PROCEEDINGS OF THE
AMERICAN MATHEMATICAL SOCIETY
Volume 97, Number 3, July 1986

DECOMPOSITIONS OF SPACES DETERMINED BY COMPACT SUBSETS

YOSHIO TANAKA

ABSTRACT. Let X be a k'-space, and let $\mathcal{F}$ be a closed cover of (locally) compact subsets of X. Then X is decomposed into a closed discrete subset and a locally compact subset if X is dominated by $\mathcal{F}$, or X has the weak topology with respect to a point-countable cover $\mathcal{F}$. Here, a cover of a space is point-countable if every point is in at most countably many elements of the cover.

Introduction. We assume that all spaces are Hausdorff, and that all maps are continuous and onto. Suppose that $f: X \to Y$ is a closed map. When X is a locally compact paracompact space, K. Morita [**14**] showed that Y is decomposed into a closed discrete subset and a locally compact subset. When X is a metric space, N. Lašnev [**9**] showed that Y is decomposed into a σ-discrete subset and a metric subset. Here, a subset of Y is *σ-discrete* if it is a countable union of closed discrete subsets of Y. However, if f is a quotient map, not every paracompact space Y is decomposed into a σ-discrete subset and a subset which is metric or locally compact, even if every $f^{-1}(y)$ is finite and X is locally compact separable metric; see Example 3.1.

Now, in terms of weak topology, let us recall some definitions related to quotient spaces of locally compact spaces. Let $\mathcal{C}$ be a cover of a space Z. Then Z *is determined by* $\mathcal{C}$ [**8**], or Z has the weak topology with respect to $\mathcal{C}$, if $F \subset Z$ is closed in Z if and only if $F \cap C$ is relatively closed in C for every $C \in \mathcal{C}$. Here we can replace "closed" by "open". A space is a *k-space* if it is determined by the cover of all compact subsets. It is well known that every k-space is characterized as a quotient image of a locally compact (paracompact) space. Let $\mathcal{F}$ be a closed cover of a space Z. Then Z *is dominated by* $\mathcal{F}$ [**10**], if the union of any subcollection $\mathcal{F}'$ of $\mathcal{F}$ is closed in Z and the union is determined by $\mathcal{F}'$. Every CW-complex is dominated by compact metric subsets [**19**]. Every locally compact paracompact space X, as well as every closed image of X, is dominated by a hereditarily closure-preserving cover of compact subsets. Here, a cover $\{C_\alpha\}$ of a space is *hereditarily closure-preserving* if $\overline{\bigcup_\alpha B_\alpha} = \bigcup_\alpha \overline{B_\alpha}$ for any $B_\alpha \subset C_\alpha$. A space Y is a *k'-space* [**3**], if whenever $y \in \overline{A}$, then $y \in \overline{A \cap C}$ for some compact subset C of Y. A space Y is *Fréchet*, if whenever $y \in \overline{A}$, there exists a sequence in A converging to y. Every locally compact and every Fréchet space is a k'-space. Every k'-space is characterized as an image of a locally compact (paracompact) space under a pseudo-open map [**2**]. Recall that a map $f: X \to Y$ is *pseudo-open* [**2**], or hereditarily quotient, if for any

Received by the editors March 15, 1985 and, in revised form, June 28, 1985.
1980 *Mathematics Subject Classification*. Primary 54B15, 54D45; Secondary 54C10, 54D50, 54E45.
Key words and phrases. Weak topology, k-space, k'-space, locally compact space, quotient map, pseudo-open map, closed map.

$y \in Y$ and any open subset U containing $f^{-1}(y)$, $y \in \operatorname{int} f(U)$; equivalently, for any $A \subset Y$, $f|f^{-1}(A)$ is quotient [**2**]. Closed maps and open maps are psuedo-open, and pseudo-open maps are quotient.

In this paper, we show that every k'-space (more generally, singly bi-k-space [**12**]) dominated by locally compact subsets, or determined by a point-countable closed cover of locally compact subsets, is decomposed into a closed discrete subset and a locally compact subset. Thus, for a k'-space Y, if Y is a CW-complex or an image of a locally compact paracompact space under a quotient map with each point-inverse Lindelöf, then Y is decomposed into a closed discrete subset and a locally compact subset.

