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THE STRUCTURE OF A CRITICAL SET OF A COMPLETE INTERSECTION SINGULARITY

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ABSTRACT. Let f: $(C^n, 0) \to (C^k, 0)$ be the germ of an isolated complete intersection singularity. The structure of strata $\mu = \text{const}$ in a critical set of f is studied. The main result is the following: if the dimension of the stratum $\mu = \mu(0)$ is $k-1$, then f coincides with a family of hypersurface singularities with constant Milnor number.

1. We give here some definitions and results, concerning isolated complete intersection singularities (details can be found e.g., in [**2**]) and state the main theorem.

Let f: $(C^n, 0) \to (C^k, 0)$, $n \geq k$, be the germ of a flat analytic mapping, $Y = f^{-1}(0)$, and let Y have an isolated singular point at the origin. Such germs f (and Y) will be called shortly SCI. The critical set $\Sigma(f)$ of f is in this case $k-1$-dimensional and $f/\Sigma_{\text{red}}(f)$ is finite. ($\Sigma(f)$ is considered with the structural ring $\mathcal{O}_{\Sigma(f)} = \mathcal{O}_n/J$, where $\mathcal{O}_n$ is the ring of germs of analytic functions at $0 \in C^n$, and $J \subset \mathcal{O}_n$ is the ideal, generated by all $(k \times k)$-minors of the Jacobian matrix of f.)

The Milnor number $\mu_f(0)$ of f (or of Y) at $0 \in C^n$ is defined as the middle Betti number of the Milnor fiber $F = f^{-1}(\xi) \cap B_\varepsilon^n$. Here B_ε^n is the ε-ball, centered at $0 \in C^n$, ξ a regular value of f, $1 \gg \varepsilon \gg |\xi|$.

Let G_k^l be the Grassman manifold of l-dimensional subspaces of C^k. There is a Zariski open subset $U_l \subset G_k^l$, such that for $L \in U_l$, f_L: $(C^n, 0) \to C^k/L \cong (C^{k-l}, 0)$ is SCI, and the Milnor number of this singularity does not depend on $L \in U_l$. This number will be denoted by $\mu_f^{k-l}(0)$, $\mu_f^k(0) = \mu_f(0)$.

Now let f: $M^n \to N^k$, $n \geq k$, be a flat mapping of regular complex manifolds, such that $f/\Sigma(f)$ is finite. Then for any $z \in M$ the germ of f at z is SCI and setting $\mu_f(z)$ to be equal to the Milnor number of this singularity, we define an integral function μ_f on M. The function μ_f is upper semicontinuous and $\mu_f(z) > 0$ if and only if $z \in \Sigma(f)$.

Let $W_\nu(f) = \{z \in M \mid \mu_f(z) \geq \nu\}$, $V_\nu(f) = \{z \in M \mid \mu_f(z) = \nu\} = W_\nu - W_{\nu+1}$. Then $W_\nu(f)$ are analytic subsets of M, $W_0(f) = M$, $W_1(f) = \Sigma(f)$, and if M is compact, $W_\nu(f) = \varnothing$ for ν sufficiently big.

We consider also analytic subsets of M, $\sigma_s(f) = \{z \in M \mid \operatorname{rank} df(z) \leq k - s\}$, $s = 1, \ldots, k$. Clearly $\Sigma(f) = \sigma_1(f) \supseteq \cdots \supseteq \sigma_k$, and since $f/\Sigma(f)$ is finite, $\dim \sigma_s \leq k - s$. (By [**1**, Corollary 1.7], if $z \in \sigma_s(f)$, then $\mu_f(z) \geq m(n, s) = \sum_{q=0}^{s-1} C_{n-s+q}^{n-s} 2^q$, and hence $\sigma_s(f) \subseteq W_{m(n,s)}(f)$.)

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Let $N' \subset N$ be an analytic submanifold of N such that f is transversal to N', $M' = f^{-1}(N')$, $f' = f|M'$: $M' \to N'$. Then, clearly, $\mu_{f'} = \mu_f|M'$, $V_\nu(f') = V_\nu(f) \cap M'$, $\sigma_s(f') = \sigma_s(f) \cap M'$.

