PROCEEDINGS OF THE
AMERICAN MATHEMATICAL SOCIETY
Volume 94, Number 4, August 1985

SHORTER NOTES

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ON THE RADON-NIKODÝM PROPERTY IN BANACH LATTICES

ELIAS SAAB

ABSTRACT. In his recent Memoirs, Talagrand showed that under Axiom L, the weak*-Radon-Nikodým property is equivalent to the Radon-Nikodým property in a Banach lattice. In this note we prove this result by a simple method without assuming Axiom L.

In [3] Talagrand introduced the following notion of the weak*-Radon-Nikodým property:

DEFINITION 1 (TALAGRAND [3]). A Banach space E has the *weak*-Radon-Nikodým property* if for every bounded linear operator $T: L^1[0,1] \to E$, there exists a Pettis integrable function $\phi: [0,1] \to E^{**}$ such that

$$T(f) = \text{Pettis-}\int_0^1 f\phi \, d\lambda \quad \text{for every } f \in L^1[0,1].$$

Let us recall the following

DEFINITION 2. A Banach space E has the *Radon-Nikodým property* (resp., *the weak-Radon-Nikodým property*) if for every bounded linear operator $T: L^1[0,1] \to E$ there exists a function $\phi: [0,1] \to E$ such that ϕ is Bochner integrable (resp., Pettis integrable) and

$$T(F) = \text{Bochner-}\int_0^1 f\phi \, d\lambda \quad \left(\text{resp., } T(f) = \text{Pettis-}\int_0^1 f\phi \, d\lambda\right)$$

for every $f \in L^1[0,1]$.

In [3, pp. 88–91] Talagrand showed that under Axiom L a Banach lattice E has the Radon-Nikodým property if and only if it has the weak*-Radon-Nikodým property. His proof, besides the fact that it uses Axiom L, is quite involved. (Axiom L states that $[0,1]$ cannot be covered by less than the continuum of closed negligible sets.)

Received by the editors October 15, 1984.
1980 *Mathematics Subject Classification*. Primary 46B22, 28B05, 46G10.

The following more general theorem is true and its proof is quite simple.

THEOREM 3. *For any Banach lattice E, the following statements are equivalent*:
(1) *The space E has the Radon-Nikodým property.*
(2) *The space E has the weak-Radon-Nikodým property.*
(3) *The space E has the weak*-Radon-Nikodým property.*

PROOF. (1) is equivalent to (2) by Ghoussoub and Saab [**1**]. So it is enough to show that (3) $\to$ (2). (3) implies that every vector measure of bounded variation with values in E has relatively compact range by a result of Stegall, and therefore c_0 does not embed in E (see [**1**]). Hence E is weakly sequentially complete and therefore E is complemented in its bidual E^{**} by a projection

$$P\colon E^{**} \to E \quad (\text{see } [\mathbf{2}, \text{p. } 34]).$$

Let $T\colon L^1[0,1] \to E$ be a bounded linear operator. By (3) there is a function $\phi\colon [0,1] \to E^{**}$, Pettis integrable such that

$$T(f) = \text{Pettis-}\int_0^1 f\phi\, d\lambda$$

for every f in $L^1[0,1]$. The function $P\phi\colon [0,1] \to E$ is Pettis integrable and

$$T(F) = P(T(f)) = \text{Pettis-}\int_0^1 f P\phi\, d\lambda.$$

So T has a Pettis kernel in E and therefore E has the weak-Radon-Nikodým property.

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DEPARTMENT OF MATHEMATICS, UNIVERSITY OF MISSOURI-COLUMBIA, COLUMBIA, MISSOURI 65211