = \bigcup \{s_\beta(A) | \beta < \alpha\}$. The sequential order of a sequential space X is the least ordinal α such that $s_\alpha(A)$ is the closure of A for every subset A of X; this order always exists and does not exceed the first uncountable ordinal ω_1.

By (S_1, s_0) we mean a space consisting of a single convergent sequence $s_1, s_2, s_3, \ldots$, together with its limit point s_0 taken as the basepoint.

(S_2, s_0) is a space obtained from S_1 by attaching to each isolated point s_n of S_1 a sequence $s_{n,1}, s_{n,2}, s_{n,3}, \ldots$ converging to s_n. S_2 may be viewed as a quotient of a disjoint union of convergent sequences; we give it the quotient topology. Inductively, we obtain the space (S_{n+1}, s_0) from (S_n, s_0) by attaching a convergent sequence to each isolated point of (S_n, s_0) and giving the resulting set the quotient topology.

Finally, let (S_ω, s_0) be the union of the sets $S_1 \subset S_2 \subset S_3 \subset \ldots$ with the weak union topology (a subset of S_ω is closed if and only if its intersection with each S_n is closed in the topology of S_n).

We shall use heavily the following facts from **[Fr$_1$]**, **[Fr$_2$]**, and **[A-Fr]**: S_n is sequential of order n; S_ω is sequential of order ω_1 and is countable, homogeneous, and zero-dimensional. A closed subspace of a sequential space (of order α) is sequential (of order $\leqslant \alpha$). A quotient of a sequential space is sequential. The pointed union (one-point union) of sequential spaces is sequential. The weak union of a nest of sequential spaces is sequential.

By a theorem of Boehme **[B]**, the cartesian product of a sequential space and a locally compact sequential space is again sequential. The local compactness condition cannot be entirely removed: the cartesian product of two sequential spaces (even of a metric space and a sequential space) need not be sequential. Let $(\mathcal{Q}, p)$ denote the rationals with basepoint $p = 0$; let (W, q) denote the union of countably many copies of the unit interval $[0, 1]$ with all the basepoints $q = 0$ identified and with the quotient topology. Then $\mathcal{Q}$ is a sequential (in fact, a metric) space and W is sequential, but $(\mathcal{Q} \times W, (p, q))$ is not sequential. To see this think of $\mathcal{Q} \times W$ as an infinite book: each leaf is $\mathcal{Q} \times I$ and the spine is $\mathcal{Q} \times \{q\}$. Pick a decreasing sequence of irrationals converging (in $\mathbf{R}$) to 0, say $\alpha_1, \alpha_2, \ldots$; in the nth leaf of the book pick a sequence $\{x_{n,i}\}_{i=1}^{\infty}$ in $\mathcal{Q} \times (0, 1]$ which converges to $(\alpha_n, 0)$ in $\mathbf{R} \times I$. Then $F = \bigcup_n \{x_{n,i}\}_{i=1}^{\infty}$ is sequentially closed, but not closed since (p, q) is in its closure.

If X is any topological space, one may impose a sequential topology on it by taking as open sets of the new topology all the sequentially open sets of the original topology. We denote this new space by $S(X)$ and call it the *sequential coreflection* of X. If X was sequential, $S(X)$ will have the same topology as X; if X was not sequential, $S(X)$ will have a strictly finer topology. For a more extensive discussion, see **[D]**. We will need the fact that if X is a topological space and A is a closed subset of X which is sequential in its inherited topology, then the topology which A inherits from $S(X)$ is the same one it inherits from X. To see this, observe that $S(X)$ has the same convergent sequences as X; the topology A inherits from X is sequential by hypothesis and the topology it inherits from $S(X)$ is sequential since A is closed in $S(X)$. Thus both topologies are completely determined by their convergent sequences which are the same.