The sign f will be omitted below in notations of $\Sigma(f)$, μ_f, $V_\nu(f)$, $W_\nu(f)$, $\sigma_s(f)$.

In the local case of SCI f: $(C^n, 0) \to (C^k, 0)$, μ, W_ν, V_ν, σ_s are germs at $0 \in C^n$. Note, that $V_{\mu(0)} = W_{\mu(0)}$ is an analytic germ.

The germ f: $(C^n, 0) \to (C^k, 0)$ is said to be equivalent to g: $(C^n, 0) \to (C^k, 0)$ if there exist germs of analytic diffeomorphisms φ: $(C^n, 0) \to (C^n, 0)$ and ψ: $(C^k, 0) \to (C^k, 0)$, such that $\psi \circ f = g \circ \varphi$.

We say f: $(C^n, 0) \to (C^k, 0)$ is a family of hypersurface singularities with $\mu = \text{const}$, if f is equivalent to the germ Φ: $(C^{k-1} \times C^{n-k+1}, 0) \to (C^{k-1} \times C, 0)$, $\Phi(t, y) = (t, h_t(y))$, with h_t: $(C^{n-k+1}, 0) \to (C, 0)$ having for any t an isolated singularity at $0 \in C^{n-k+1}$ with constant Milnor number.

Note, that for $n - k \neq 2$ a family of hypersurface singularities with $\mu = \text{const}$ is topologically trivial by [7].

THEOREM 1.1. *Let f: $(C^n, 0) \to (C^k, 0)$ be SCI, and let* $\dim_0 V_{\mu(0)} = k - 1$. *Then f is a family of hypersurface singularities with $\mu = \mu(0)$.*

COROLLARY 1.2. *If for $z \in \Sigma_{\text{red}}$, $\dim_z V_{\mu(z)} = k - 1$, then in a neighborhood of z $\Sigma_{\text{red}} = V_{\mu(z)}$ is smooth, and the restriction f/Σ_{red} is regular at z. In particular, $z \in \Sigma_{\text{red}} \setminus \sigma_2$.*

Originally, Theorem 1.1 was obtained under the additional restriction rank $df(0) \geqslant k - 3$. The author would like to thank G. M. Greuel, who noted that a modification of arguments gives a proof of a present version of the theorem.

2. In this section we state some results of **[8]**, concerning the global topology of strata V_ν, which are used below.

Let f: $M^n \to N^k$, μ, V_ν, W_ν be as above, with M and N compact.

THEOREM 2.1.

$$\sum_{\nu \geqslant 1} \nu \cdot \chi(V_\nu) = \sum_{\nu \geqslant 1} \chi(W_\nu) = (-1)^{n-k}[\chi(\mathfrak{F}) \cdot \chi(N) - \chi(M)],$$

where χ denotes the topological Euler characteristic and $\mathfrak{F}$ is a generic (nonsingular) fiber of f.

This theorem remains true under more weak conditions. We do not state here the general version, but give some corollaries, concerning the local structure of SCI.

Let f: $(C^n, 0) \to (C^k, 0)$ be a germ of SCI, B_ε^n (resp. B_δ^k) an open ε (resp. δ)-ball, centered at $0 \in C^n$ (at $0 \in C^k$), $1 \gg \varepsilon \gg \delta > 0$. Let S be an analytic submanifold of B_δ^k, such that f is transversal to S, $Z = B_\varepsilon^n \cap f^{-1}(S)$, $V_\nu' = V_\nu \cap Z$. Theorem 2.1 holds for f/Z: $Z \to S$, and since the generic fiber $\mathfrak{F}$ of f/Z coincides with the Milnor fiber F of f, $\chi(\mathfrak{F}) = 1 + (-1)^{n-k}\mu(0)$. Thus we have

COROLLARY 2.2.

