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SPACES OF AANR'S

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ABSTRACT. For a metric space X, let $\mathrm{AANR}_C(X)$ denote the hyperspace of all nonempty approximative absolute neighborhood retracts in the sense of Clapp in X topologized with the metric of continuity. We show that $\mathrm{AANR}_C(X)$ is topologically complete iff X is topologically complete. Some subsets of the first Baire category in $\mathrm{AANR}_C(X)$ for a Q-manifold X are identified. For example, the collection $\mathrm{AANR}_N(X)$ of all nonempty approximative absolute neighborhood retracts in the sense of Noguchi in X is such a subset.

Introduction. In 1953 Noguchi [N] introduced a generalization of the notion of an absolute neighborhood retract (ANR) which is now called an approximative absolute neighborhood retract in the sense of Noguchi (AANR_N). A further generalization was given in 1971 by Clapp [C] and is now known as an approximative absolute neighborhood retract in the sense of Clapp (AANR_C). The importance of these generalizations is that they share many properties with ANR's (for example, certain fixed point properties). On the other side, they include compacta with local pathologies.

Recently, several authors studied AANR_C's using techniques of shape theory. In particular, Borsuk [**B2**] described them as NE-sets, Mardešić [**M**] characterized them as approximate polyhedra, and the author [**Č2**] observed that AANR_C's coincide with P-e-movable compacta, where P denotes the class of all finite polyhedra.

The main results of this paper, described in the above abstract, are obtained in the following way. First we define the notion of a P-e-movably regular convergence for compacta in a metric space X. The limit of a P-e-movably regularly convergent sequence must be P-e-movable (i.e., an AANR_C) so that a sequence $\{A_n\}$ in $\mathrm{AANR}_C(X)$ converges P-e-movably regularly to $A_0 \in \mathrm{AANR}_C(X)$ iff $\lim d_C(A_n, A_0) = 0$, where d_C is Borsuk's metric of continuity [**B1**]. Then we apply the method of investigating topological properties of the hyperspace of all P-movable compacta in X with the topology induced by P-movably regular convergence from [**Č1**] to $\mathrm{AANR}_C(X)$ using P-e-movably regular convergence. Hence, in essence, our proof of the statement about topological completeness relies on Begle's method in [**Be**].

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We assume that the reader is familiar with the theory of shape **[B3]** and with the infinite-dimensional topology **[Ch]**.

Throughout the paper P will denote the class of all finite polyhedra. A map will be called a *P-map* provided its domain is a member of P.

We shall say that maps f and g of a space Z into a metric space (X, d) are *ε-close* provided $d(f(z), g(z)) < \varepsilon$ for every $z \in Z$. If Z is a subset of X and f is ε-close to the inclusion $i_{Z,X}$ of Z into X, we call f an *ε-map*. For compacta A and B in X define $d_C(A, B) = \inf\{\varepsilon | \exists\ \varepsilon\text{-maps } A \to B \text{ and } B \to A\}$.

If not stated otherwise, we reserve X for an arbitrary metric space with a fixed metric d; $A_0, A_1, A_2, \ldots$ are compact subsets of X; d_H is the Hausdorff metric and d_C is the metric of continuity (defined above) on the hyperspace 2^X of all nonempty compacta in X; M is an ANR for the class of all metrizable spaces which contains X metrically; a neighborhood means an open neighborhood; and $N_\varepsilon(A_0)$ denotes the ε-neighborhood of A_0 in M.

2. P-e-movably regular convergence. Recall **[C]** that a compactum A is an AANR_C provided for every embedding of A into a metric space X and every $\varepsilon > 0$, there is a neighborhood V of A in X and a map $r\colon V \to A$ such that the restriction $r|A$ is an ε-map. We proved in **[Č2]** that a compactum A is an AANR_C iff A is P-e-movable, i.e., iff for some (and hence for every) embedding of A into an ANR M the following holds. For each $\varepsilon > 0$ there is a neighborhood V of A in M such that every P-map $f\colon K \to V$ is ε-close to a map $f'\colon K \to A$.