2. Free topological groups and k_ω-spaces. Let (X, p) be a Tychonoff space with basepoint p. The *Graev free topological group* over (X, p) is a topological group $F_G(X, p)$ which is algebraically the free group on $X \setminus \{p\}$ and whose topology is the finest topology compatible with the group structure making the "insertion of generators" η: $(X, p) \to F_G(X, p)$ continuous ($\eta(p)$ is the group identity e). $F_G(X, p)$ has the usual properties associated with the word "free"; in particular, any continuous pointed map f: $(X, p) \to (G, e)$ into a topological group extends uniquely to a continuous homomorphism $\hat{f}$: $F_G(X, p) \to (G, e)$.

While the topology of $F_G(X, p)$ can be unpleasant in general **[F-O-T]**, **[H-M]**, it is tractable if X is a k_ω-space. A topological space is called a *k_ω-space* when it is the weak union of an increasing sequence of compact Hausdorff subspaces. The k_ω-spaces are very well behaved **[S]**, **[Fr-T]**, **[$\mathbf{O_1}$]**: they are closed hereditary, finitely productive, preserved by countable disjoint (or pointed) unions, and preserved by Hausdorff quotients.

The spaces S_n and S_ω are k_ω-spaces.

If (X, p) is a k_ω-space, then $F_G(X, p)$ is topologically (and algebraically) the weak union of the subspaces $(F_G(X, p))_n$ consisting of words of reduced length $< n$. Further, each of the subspaces $(F_G(X, p))_n$ is the quotient of the product $(X \cup_p X)^n$ (the cartesian product of n factors, each a pointed union of two copies of X) in a natural way. For details, see **[$\mathbf{O_1}$]**.

If G is a topological group and A is a subset of G, we say that *A generates G* provided that A algebraically generates G and that G has the finest topology compatible with both the group structure and the original topology on A (that is, any strictly finer group topology on G would induce a strictly finer topology on the subset A). Theorem 1 of **[MMO]** states that if A generates G and A is a k_ω-space, then G is also a k_ω-space in a natural way. We will need the following related result:

LEMMA 2.1. *Let G be a topological group and A a subset which generates it and contains the identity. Suppose A is a k_ω-space. Then the map $\hat{f}$: $F_G(A, e) \to (G, e)$ which extends the inclusion f: $A \hookrightarrow G$ is a quotient map.*

PROOF. Write $A = \bigcup_i A_i$, where the basepoint is in A_1 and the A_i's form an increasing sequence of compact Hausdorff subspaces of A which determines the topology of A. Then by the formulation in **[MMO]**, the ith compact Hausdorff subset of $F_G(A, e)$ (in a sequence determining the topology) may be taken to be the set of words which are products of at most i elements of $\eta(A_i)$, or their inverses, and the ith compact subset of G, in the quotient topology under $\hat{f}$, may be taken to be the set of products of at most i elements of $\hat{f}\eta(A_i)$, or their inverses. However, this is precisely the topology that G already has as a group generated by A, according to the proof of Theorem 1 of **[MMO]**.

3. Sequential conditions in free groups.

THEOREM 3.1. *A k_ω-space (X, p) is sequential if and only if its Graev free topological group, $F_G(X, p)$, is sequential.*

PROOF. Since (X, p) can be embedded as a closed subspace of $F_G(X, p)$, it must be sequential if $F_G(X, p)$ is. On the other hand, suppose (X, p) is sequential; write $X = \bigcup_i X_i$, where p is in X_1 and the X_i's form an increasing sequence of compact Hausdorff subspaces of X which determines the topology of X. Then each of the spaces $X_i \cup_p X_i$ is sequential and compact, so (by the theorem of Boehme) each $(X_i \cup_p X_i)^2$, and by induction each $(X_i \cup_p X_i)^n$, is sequential. Thus the weak union (in fact, k_ω-decomposition) $(X \cup_p X)^n = \bigcup_i (X_i \cup_p X_i)^n$ is sequential for each n. It follows that the quotients $(F_G(X, p))_n$ are sequential, and so the weak union $F_G(X, p)$ must be also.

COROLLARY 3.2. *Let G be a topological group generated (in our special sense) by a subset A. If A is a sequential k_ω-space, then so is G.*

PROOF. G is a k_ω-space by **[MMO]**. It is sequential since by Lemma 2.1, it is a quotient of the sequential space $F_G(A, p)$.