$$\sum_{\nu=1}^{\mu(0)} \nu \cdot \chi(V_\nu') = \mu(0)\chi(S) + (-1)^{n-k}[\chi(S) - \chi(Z)].$$

Now let $L \in U_l \subset G_k^l$ be a l-dimensional subspace of C^k, $\xi \in C^k/L$ a sufficiently small regular value of f_L: $(C^n, 0) \to C^k/L$, and let S be a parallel translation $L + \xi$ of L, intersected with B_δ^k, Z, V_ν' as above. In this case $\chi(S) = 1$, $\chi(Z) = 1 + (-1)^{n-k+l}\mu^{k-l}(0)$, and we obtain

COROLLARY 2.3.

$$\sum_{\nu=1}^{\mu(0)} \nu \cdot \chi(V_\nu') = \mu(0) + (-1)^{l+1}\mu^{k-l}(0).$$

Finally, if we consider some small deformation f' of f as the section of an appropriate family, then for corresponding strata V_ν' of f' we obtain

COROLLARY 2.4.

$$\sum_{\nu=1}^{\mu(0)} \nu \cdot \chi(V_\nu') = \mu(0).$$

3. In this section we prove Theorem 1.1 for singularities of codimension 2. Let f: $(C^n, 0) \to (C^k, 0)$ be SCI. Let Σ_i, $i = 1, \ldots, l$, be irreducible components of Σ_{red} and let k_i be the multiplicity of Σ_i at 0. For each i the function μ is constant on a Zariski open subset of Σ_i and its value on this subset we denote by μ_i.

LEMMA 3.1. *Let* rank $df(0) \geq k - 2$. *Then*

$$\mu(0) \geq \sum_{i=1}^{l} k_i \mu_i. \tag{1}$$

If rank $df(0) = k - 2$, *then the strict inequality holds.*

PROOF. Let L be a generic plane in C^k, passing through the origin. Then $Z = f^{-1}(L)$ is nonsingular, $\mu_{f/Z} = \mu_f/Z$ and $\Sigma(f/Z) = \Sigma(f) \cap Z$. Clearly, the multiplicity of $\Sigma_i \cap Z$ at 0 is greater than or equal to k_i, and hence it is sufficient to prove lemma for f/Z: $Z \to L$. Thus we can restrict ourself to the case f: $(C^n, 0) \to (C^2, 0)$. Let f be given by $w_1 = f_1(z_1, \ldots, z_n)$, $w_2 = f_2(z_1, \ldots, z_n)$ in coordinates $z_1, \ldots, z_n$ in C^n, w_1, w_2 in C^2.

Performing a linear coordinate change in C^2 we can assume that f_1 has an isolated singularity at the origin. Then the Milnor number $\mu(0)$ can be computed by the following formula (see [**3**, **2**]):

$$\mu(0) = \dim_C \mathcal{O}_n/\{f_1, J\} - \dim_C \mathcal{O}_n/J_1, \tag{2}$$

where J_1 is the ideal, generated by $\partial f_1/\partial z_1, \ldots, \partial f_1/\partial z_n$. $\mathcal{O}_\Sigma = \mathcal{O}_n/J$ is a Cohen-Macaulay ring and f_1 is a parameter in $\mathcal{O}_\Sigma$ (see e.g. [**2**]), hence

$$\dim_C \mathcal{O}_n/\{f_1, J\} = (\Sigma, \{f_1 = 0\})_0 = \sum_{z \in \Sigma \cap \{f_1 = \delta\}} (\Sigma, \{f_1 = \delta\})_z \tag{3}$$

for any small δ, where (,) is the intersection multiplicity.

LEMMA 3.2. *For* $z \in \Sigma_i \setminus \{0\}$, $(\Sigma, \{f_1 = f_1(z)\})_z = \mu_i$.

PROOF. This follows from the formula (2) applied at z, since f_1 is nonsingular at z and then the second term in (2) is zero. (Clearly, the intersection multiplicity of Σ with any regular hypersurface, transversal to Σ at z is also μ_i.) $\square$

Thus, from (3) we have

$$\dim_C \mathcal{O}_n / \{f_1, J\} = \sum_{i=1}^{l} r_i \mu_i, \tag{4}$$

where $r_i = (\Sigma_i, \{f_1 = 0\})_0$.