1. Spaces determined by a point-countable cover of compact subsets. Let Y be a space. For a cover $\mathcal{C}$ of Y, let us consider the following condition $(*)$ with respect to $\mathcal{C}$:

$(*)$ If $y \in \overline{A}$, then $y \in \overline{A \cap C}$ with $y \in C$ for some $C \in \mathcal{C}$.

LEMMA 1.1. *Let Y be a k-space, and $\mathcal{C}$ be a point-countable cover of Y. If Y satisfies $(*)$ with respect to $\mathcal{C}$, then $Y_0 = \{y \in Y; y \notin \operatorname{int} \bigcup \mathcal{C}'$ for any finite $\mathcal{C}' \subset \mathcal{C}\}$ is discrete in Y.*

PROOF. Suppose that Y_0 is not discrete in Y. Then some $A \subset Y_0$ is not closed in Y. Since Y is a k-space, there exists a compact subset K such that $K \cap A$ is not closed. Since $K \cap A$ is an infinite subset of K, there exists an infinite subset $\{y_n;\ n \in N\}$ of $K \cap A$ accumulating at some $y \in Y$ with $y_n \neq y$. Let V_n be a neighborhood of y_n with $\overline{V_n} \not\ni y$. Let $\{C \in \mathcal{C};\ C \ni y\} = \{C_1, C_2, \ldots\}$. For each $n \in N$, let $B_n = \bigcup_{m \leq n} C_m$, and let $A_n = V_n - B_n$. Since each $y_n \in Y_0$, $y_n \in \overline{A_n}$. Thus, $y \in \overline{\bigcup_{n \in N} A_n}$. By condition $(*)$, $y \in \overline{(\bigcup_{n \in N} A_n) \cap C_i}$ for some $i \in N$. Thus, $y \in \overline{A_j}$ for some $j < i$, hence $y \in \overline{V_j}$. This contradiction completes the proof of the lemma.

According to E. Michael [**12**], a space is *singly bi-k*, if it is a pseudo-open image of a paracompact M-space. Here, a space is *paracompact M-space* if it admits a perfect map onto a metric space. k'-spaces are singly bi-k, and singly bi-k-spaces are k-spaces [**12**].

LEMMA 1.2. *If Y is a singly bi-k-space determined by a point-countable closed cover $\mathcal{C}$, then Y satisfies $(*)$ with respect to $\mathcal{C}$.*

PROOF. Let $y \in \overline{A}$. Since Y is singly bi-k, by [**12**, Definition 5.E.1], there exists a sequence $\{A_n; n \in N\}$ such that $y \in \overline{A \cap A_n}$ for all n, and if $y_n \in A_n$, then $\{y_n; n \in N\}$ has an accumulation point in Y. Thus, as in the proof of [**17**, Lemma 6], some A_n is covered by a finite subcollection $\mathcal{C}'$ of $\mathcal{C}$. Then $y \in \overline{A \cap C}$ with $y \in C$ for some $C \in \mathcal{C}'$. Then Y satisfies $(*)$ with respect to $\mathcal{C}$.

From Lemmas 1.1 and 1.2, we have

THEOREM 1.3. *If Y is a singly bi-k-space determined by a point-countable closed cover of locally compact subsets, then Y is decomposed into a closed discrete subset and a locally compact subset.*

Example 3.1 shows that the singly bi-k-ness of Y is essential in Theorem 1.3, and also in Theorem 1.4 below.

Recall that a space is *σ-metric* if it is a countable union of closed metric subsets.

THEOREM 1.4. *Let Y be a singly bi-k-space determined by a point-countable closed cover $\{Y_\alpha\}$ of metric subsets. Then* (1) *and* (2) *below hold.*

(1) *Y is decomposed into a closed discrete subset and a subset with a point-countable base.*

(2) *If Y is a paracompact space, or a regular space with each Y_α locally separable, then Y is a σ-metric space decomposed into a closed discrete subset and a metric subset.*

PROOF. (1) By Lemmas 1.1 and 1.2, Y is decomposed into a closed discrete subset and a locally metric subset Y' of Y. Since Y' is open in Y, Y' is determined by a point-countable closed cover $\{Y_\alpha \cap Y'\}$ of Y'. Let $X = \sum_\alpha (Y_\alpha \cap Y')$, where $\sum$ denotes topological sum. Then X is metric, and the obvious map of X onto Y' is a quotient and s-map (i.e., each point-inverse is separable). Since Y' is first countable, by [**6**, Theorem 1′] Y' has a point-countable base.