The free product $G * H$ of two topological groups **[Gr₂]**, **[O₂]** is generated by $G \cup_e H$. Hence:

COROLLARY 3.3. *Let G and H be topological groups which are sequential k_ω-spaces. Then their free product $G * H$ is also a sequential k_ω-space.*

From Theorem 3.1 we see that there are free topological groups which are sequential. Naturally, one wonders what happens when the "k_ω" requirement is dropped. As before, if the Graev free topological group $F_G(X, p)$ is sequential, then (X, p) must be sequential. The following example shows that the implication in the other direction fails:

EXAMPLE 3.4. Let X be the pointed union of $(\mathfrak{Q}, p)$ with (W, q). Since $\mathfrak{Q}$ and W are sequential, so is X. But by **[F-O-T]**, $F_G(X, p)$ contains a closed subspace homeomorphic to $\mathfrak{Q} \times W$; since $\mathfrak{Q} \times W$ is not sequential, neither is $F_G(X, p)$.

In Theorem 6.6 of **[D]**, Dudley proves the following: let P denote the set of all real functions of a real variable, with the topology of pointwise convergence. Let $S(P)$ denote the sequential coreflection of P. P is of course a topological group; however, assuming the continuum hypothesis, $S(P)$ is not a topological group: addition of functions is discontinuous. If we use the space $F_G(X, p)$ of the above example, we do not need the continuum hypothesis to obtain an example of a topological group whose sequential coreflection is not a compatible topology:

EXAMPLE 3.5. Let X be as in Example 3.4. $F_G(X, p)$ is a topological group. Since it is not sequential, its sequential coreflection $S(F_G(X, p))$ has a strictly finer topology, which, however, induces the original topology on the closed sequential subset $\eta(X)$. However, the free topology on $F_G(X, p)$ is the finest such topology compatible with the group operation; hence, the group operation is discontinuous in the topology of $S(F_G(X, p))$.

We now turn to the problem of determining the sequential order of a free topological group.

LEMMA 3.6. *Let* (X, p) *be a Tychonoff space and suppose there is a sequence of distinct terms* $x_1, x_2, x_3, \ldots$ *in* X *converging to* p. *Let* T *be the set* $\{p, x_1, x_2, \ldots\}$. *Then* $F_G(T, p)$ *is contained in* $F_G(X, p)$ *as a closed subgroup.*

PROOF. For a similar result, see Proposition 5.3 of **[O$_1$]**. We give an argument using a method of proof developed in **[H-M-T]**. Let βX be the Stone-Čech compactification of X. The inclusion $T \subset X \subset \beta X$ is an inclusion of the compact set T in the compact space βX and yields continuous homomorphisms $F_G(T, p) \to F_G(X, p) \to F_G(\beta X, p)$ where it may be easily checked (since $F_G(\beta X, p)$ is a k_ω-space) that $F_G(T, p) \to F_G(\beta X, p)$ is a closed embedding. Hence the image of $F_G(T, p)$ is closed in $F_G(X, p)$.

THEOREM 3.7. *Let* (X, p) *be a Tychonoff space and suppose there is a sequence of distinct terms* $x_1, x_2, x_3, \ldots$ *in* X *converging to* p. *Then there is a closed embedding* $f\colon (S_\omega, s_0) \to F_G(X, p)$.

PROOF. In view of Lemma 3.6, it will suffice to produce a closed embedding $f\colon (S_\omega, s_0) \to F_G(T, p)$. Enumerate the sequences of S_ω as follows: denote the single sequence of S_1 by $t_1 = s_1, s_2, s_3, \ldots$. Denote the sequence of S_2 converging to s_1 by $t_2 = s_{1,1}, s_{1,2}, s_{1,3}, \ldots$. Use a diagonalization process to enumerate all the sequences of S_ω; the limits of the sequences $t_3, t_4, t_5, t_6, t_7, t_8, \ldots$ are respectively $s_{1,1}, s_2, s_{1,1,1}, s_{1,2}, s_{2,1} s_3, \ldots$. The basic idea in constructing the function f is to map each sequence t_i into the set of words of $F_G(T, p)$ which have reduced length precisely i. Let $f(s_0) = p$. Let $f(t_1) = T \setminus \{p\}$, with $f(s_n) = x_n$. Let $f(t_2) = x_1x_1, x_1x_2, x_1x_3, \ldots$; let $f(t_3) = x_1x_1x_1, x_1x_1x_2, x_1x_1x_3, \ldots$; and let