From now on we assume that the coordinate system $z_1, \ldots, z_n$ in C^n is chosen generically and in particular the following is true:

(1) $k_i = k_{i1} \leqslant k_{ij}$, $i = 1, \ldots, l$, $j = 2, \ldots, n$,
where $k_{ij} = (\Sigma_i, \{z_j = 0\})_0$;

(2) $p_i = p_{i1} \leqslant p_{ij}$, $i = 1, \ldots, l$, $j = 2, \ldots, n$,
where $p_{ij} = (\Sigma_i, \{\partial f_1 / \partial z_j = 0\})_0$.

Since the singularity of f_1 at 0 is isolated, from (2) it follows that p_i are finite, $i = 1, \ldots, l$, i.e. $\partial f_1 / \partial z_1$ is a parameter in $\mathcal{O}_\Sigma$.

LEMMA 3.3. $r_i \geqslant k_i + p_i$, $i = 1, \ldots, l$.

PROOF. Let n_i: $(C, 0) \to (\Sigma_i, 0)$ be a normalization of Σ_i and τ a coordinate in C. We have

$$z_j \circ n_i = \alpha_{ij} \tau^{k_{ij}} + \ldots,$$
$$f_1 \circ n_i = \beta_i \tau^{r_i} + \ldots,$$
$$\partial f_1 / \partial z_j \circ n_i = \gamma_{ij} \tau^{p_{ij}} + \ldots,$$

where α_{ij}, β_i, $\gamma_{ij} \neq 0$. Then

$$\frac{d(f_1 \circ n_i)}{d\tau} = r_i \beta_i \tau^{r_i - 1} + \ldots = \sum_{j=1}^{n} \frac{\partial f_1}{\partial z_j} \cdot \frac{d(z_j \circ n_i)}{d\tau}$$
$$= \sum_{j=1}^{n} \left(\gamma_{ij} \tau^{p_{ij}} + \ldots\right) \cdot \left(\alpha_{ij} k_{ij} \tau^{k_{ij} - 1} + \ldots\right).$$

By conditions (1) and (2) above the leading term on the right-hand side series is of degree $\geqslant k_i + p_i - 1$ and since $r_i \neq 0$ ($f_1(0) = 0$) it follows that $r_i \geqslant k_i + p_i$. $\square$

Now from (2), (4) and Lemma 3.3

$$\mu(0) \geqslant \sum_{i=1}^{l} k_i \mu_i + \sum_{i=1}^{l} p_i \mu_i - \dim_C \mathcal{O}_n / J_1.$$

In turn, applying Lemma 3.2 and the fact that $\partial f_1 / \partial z_1$ is a parameter in a Cohen-Macaulay ring $\mathcal{O}_\Sigma$ we obtain

$$\sum_{i=1}^{l} p_i \mu_i = \dim_C \mathcal{O}_n / \{\partial f_1 / \partial z_1, J\}.$$

Clearly $\{\partial f_1 / \partial z_1, J\} \subset J_1$, then $\sum_{i=1}^{l} p_i \mu_i \geqslant \dim_C \mathcal{O}_n / J_1$ and hence

$$\mu(0) \geqslant \sum_{i=1}^{l} k_i \mu_i.$$

Now let rank $df(0) = 0$. Then $J \subset \mathfrak{m} J_1$, where $\mathfrak{m}$ is the maximal ideal of $\mathcal{O}_n$. For corresponding ideals $\hat{J}$, $\hat{J}_1$ and $\hat{\mathfrak{m}}$ in the quotient ring $\mathcal{O}_n/\{\partial f_1/\partial z_1\}$ also $\hat{J} \subset \hat{\mathfrak{m}} \hat{J}_1$, then by Nakayama lemma $\hat{J} \underset{\neq}{\subset} \hat{J}_1$ and hence $\{\partial f_1/\partial z_1, J\} \underset{\neq}{\subset} J_1$, which proves the strict inequality in (1). □