(2.1) DEFINITION. A sequence $A_1, A_2, \ldots$ of compacta in a metric space X which lies in an ANR M is said to *converge P-e-movably regularly in M* to a compactum A_0 in X provided

(i) $\lim d_H(A_n, A_0) = 0$, and

(ii) for every $\varepsilon > 0$ there is a neighborhood V of A_0 in M and an index k_0 such that for every P-map $f\colon K \to V$ and every $k \geq k_0$, there is a map $f'\colon K \to A_k$ which is ε-close to f.

The definition (2.1) is shape theoretic in the sense that P-e-movably regular convergence is independent of the choice of M. If a sequence $\{A_n\}$ of compacta in X converges P-e-movably regulary to a compactum A_0 in X in some (and hence in every) ANR containing X we shall write $A_n - mo_P^e \to A_0$.

(2.2) LEMMA. *Let $A_n - mo_P^e \to A_0$. Then A_0 is P-e-movable regardless of the nature of A_n's.*

PROOF. Let an $\varepsilon > 0$ be given. By **[Č2]**, it suffices to prove that there exists a neighborhood V of A_0 in M such that for every P-map $f\colon K \to V$ and every neighborhood W of A_0 in M, there is a map $f'\colon K \to W$ which is ε-close to f. This property will have any neighborhood V of A_0 in M satisfying (ii) in (2.1). Indeed, if W is an arbitrary neighborhood of A_0 in M, then there is an index $k \geq k_0$ such that $A_k \subset W$. The choice of V and k_0 implies that every P-map $f\colon K \to V$ is ε-close to a map $f'\colon K \to A_k$ and hence to a map of K into W.

(2.3) LEMMA. *$A_n - mo_P^e \to A_0$ iff A_0 is P-e-movable and* $\lim d_C(A_n, A_0) = 0$.

PROOF. Let A_0 be P-e-movable and suppose $\lim d_C(A_n, A_0) = 0$. For any $\varepsilon > 0$ there is a neighborhood V of A_0 in M and an index n_0 such that every P-map f: $K \to V$ is $(\varepsilon/2)$-close to a map f': $K \to A_0$ and $d_C(A_n, A_0) < \varepsilon/2$ for all $n > n_0$. For each $n \geqslant n_0$, pick an $(\varepsilon/2)$-map g_n: $A_0 \to A_n$. Then f is ε-close to each of the compositions $g_n \circ f'$: $K \to A_n$ $(n \geqslant n_0)$. Hence, $A_n - mo_P^e \to A_0$.

Conversely, suppose that $A_n - mo_P^e \to A_0$. Without loss of generality, we can assume that X is a subset of the Hilbert cube Q. By Lemma (2.2), A_0 is P-e-movable. Hence, for every $\varepsilon > 0$, there is a neighborhood V of A_0 in Q and an index n_0 such that, for $n = 0$ and $n > n_0$ and for every P-map f: $K \to V$, $A_n \subset V$ and there is a map f': $K \to A_n$ which is $(\varepsilon/2)$-close to f. On the other hand, for each such n, there are $(\varepsilon/2)$-maps g_n: $A_n \to V$ with $K_n = g_n(A_n)$ a finite polyhedron. The choice of V and n_0 implies that the P-map $i_0 = i_{K_0, V}$: $K_0 \to V$ is $(\varepsilon/2)$-close to a map i'_{0n}: $K_0 \to A_n$ $(n \geqslant n_0)$ and that each P-map $i_n = i_{K_n, V}$: $K_n \to V$ $(n > n_0)$ is $(\varepsilon/2)$-close to a map i'_{n0}: $K_n \to A_0$. Clearly, for every $n > n_0$, $i'_{0n} \circ g_0$: $A_0 \to A_n$ and $i'_{n0} \circ g_n$: $A_n \to A_0$ are ε-maps. Hence, $d_C(A_n, A_0) < \varepsilon$ for each $n > n_0$. Since ε was arbitrary, $\lim d_C(A_n, A_0) = 0$.