$$f(t_4) = x_2x_1x_1x_1, x_2x_2x_2x_2, x_2x_3x_3x_3, \ldots.$$

Inductively, if the sequence t_i converges to $s_{j,k,\ldots,m}$, then $f(t_i) = f(s_{j,k,\ldots,m})x_1^r$, $f(s_{j,k,\ldots,m})x_2^r, \ldots$, where $r = i - (j + k + \ldots + m)$. Note that in view of the way the $s_{j,k,\ldots}$ were enumerated and the way the i are defined, the exponents r are always positive, and the function f is clearly one-to-one. Since f was chosen to preserve sequential convergence and S_ω is sequential, f is continuous. That f is a closed embedding follows readily from the fact that the intersection of its image with each $(F_G(T, p))_n$ consists of precisely n convergent sequences with their limits.

COROLLARY 3.8. *Let* (X, p) *be a Tychonoff space and suppose it contains some point which is the limit of a nonconstant sequence. Then* $F_G(X, p)$ *contains a closed subspace homeomorphic to* (S_ω, s_0).

PROOF. A convergent subsequence of distinct terms may be extracted from the given sequence in X. If the limit point is p, Theorem 3.7 applies. But by **[Gr$_1$]**, $F_G(X, p)$ is (up to isomorphism of topological groups) independent of the choice of basepoint p in X.

Since (S_ω, s_0) is sequential of order ω_1, we obtain:

THEOREM 3.9. *Let (X, p) be any Tychonoff space, and suppose $F_G(X, p)$ is sequential but not discrete. Then it is sequential of order ω_1.*

PROOF. As was noted before Example 3.4, X is sequential, and X is not discrete (or $F_G(X, p)$ would be also). Hence X contains a nonconstant convergent sequence, and $F_G(X, p)$ contains a copy of (S_ω, s_0). Hence the sequential order of $F_G(X, p)$ is at least ω_1; since that is the maximum possible sequential order of any space, it must be exactly ω_1.

It would be nice to take Corollary 3.8 one step further, leading to the following:

Question 3.10. Let $F_G(X, p)$ contain some nontrivial convergent sequence. Must it contain a copy of (S_ω, s_0)? A closed copy?

An affirmative answer would follow from an affirmative answer to:

Question 3.11. Let $F_G(X, p)$ contain a nontrivial convergent sequence. Need (X, p) contain a nontrivial convergent sequence?

4. Nongroupability of S_ω. Since we have now embedded (S_ω, s_0) as a closed subspace of $F_G(X, p)$ for many spaces (X, p), it is natural to ask if it can be embedded as a subgroup. Clearly, the embedding given here does not make it a subgroup, since it includes all elements of $\eta(T) \subset F_G(T, p)$ but not all products of three such elements. In fact, no embedding can make it a subgroup. The following result is even stronger:

THEOREM 4.1. *There is no topological group homeomorphic to (S_ω, s_0).*

Loosely, our strategy will be to show that while diagonals generally fail to converge in S_ω, in a topological group diagonals generally do converge. We need this lemma:

LEMMA 4.2. *In S_ω let y_0 be a fixed but arbitrary point. Let $y_1, y_2, y_3, \ldots$ be a sequence converging to y_0. For each i let $y_{i,1}, y_{i,2}, y_{i,3}, \ldots$ be a sequence converging to y_i. Suppose all the points $y_0, y_i, y_{i,j}$ are distinct. Then there is a function f: $\mathbf{N} \to \mathbf{N}$ ($\mathbf{N}$ is the natural numbers) with $f(i) > i$ for all i, such that the sequence $y_{i,f(i)}$, $i = 1, 2, 3, \ldots$ fails to converge.*