REMARK. Let $(Y, 0)$ be SCI, $f = (f_1, f_2)$: $(Y, 0) \to (C^2, 0)$ a flat mapping, such that $f^{-1}(0)$ and $f_1^{-1}(0)$ also have isolated singularities at 0. Lemmas 3.2 and 3.3 hold in this case and thus we have

$$\mu_f(0) + \mu_{f_1}(0) \geqslant \sum_{i=1}^{l} (k_i + p_i)\mu_i.$$

PROOF OF THEOREM 1.1 FOR THE CASE rank $df(0) \geqslant k - 2$. If $\dim_0 V_\mu(0) = k - 1$, then at least one of μ_i is equal to $\mu(0)$. From Lemma 3.1 it follows immediately that $l = 1$, $k_1 = 1$, $\mu_1 = \mu(0)$ and rank $df(0) = k - 1$. In other words, Σ_{red} is nonsingular and μ is constant on it. Now for a generic line λ in C^k, $Y' = f^{-1}(\lambda)$ is nonsingular and $(\Sigma_{\text{red}}, Y')_0 = 1$ (for otherwise a hypersurface singularity f/Y' can be split into more than one singularity with the same Milnor number). Hence f/Σ_{red} is regular. By the implicit function theorem we can choose coordinates $t_1, \ldots, t_{k-1}, z_1, \ldots, z_{n-k+1}$ in C^n and $t'_1, \ldots, t'_{k-1}$, w on C^k such that Σ_{red} is defined by $z_1 = \cdots = z_{n-k+1} = 0$, and $t_i = t'_i \circ f$, $i = 1, \ldots, k-1$, i.e., f is equivalent to Φ: $(C^{k-1} \times C^{n-k+1}, 0) \to (C^{k-1} \times C, 0)$, $\Phi(t, z) = (t, h_t(z))$. □

4. In this section we complete the proof of Theorem 1.1. Let f: $(C^n, 0) \to (C^k, 0)$ be SCI, and let $\dim_0 V_{\mu(0)} = k - 1$. Then one of irreducible components of Σ_{red}, say Σ_1, is contained in $V_{\mu(0)}$. We shall prove, that $\Sigma_{\text{red}} = \Sigma_1$. Indeed, assume that Σ_2 is another component of Σ_{red}.

Choose a plane $L \in U_2 \subset G_k^2$ in C^k and let as in Corollary 2.3, ξ be a sufficiently small regular value of f_L: $(C^n, 0) \to C^k/L$, $L_\xi = L + \xi$, $S = L_\xi \cap B_\delta^k$, $Z = f^{-1}(S) \cap B_\varepsilon^n$, $f' = f/Z$, $\Sigma' = \Sigma \cap Z$. Let $\omega_1 = Z \cap \Sigma_1$, $\omega_2 = Z \cap \Sigma_2$. Since $\mu_{f'} = \mu_f/Z$ is constant on ω_1, we see (by already proved part of Theorem 1.1) that the germ of Σ' at any point $z \in \omega_1$ coincides with ω_1. In particular, $\omega_1 \cap \omega_2 = \varnothing$.

Now $f(\Sigma_1)$ and $f(\Sigma_2)$ are two hypersurfaces in C^k and their intersection Q is at least $(k-2)$-dimensional. Then $S \cap Q \neq \varnothing$ (L is chosen generically). Take some $t_0 \in S \cap Q$. We see, that $f^{-1}(t_0) \cap \Sigma$ contains at least two different points $z_1 \in \omega_1$ and $z_2 \in \omega_2$ with $\mu(z_1) = \mu(0)$, $\mu(z_2) > 0$. But this contradicts to the following inequality:

If for $t \in C^k$, $f^{-1}(t) \cap \Sigma = \{z_1, \ldots, z_q\}$, then $\Sigma_{i=1}^q \mu(z_q) \leqslant \mu(0)$. (Proof: let an analytic set $\{z_1, \ldots, z_q\}$ be defined by equations $h_1 = 0, \ldots, h_s = 0$. Considering the Morse function $\varphi = |h_1|^2 + \cdots + |h_s|^2$ on $f^{-1}(t')$, where t' is a regular value of f near t, we see, that the Milnor fiber $f^{-1}(t')$ can be obtained from the union of Milnor fibers of f at $z_1, \ldots, z_q$, by attaching cells of dimensions $\leqslant m - k$.)