(2) For $y \in Y$, let $A = \bigcup\{Y_\alpha; Y_\alpha \ni y\}$. Since Y is singly bi-k, by Lemma 1.2, $y \notin \overline{X - A}$, hence $y \in \operatorname{int} A$. Thus Y is a locally σ-metric space. In case where Y is paracompact, Y is a σ-metric space. Then the open subset Y' in (1) is an F_σ-set of Y. Hence Y' is paracompact. Since Y' is locally metric, Y' is metric. In case where Y is regular and each Y_α is locally separable, each Y_α is determined by a locally finite closed cover $\{Y_{\alpha\beta}; \beta\}$ of separable metric subsets. Hence Y is determined by a point-countable closed cover $\{Y_{\alpha\beta}; \alpha, \beta\}$ of separable metric subsets. But Y is singly bi-k, hence is Fréchet by Lemma 1.2. Then, since Y is regular, Y is paracompact by [**8**, Corollary 8.9]. As seen above, then, Y is σ-metric and Y_1 is metric. Therefore, in any case, Y is a σ-metric space decomposed into a closed discrete subset and a metric subset.

COROLLARY 1.5. *Let $f: X \to Y$ be a quotient s-map such that X is locally compact metric. If Y is a regular k'-space, then Y is a σ-metric space decomposed into a closed discrete subset and a* (*locally compact*) *metric subset.*

PROOF. Since X is determined by a locally finite cover $\{X_\alpha\}$ of compact metric subsets, Y is determined by a point-countable cover $\{f(X_\alpha)\}$ of compact metric subsets. Thus the corollary follows from Theorem 1.4(2).

The local compactness of X in Corollary 1.5 is essential. Indeed, there exists a closed image Y of a separable metric space such that Y is not σ-metric [**7**], hence is not decomposed into a closed discrete subset and a metric subset. Also, there exists a paracompact space X which is an open s-image of a metric space such that X is not a σ-space (hence, not σ-metric), nor is X decomposed into a σ-discrete subset and a metric subset; see Example 3.2.

Recall that a space X is *meta-Lindelöf* if every open cover of X has a point-countable open refinement. Every meta-compact space is meta-Lindelöf.

LEMMA 1.6. *Let $f: X \to Y$ be a quotient map with X locally compact. Suppose that* (a) *or* (b) *below holds.*

(a) *X is meta-Lindelöf and every $f^{-1}(y)$ is separable.*

(b) *X is paracompact and every $f^{-1}(y)$ is Lindelöf.*

If Y is a singly bi-k-space; equivalently, f is pseudo-open, then Y is determined by a point-countable cover $\mathcal{C}$ such that each element of $\mathcal{C}$ has a compact closure, and Y satisfies $(*)$ *with respect to $\mathcal{C}$.*

PROOF. Let us assume case (a) (the proof for case (b) is similar). Then X is determined by a point-countable open cover $\{G_\alpha\}$ such that $\overline{G_\alpha}$ are compact. Since f is a quotient s-map, Y is determined by a point-countable cover $\mathcal{C} = \{f(\overline{G_\alpha})\}$. To show that f is pseudo-open, let $y \in Y$ and U be any open subset containing $f^{-1}(y)$. Suppose that $y \notin \operatorname{int} f(U)$, hence $y \in \overline{Y - f(U)}$. Since Y is singly bi-k, as in the proof of Lemma 1.2 (cf. [**17**, Lemma 6]), $y \in \overline{(Y - f(U)) \cap f(\overline{G_{\alpha_0}})}$ for some α_0. Thus $y \in \overline{K - f(U)}$, where $K = f(\overline{G_{\alpha_0}})$. Let $g = f|\overline{G_{\alpha_0}}$. Since $\overline{G_{\alpha_0}}$ is compact, g is closed. But $g^{-1}(y) \subset U \cap \overline{G_{\alpha_0}}$. Thus $y \in \operatorname{int}_K g(U \cap \overline{G_{\alpha_0}})$, hence $y \notin \overline{K - f(U)}$. This is a contradiction. Thus f is pseudo-open. To show that Y satisfies $(*)$ with respect to $\mathcal{C}$, let $y \in \overline{A}$. Since f is pseudo-open, $f^{-1}(y) \cap \overline{f^{-1}(A)} \neq \varnothing$. Let $x \in f^{-1}(y) \cap \overline{f^{-1}(A)}$, and let $x \in G_{\alpha_1}$. Then $x \in \overline{f^{-1}(A) \cap G_{\alpha_1}}$, thus $y \in \overline{A \cap f(G_{\alpha_1})}$ and $y \in f(\overline{G_{\alpha_1}})$. Thus Y satisfies $(*)$ with respect to $\mathcal{C}$.