PROOF. We must describe the topology of S_ω in more detail. Consider the labelling of sequences in S_ω given in the proof of Theorem 3.7; let T_n denote the sequence t_n and its limit. Instead of considering S_ω to be the weak union of the spaces S_n, we may regard it as a quotient of the disjoint union of the sequences T_n. With this viewpoint, we see that a subset A of S_ω is closed if and only if every intersection $A \cap T_n$ is closed. If $n \neq m$, T_n and T_m intersect in at most one point, and if there is such a point it is the (unique) limit point of exactly one of T_n or T_m. Each point of S_ω is the limit point of exactly one T_n and, except for s_0, a nonlimit point of exactly one other. A sequence converges only if it is eventually in some T_n, and then, if it is not eventually constant, it converges to the limit point of that T_n.

Now, y_0 is the limit point of some T_n, say T_{n_0}, and the sequence $y_1, y_2, y_3, \ldots$ is eventually in T_{n_0}. Each y_i is the limit point of some T_n, say T_{n_i}, and the sequence $y_{i,1}, y_{i,2}, y_{i,3}, \ldots$ is eventually in T_{n_i}. Pick f so that $f(i) > i$ and $y_{i,f(i)} \in T_{n_i}$ for each i.

Now $y_{i,f(i)}$ is a nonlimit point of T_{n_i} for each i; hence at most two $y_{i,f(i)}$ lie in any T_n, the sequence $y_{i,f(i)}$, $i = 1, 2, 3, \ldots,$ is not eventually in any T_n, and so this sequence cannot converge.

PROOF OF THEOREM 4.1. Suppose S_ω to be a topological group. Let x_0 be a point in S_ω and let $x_1, x_2, x_3, \ldots$ be a sequence converging to x_0 such that all the points $x_0, x_1, x_2, \ldots$ are distinct and none is the group identity. Denote the multiplication on S_ω by $m(,)$.

We shall select some points in S_ω to fill the roles of the points in the statement of Lemma 4.2. Let y_0 be $m(x_0, x_0)$. Let y_i be $m(x_0, x_i)$ for $i = 1, 2, 3, \ldots$. Then y_0 and the y_i are distinct and $\lim y_i = y_0$. We next choose the $y_{1,j}$, $j = 1, 2, 3, \ldots$ as follows: the sequence $m(x_j, x_1)$, $j = 1, 2, 3, \ldots,$ converges to $m(x_0, x_1)$. It is eventually disjoint from the sequence $m(x_0, x_i)$ because the two sequences have different limits. Hence there is a positive integer k_1 such that for $j > k_1 + 1$, the points $m(x_j, x_1)$ are distinct from all points chosen thus far. Pick as $y_{1,j}$ the point $m(x_{k_1+j}, x_1)$. Having picked sequences $y_{i,j}$, $j = 1, 2, 3, \ldots,$ for $i < n$, proceed by induction: Note that $m(x_j, x_n)$, $j = 1, 2, 3, \ldots,$ converges to $m(x_0, x_n)$ and is eventually disjoint (for $j > k_n$) from the set consisting of y_0, the y_i's, and all the previously chosen $y_{i,j}$'s; pick k_n sufficiently large and let $y_{n,j} = m(x_{k_n+j}, x_n)$.

The y_0, y_i's, and $y_{i,j}$'s we have selected from S_ω meet the conditions of Lemma 4.2. Now let $f\colon \mathbf{N} \to \mathbf{N}$ be any function with $f(i) > i$ for all i. $\lim x_i = x_0$ (all limits are taken as $i \to \infty$); also $\lim x_{f(i)+k_i} = x_0$ since if $i > M$, $f(i) + k_i > M$. Hence in $S_\omega \times S_\omega$, $\lim(x_{f(i)+k_i}, x_i) = (x_0, x_0)$. Now by continuity of multiplication in S_ω,

$$\lim y_{i,f(i)} = \lim m(x_{f(i)+k_i}, x_i) = m(x_0, x_0) = y_0$$

contradicting Lemma 4.2 and completing the proof of Theorem 4.1.

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