3. The hyperspace $\mathrm{AANR}_C(X)$. The collection of all AANR_C's (or, equivalently, of all P-e-movable compacta) in a metric space X can be made into a hyperspace $\mathrm{AANR}_C(X)$ by defining the notion of convergence in $\mathrm{AANR}_C(X)$ be means of P-e-movably regular convergence. By Lemma (2.3), $\mathrm{AANR}_C(X)$ is a metric space because its topology is induced by the metric of continuity. In this section we shall use Begle's method in **[Be]** to define a new metric d_{mo}^e on $\mathrm{AANR}_C(X)$ which is equivalent to d_C. Both d_C and d_{mo}^e need not be complete (see **[B1**, Example 5]). However, the results in **[Be]** imply that the metric d_{mo}^e (and therefore also the metric d_C) is equivalent to a complete metric on $\mathrm{AANR}_C(X)$ when X is a complete metric space. Since X is clearly homeomorphic with a closed subset of $\mathrm{AANR}_C(X)$, we get that $\mathrm{AANR}_C(X)$ is topologically complete iff X is topologically complete. We can also conclude that $\mathrm{AANR}_C(X)$ is a separable metric space when X is a separable metric space (this was first observed in **[C]**).

In this section we assume that X lies in an AR space M of diameter 1.

(3.1) DEFINITION. For a compact subset A of M and an $\varepsilon > 0$, let $\delta(\varepsilon, A)$ be the least upper bound of all δ, $0 \leqslant \delta \leqslant \varepsilon$, such that every P-map f: $K \to N_\delta(A)$ is ε-close to a map f': $K \to A$.

It is clear that for each compactum A in M, $\delta(\varepsilon, A)$ always exists and is a nonnegative, monotone, nondecreasing, and hence measurable, function on the half-open interval $I^* = (0, 1]$. If A is P-e-movable, then $\delta(\varepsilon, A) > 0$ everywhere in I^* and conversely.

The following three lemmas resemble Lemmas (3.2), (3.3), and (3.4) in **[Č1]**, respectively. In order to prove them, one must simply require in the corresponding proofs in **[Č1]** that maps are small. We shall illustrate those changes by presenting the proof of Lemma (3.4).

(3.2) LEMMA. *If* $\lim d_H(A_n, A_0) = 0$, *then* $A_n - mo_P^e \to A_0$ *iff* $\liminf \delta(\varepsilon, A_n) > 0$ *for each* ε *in* I^*.

(3.3) LEMMA. *If* $\lim d_H(A_n, A_0) = 0$ *and* $A_0 \in \mathrm{AANR}_C(X)$, *then* $\limsup \delta(\varepsilon_0, A_n) \leqslant \delta(\varepsilon_0, A_0)$ *for all* $\varepsilon_0 \in I^*$ *for which* $\delta(\varepsilon, A_0)$ *is right-hand continuous at* ε_0.

(3.4) LEMMA. *If* $A_n - mo_P^e \to A_0$, *then* $\delta(\varepsilon_0, A_0) \leqslant \liminf \delta(\varepsilon_0, A_n)$ *for all* $\varepsilon_0 \in I^*$ *for which* $\delta(\varepsilon, A_0)$ *is left-hand continuous at* ε_0.

PROOF. Let us consider a point $\varepsilon_0 \in I^*$ at which the function $\delta(\varepsilon, A_0)$ is left-hand continuous. Suppose that $\delta(\varepsilon_0, A_0) > \liminf \delta(\varepsilon_0, A_n)$. Then there is an e, $0 < e < \varepsilon_0$, and a subsequence $\{A_{n_i}\}$ of $\{A_n\}$ such that $\delta(\varepsilon_0, A_{n_i}) + e < \delta(\varepsilon_0, A_0) - e$ for all $i > 0$. Since the function $\delta(\varepsilon, A_0)$ is left-hand continuous at ε_0, there is a number d, $0 < 2d < e$, such that $\delta(\varepsilon, A_0) \in (\delta(\varepsilon_0, A_0) - e, \delta(\varepsilon_0, A_0) + e)$ whenever $\varepsilon \in (\varepsilon_0 - 2d, \varepsilon_0] \cap I^*$. In particular, $\delta(\varepsilon_0 - d, A_0) > \delta(\varepsilon_0, A_{n_i}) + e$ for all $i > 0$.