By Lemmas 1.1 and 1.6, we have the following theorem. Example 3.1 shows that "pseudo-open map" cannot be weakened to "quotient map", and Example 3.3 shows that the condition of $f^{-1}(y)$ is essential in the theorem, even if X is locally compact metric.

THEOREM 1.7. *Let $f: X \to Y$ be a pseudo-open map, or a quotient map with Y a k'-space. If X is a locally compact space, and* (a) *or* (b) *of Lemma* 1.6 *holds, then Y is decomposed into a closed discrete subset and a locally compact subset.*

2. Spaces dominated by compact subsets.

LEMMA 2.1. *Let X be dominated by a closed cover $\{X_\alpha\}$, and let $Y_\alpha = \overline{X_\alpha - \bigcup_{\beta<\alpha} X_\beta}$. If X is singly bi-k, then $\{Y_\alpha\}$ is a hereditarily closure-preserving closed cover of X.*

PROOF. Suppose that $\{Y_\alpha\}$ is not hereditarily closure-preserving. Then there exist closed subsets A_α of Y_α such that $\bigcup_\alpha A_\alpha$ is not closed in X. Since X is a k-space, there exists a compact subset K of X such that $(\bigcup_\alpha A_\alpha) \cap K$ is not closed in K. Then there exists an infinite subset $\{x_n; n \in N\}$ of X such that $x_n \in A_{\alpha_n} \cap K$ with $\alpha_n < \alpha_{n+1}$. Let $x \in X$ be an accumulation point of $\{x_n; n \in N\}$, which we may suppose different from all x_n. Let each V_n be a neighborhood of x_n with $\overline{V_n} \not\ni x$. Let $B_n = V_n \cap (X_{\alpha_n} - \bigcup_{\beta<\alpha_n} X_\beta)$, $B = \bigcup_{n\in N} B_n$, and let $C = \bigcup_{n \in N} X_{\alpha_n}$. Then $x \in \overline{B}^C$. But C is a singly bi-k-space determined by a countable closed cover $\{X_{\alpha_n}; n \in N\}$. Thus, by Lemma 1.2, $x \in \overline{B \cap X_{\alpha_i}}$ for some $i \in N$. Hence $x \in \overline{B_j}$ for some $j \leq i$, thus $x \in \overline{V_j}$. This contradiction completes the proof of the lemma.

Since every k'-space is singly bi-k, the "only if" part of the following follows from Lemma 2.1. The "if" part is easy.

COROLLARY 2.2. *Let X be dominated by a cover $\{X_\alpha\}$ of compact subsets, and let $Y_\alpha = \overline{X_\alpha - \bigcup_{\beta<\alpha} X_\beta}$. Then X is a k'-space if and only if $\{Y_\alpha\}$ is hereditarily closure-preserving.*

LEMMA 2.3. *Let $\mathcal{F}$ be a hereditarily closure-preserving closed cover of a space Y. If Y is a k-space, then $Y_0 = \{y \in Y;\ \mathcal{F}$ is not locally finite at $y\}$ is discrete in Y.*

PROOF. Suppose that Y_0 is not discrete in Y. Then some $A \subset Y_0$ is not closed in Y. Since Y is a k-space, there exists a compact subset K such that

$K \cap A$ is not closed. Thus there exists an infinite subset $\{y_n; n \in N\}$ of $K \cap A$ accumulating at some $y \in Y$. Since $y_n \in Y_0$, $\mathcal{F}$ is not point-finite at y_n, so there exists $\{F_n; n \in N\} \subset \mathcal{F}$ with $y_n \in F_n$. Thus $\{y_n; n \in N\}$ is discrete in Y, a contradiction.