Thus, Σ_{red} coincides with $V_{\mu(0)}$ and is irreducible.

Now, let us prove, that rank $df(0) = k - 1$. Indeed, assume, that $r = \text{rank } df(0) < k - 1$. Choose a coordinate system $w_1, \ldots, w_k$ in C^k, such that

(a) rank $d(\pi \circ f)(0) = r$, where π: $C^k \to C^r$, $\pi(w_1, \ldots, w_k) = (w_1, \ldots, w_r)$,

(b) a generic hyperplane L in $C^{k-r} = \{(0, \ldots, 0, w_{r+1}, \ldots, w_k)\}$, considered as a $k - r - 1$ dimensional subspace in C^k, belongs to $U_{k-r-1} \subset G_k^{k-r-1}$.

Now choose coordinates $z_1,\dots,z_n$ in C^n such that $f(z_1,\dots,z_n)=(w_1,\dots,w_k)=(z_1,\dots,z_r,f_{r+1},\dots,f_k)$. Let $h=(f_{r+1},\dots,f_k)/C^{n-r}$: $C^{n-r}\to C^{k-r}$. We have rank $dh(0)=0$, and $k-r\geqslant 2$.

We apply to f Corollary 2.3, taking L with $\dim L=k-r-1$ to be a hyperplane in C^{k-r} (according to (b)). Since $\Sigma_{\text{red}}=V_{\mu(0)}$, $V'_\nu=\varnothing$ for $\nu\neq\mu(0)$ and Corollary 2.3 gives

$$\mu(0)\chi(V'_{\mu(0)})=\mu(0)+(-1)^{k-r}\mu^{r+1}(0),\quad\text{or}$$
$$\mu(0)\left[\chi(V'_{\mu(0)})-1\right]=(-1)^{k-r}\mu^{r+1}(0).\tag{4.1}$$

By the construction, we have $\mu(0)=\mu_f(0)=\mu_h(0)$; $\mu^{r+1}(0)=\mu_f^{r+1}(0)=\mu_h^1(0)$, and by [**1**, Corollary 1.6], we obtain

$$\mu(0)>\mu^{r+1}(0).\tag{4.2}$$

Now, (4.1) and (4.2) lead to a contradiction: if $\chi(V'_{\mu(0)})=1$, by (4.1), $\mu^{r+1}(0)=0$ and hence rank $df(0)\geqslant r+1$, which contradicts the assumption. If $\chi(V'_{\mu(0)})\geqslant 2$, by (4.1), $k-r$ must be even and $\mu^{r+1}(0)\geqslant\mu(0)$, which contradicts (4.2). Finally, if $\chi(V'_{\mu(0)})\leqslant 0$, then by (4.1), $k-r$ is odd and once more $\mu^{r+1}(0)\geqslant\mu(0)$. This contradiction proves that rank $df(0)=k-1$ and in this case Theorem 1.1 was already proved in §3. □

REMARK. Corollary 1.2 shows, that f is topologically equisingular along the strata $\mu=$ const of the maximal dimension.

If the dimension of stratum $\mu=$ const is strictly less than $k-1$, this is not true, even for rank $df=k-1$ (see [**5**]). rank df can also change in such deformations (see [**6**, Example 4.6.3]). However, the following kind-of-topological triviality can be proved by method of [**4**]: in a family of SCI with $\mu=$ const (and $n-k\neq 2$) the Milnor fiber and the knot $Y\cap S_\varepsilon^{n-1}$, where $S_\varepsilon^{n-1}=\partial B_\varepsilon^n$, do not change their topological type.

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