Now, we select a δ, $0 < \delta < d/2$, and a j_0 such that for every P-map f: $K \to N_\delta(A_0)$ and every $j \geqslant j_0$ there is a map f': $K \to A_j$ which is $(\varepsilon_0 - d)$-close to f. Finally, pick k so that $n_k \geqslant j_0$ and $d_C(A_{n_k}, A_0) < \delta$. Since $N_{\delta(\varepsilon_0, A_{n_k}) + (e/2)}(A_{n_k}) \subset N_{\delta(\varepsilon_0 - d, A_0)}(A_0)$, every P-map f: $K \to N_{\delta(\varepsilon_0, A_{n_k}) + (e/2)}(A_{n_k})$ is ε_0-close to a map f': $K \to A_{n_k}$. This however contradicts the definition of $\delta(\varepsilon_0, A_{n_k})$ and therefore proves the lemma.

From Lemmas (3.3) and (3.4) we have the following theorem resembling Theorem 1 in **[Be]**.

(3.5) THEOREM. *If* $A_n - mo_P^e \to A_0$, *then* $\lim \delta(\varepsilon, A_n)$ *exists and equals* $\delta(\varepsilon, A_0)$ *almost everywhere in* I^*.

We are now ready to introduce the metric d_{mo}^e on the space $\mathrm{AANR}_C(X)$. Let E be the Banach space of all bounded measurable functions on the interval I^*, the norm of an element f in E being defined as

$$\|f\| = \int_0^1 |f| \, d\varepsilon.$$

We can consider $\mathrm{AANR}_C(X)$ as a subset of $2^X \times E$ if we identify each element A of $\mathrm{AANR}_C(X)$ with the pair $(A, \delta(\varepsilon, A))$ in $2^X \times E$. Hence a metric is defined in $\mathrm{AANR}_C(X)$ by letting the distance between two points in $\mathrm{AANR}_C(X)$ be the distance between the corresponding points in $2^X \times E$. Specifically,

$$d_{mo}^e(A, B) = \left[d_H^2(A, B) + \left(\int_0^1 |\delta(\varepsilon, A) - \delta(\varepsilon, B)| \, d\varepsilon \right)^2 \right]^{1/2}.$$

With obvious modifications the arguments on pages **444-446** in **[Be]** (using Lemmas (3.2), (3.3), (2.2), and (3.4) instead of Begle's Lemmas 1, 3, 4, and 5, respectively) show that this metric induces the same topology on $\mathrm{AANR}_C(X)$ as the metric of continuity (or, equivalently, as that defined in terms of P-e-movably regular convergence) and proves the following two theorems.

(3.6) THEOREM. *For a metric space* X, *the following are equivalent.*

(i) X *is topologically complete.*

(ii) $\mathrm{AANR}_C(X)$ *is topologically complete.*

(iii) $\mathrm{AANR}_C(X)$ *is a* G_δ *in* $2^X \times E$.

(3.7) THEOREM [C, THEOREM 3.2]. *If X is a separable metric space, then* $\mathrm{AANR}_{\mathrm{C}}(X)$ *is a separable metric space.*

(3.8) COROLLARY. *The hyperspace* $\mathrm{AAR}(X)$ *of all approximative absolute retracts* [C] *in a metric space X topologized by the metric of continuity is topologically complete iff X is topologically complete.*

PROOF. Since an $\mathrm{AANR}_{\mathrm{C}}$ is an AAR iff it has trivial shape **[Bo]**, $\mathrm{AAR}(X)$ is a closed subset of $\mathrm{AANR}_{\mathrm{C}}(X)$ **[ČŠ]**.