THEOREM 2.4. *Let Y be a singly bi-k-space dominated by a closed cover of locally compact subsets (resp. metric subsets). Then Y is decomposed into a closed discrete subset and a locally compact subset (resp. metric subset).*

PROOF. This theorem follows from Lemmas 2.1 and 2.3. For the parenthetic part, use the well-known fact that every space dominated by metric subsets is hereditarily paracompact [**10** or **13**].

In the following corollary, the k'-ness of Y is essential; see Example 3.4.

COROLLARY 2.5. *Let Y be a CW-complex. If Y is a k'-space, then Y is decomposed into a closed discrete subset and a locally compact metric subset.*

THEOREM 2.6. *The following are equivalent:*

(1) *Y is a pseudo-open image of a paracompact M-space (or a locally compact space), and Y is dominated by compact subsets.*

(2) *Y is a closed image of a locally compact paracompact space.*

Proof. (2) $\to$ (1) is straightforward. (1) $\to$ (2) follows from Lemma 2.1 and the fact that, for every hereditarily closure-preserving closed cover $\{Y_\alpha\}$ of Y, the obvious map of $\sum_\alpha Y_\alpha$ onto Y is closed.

THEOREM 2.7. *The following are equivalent:*

(1) *Y is a pseudo-open image of a locally compact Lindelöf space.*

(2) *Y is a closed image of a locally compact Lindelöf space.*

PROOF. It suffices to prove (1) $\to$ (2). Since Y is a quotient image of a locally compact Lindelöf space, by [**15**], Y is determined by a countable cover $\mathcal{C}$ of compact subsets. Here we can assume that $\mathcal{C}$ is increasing, so Y is dominated by $\mathcal{C}$, while, Y is a k'-space. Then (1) $\to$ (2) can be given using Lemma 2.1.

3. Examples. A space Y is a *q-space* [**11**] if each point of Y has a sequence $\{U_n; n \in N\}$ of open neighborhoods such that, if $y_n \in U_n$, then $\{y_n; n \in N\}$ has an accumulation point in Y. First countable spaces and locally compact spaces, more generally spaces of pointwise-countable type [**3**], are q-spaces.

EXAMPLE 3.1. A regular Lindelöf space Y is determined by a point-finite, countable cover $\{Z_n; n \in \omega\}$ of compact metric subsets, hence Y is a quotient finite-to-one image of a locally compact separable metric space $\sum_{n\in\omega} Z_n$. But, for any σ-discrete subset Y_0 of Y, $Y - Y_0$ is not a q-space.

PROOF. For each $n \in \omega$, define a subspace Z_n of $\mathbf{R}^3$ by $Z_0 = \{0\} \times A$, and $Z_n = A \times \{1/n\}$, where $A = [0,1] \times [0,1]$. Let $Y = \bigcup_{n\in\omega} Z_n$, and let $U \subset Y$ be open in Y whenever every $U \cap Z_n$ is open in Z_n. Then Y is a space determined by a point-finite, countable cover $\{Z_n; n \in \omega\}$ of compact metric subsets. For any σ-discrtete subset Y_0 of Y, let $Y_1 = Y - Y_0$. Since each $Z_n \cap Y_0$ is countable (= at most countable), there exists $y_0 = (0,t,0) \in Y_1$ such that $(0,t,1/n) \in Y_1$ for all $n \in N$. But no sequences $\{z_n; n \in N\}$ with $z_n \in Z_n - Z_0$ have accumulation points in Y. Thus Y_1 is not a q-space.

EXAMPLE 3.2. A regular Lindelöf space X which is an open s-image of a metric space, but for any σ-discrete subset X_0 of X, $X - X_0$ is neither a σ-space nor a p-space in the sense of A. V. Arhangel'skiĭ [**1**].