(3.9) REMARK. Applying Theorem 1 in **[K]** it is possible to exhibit for a complete metric space X a complete metric $\bar{d}^e_{mo}$ on $\mathrm{AANR}_{\mathrm{C}}(X)$ which is equivalent with both d_C and d^e_{mo}. One such metric is defined by

$$\bar{d}^e_{mo}(A, B) = d^e_{mo}(A, B) + \sum_{n=1}^{\infty} \frac{1}{2^n} \cdot \frac{|g_n(A) - g_n(B)|}{|g_n(A) - g_n(B)| + |g_n(A) \cdot g_n(B)|},$$

where $g_n(A) = \int_0^{1/n} \delta(\varepsilon, A)\, d\varepsilon$.

(3.10) REMARK. All of the results in §§2 and 3 (except (2.3)) are also true when the class P is replaced by an arbitrary class C of topological spaces. Hence, there are versions of Theorems (3.6) and (3.7) for the hyperspace $mo^e_C(X)$ of all C-e-movable compacta **[Č2]** in a metric space X. It might appear to the reader that to consider C-e-movable instead of P-e-movable compacta (or $\mathrm{AANR}_{\mathrm{C}}$'s) is an uninteresting generalization. However, the following theorem from **[Č2]** shows that this maybe is not so.

(3.11) If a tree-like continuum X is T-e-movable, where T denotes the class of all compact trees, then X has the fixed point property.

This result implies that the question as to whether every tree-like plane continuum has the fixed point property will have an affirmative answer provided the following conjecture is true.

(3.12) CONJECTURE. Every tree-like plane continuum is T-e-movable.

We proved in (3.6) and (3.7) that the hyperspace of all T-e-movable plane compacta can be organized into a complete separable metric space.

(3.13) REMARK. An interesting consequence of the method of proof of Theorem (3.6) is that the hyperspace $\mathrm{AANR}_{\mathrm{N}}(X)$ of all $\mathrm{AANR}_{\mathrm{N}}$'s in a complete metric space X can be topologized as a complete metric space. Indeed, since a compactum is an $\mathrm{AANR}_{\mathrm{N}}$ iff it is an $\mathrm{AANR}_{\mathrm{C}}$ and an FANR **[Bo]** and $\mathrm{AANR}_{\mathrm{C}}$'s are movable **[Bo]**, we see that a compactum is an $\mathrm{AANR}_{\mathrm{N}}$ iff it is an $\mathrm{AANR}_{\mathrm{C}}$ and is calm **[ČŠ]**. Using the author's results in **[Č3]** and Theorem (3.6), it is easy to check that the following metric d_{No} on $\mathrm{AANR}_{\mathrm{N}}(X)$ is equivalent to a complete metric when X is a complete metric space. For $A, B \in \mathrm{AANR}_{\mathrm{N}}(X)$, define

$$d_{No}(A, B) = \left[(d^e_{mo}(A, B))^2 + (d_{ca}(A, B))^2 \right]^{1/2},$$

where d_{ca} is the metric of P-calmly regular convergence from **[Č3]**.

4. Subsets of $\mathrm{AANR}_{\mathrm{C}}(X)$ of the first category. In this section we shall make the assumption that X is a Q-manifold. Since every Q-manifold is topologically complete, $\mathrm{AANR}_{\mathrm{C}}(X)$ is, by (3.6), topologically complete. Hence, a statement about the Baire category of a subset of $\mathrm{AANR_C}(X)$ is meaningful. The following results, of this type, are of some interest.

(4.1) THEOREM. *The collection of all* $\mathrm{AANR_C}$*'s of finite fundamental dimension in* X *is of the first Baire category in* $\mathrm{AANR_C}(X)$.

PROOF. Let $mo_n^e(X)$ denote all $\mathrm{AANR_C}$'s in X of fundamental dimension $\leqslant n$ and let $mo_*^e(X) = \bigcup_{n=0}^{\infty} mo_n^e(X)$. Clearly, $A \in \mathrm{AANR_C}(X)$ has finite fundamental dimension iff $A \in mo_*^e(X)$. In order to prove that $mo_*^e(X)$ is of the first Baire category it suffices to show that each $B_n = \mathrm{AANR_C}(X) - mo_n^e(X)$ is both open and dense in $\mathrm{AANR_C}(X)$. It was proved in [ČŠ] that $mo_n^e(X)$ is closed in $(2^X, d_C)$. Hence, B_n is open in $\mathrm{AANR_C}(X)$. Let $\mathrm{AANR_C^Z}(X)$ denote all $\mathrm{AANR_C}$'s in X which are Z-sets in X. Since, by the Mapping Replacement Theorem [**Ch**], $\mathrm{AANR_C^Z}(X)$ is dense in $\mathrm{AANR_C}(X)$, B_n will be dense in $\mathrm{AANR_C}(X)$ provided $B_n^Z = B_n \cap \mathrm{AANR_C^Z}(X)$ is dense in $\mathrm{AANR_C^Z}(X)$. To prove this, consider an $A \in \mathrm{AANR_C^Z}(X)$ and let $p \in A$. For every integer $k > 0$, pick a Z-set $\mathrm{AANR_C}$ A_k' in X with infinite fundamental dimensional which does not intersect A and lies in $N_{1/k}(p)$. An example of an $\mathrm{AANR_C}$ which is not of finite fundamental dimension is a one-point compactification of an infinite string of spheres of higher and higher dimension with two adjacent intersecting only in a single point. This space is an $\mathrm{AANR_C}$ by Theorem 2.3 in [**C**]. Let $A_k = A \cup A_k'$. By [**B2**, Theorem (11.1)], each A_k is also an $\mathrm{AANR_C}$. Clearly, $\lim d_C(A_k, A) = 0$ so that the sequence $\{A_k\}$ in B_n^Z converges in $\mathrm{AANR_C^Z}(X)$ to A.

(4.2) THEOREM. *The collection of all* $\mathrm{AANR_C}$*'s in* X *with some Betti number finite is of the first Baire category in* $\mathrm{AANR_C}(X)$.

PROOF. Let $mo_{B(n,m)}^e(X)$ denote all $\mathrm{AANR_C}$'s A in X with the nth Bettti number $p^n(A) \leqslant m$ and let $mo_{B(*)}^e(X) = \bigcup_{n,m=0}^{\infty} mo_{B(n,m)}^e(X)$. Then some Betti number of $A \in \mathrm{AANR_C}(X)$ is finite iff $A \in mo_{B(*)}^e(X)$. Put $B_{n,m} = \mathrm{AANR_C}(X) - mo_{B(n,m)}^e(X)$. As in (4.1), it suffices to prove that $B_{n,m}^Z = B_{n,m} \cap \mathrm{AANR_C^Z}(X)$ is dense in $\mathrm{AANR_C^Z}(X)$ because $B_{n,m}$ is open in $\mathrm{AANR_C}(X)$ by [**B1**, §3]. This can be done as in the proof of (4.1). The only difference is that now we take for A_k' a compactum such that $p^n(A_k') > m$.

(4.3) COROLLARY. *The collection* $\mathrm{AANR_N}(X)$ *of all approximative absolute neighborhood retracts in the sense of Noguchi* [**N**] *in* X *is of the first Baire category in* $\mathrm{AANR_C}(X)$.

PROOF. Gmurczyk [**G**] proved that all Betti numbers of an $\mathrm{AANR_N}$ are finite. Hence, $\mathrm{AANR_N}(X) \subset mo_{B(*)}^e(X)$ and we can apply (4.2).

(4.4) COROLLARY. *The collection of all compact* ANR*'s in* X *is of the first Baire category in* $\mathrm{AANR_C}(X)$.

PROOF. Recall [**N**] that every compactt ANR is an $\mathrm{AANR_N}$.

Using similar techniques one can also prove the following theorem.

(4.5) THEOREM. *The collection of all* AANR_C*'s in* X *with some shape group finitely presented is of the first Baire category in* $\mathrm{AANR}_C(X)$.

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