PROOF. In view of the proof of [**4**, Problem 285, p. 146], there exists a subset A of $[0,1]$ such that A and its complement have size 2^ω, and A does not contain $C - D$ for any uncountable compact subset C of $[0,1]$ and for any countable subset D of $[0,1]$. Let X be the space obtained from $[0,1]$ by isolating the points of A. Since X is a regular space with a point-countable base, X is an open s-image of a metric space by [**16**]. Now, let X_0 be any σ-discrete subset of X, and let $X_1 = X - X_0$. Since X is Lindelöf, X_0 is countable. Thus X_1 is not separable. But X_1 is Lindelöf. Hence X_1 is not a σ-space. Also, X_1 is not a p-space, for every paracompact p-space with a point-countable base is metric [**5**].

EXAMPLE 3.3. A regular Lindelöf space Y which is a pseudo-open image of a locally compact metric space, but for any σ-discrete subset Y_0 of Y, $Y - Y_0$ is neither first countable nor a p-space.

PROOF. Let $T = D \cup \{\infty\}$ be the one point compactification of an uncountable discrete space D. Let $Z = T \times T$. Suppose that Z is decomposed into a σ-discrete subset Z_0 and a subset Z_1 of Z. Since Z_0 is countable, there exists $d \in D$ such that $(\infty, d) \in Z_1$, but Z_1 has no countable bases at the point (∞, d). Then Z_1 is not first countable. Now, let Y be a topological sum of Z and the space X of Example 3.2. Since Y is Fréchet, Y is a pseudo-open image of a locally compact metric space by [**2**]. However, for any σ-discrete subset Y_0 of Y, $Y - Y_0$ is neither first countable nor a p-space.

EXAMPLE 3.4. A countable CW-complex X such that for any σ-discrete subset X_0 of X, $X - X_0$ is not a q-space.

PROOF. For each $n \in N$, let L_n be a copy $\square A_nB_nC_nD_n$ of a rectangle $\square ABCD$. Let X be the space obtained from $\sum_{n\in N} L_n$ by identifying all of segments $\overline{A_nB_n}$ to a segment. Then X is a countable CW-complex satisfying the desired property.

REFERENCES

1. A. V. Arhangel'skiĭ, *On a class of spaces containing all metric and all locally bicompact spaces*, Soviet Math. Dokl. **4** (1963), 1051–1055.
2. ——, *Some types of factor mappings, and the relations between classes of topological spaces*, Soviet Math. Dokl. **4** (1963), 1726–1729.
3. ——, *Bicompact sets and the topology of spaces*, Trans. Moscow Math. Soc. **13** (1965), 1–62.
4. A. V. Arkhangel'skiĭ and V. I. Ponomarev, *Fundamentals of general topology*, Reidel, Dordrecht, 1984.
5. V. V. Filippov, *On feathered paracompacta*, Soviet Math. Dokl. **9** (1968), 161–164.
6. ——, *On factor s-mappings*, Soviet Math. Dokl. **9** (1968), 1055–1057.
7. B. Fitzpatric, Jr., *Some topologically complete spaces*, Topology Appl. **1** (1971), 101–103.
8. G. Gruenhage, E. Michael and Y. Tanaka, *Spaces determined by point-countable covers*, Pacific J. Math. **113** (1984), 303–332.
9. N. Lašnev, *Continuous decompositions and closed mappings of metric spaces*, Soviet Math. Dokl. **6** (1965), 1504–1506.
10. E. Michael, *Continuous selections*. I, Ann. of Math. (2) **63** (1956) 361–382.
11. ——, *A note on closed maps and compact sets*, Israel J. Math. **3** (1964), 173–176.
12. ——, *A quintuple quotient quest*, Topology Appl. **2** (1972), 91–138.
13. K. Morita, *On spaces having the weak topology with respect to closed coverings*, Proc. Japan Acad. **29** (1954), 537–543.

14. ——, *On closed mappings*, Proc. Japan Acad. **32** (1956), 539–543.
15. ——, *On decomposition spaces of locally compact spaces*, Proc. Japan Acad. **32** (1956), 544–548.
16. V. I. Ponomarev, *Axioms of countability and continuous mappings*, Bull. Acad. Polon. Sci. Sér. Math. Astronom. Phys. **8** (1960), 127–134. (Russian)
17. Y. Tanaka, *Point-countable k-systems and products of k-spaces*, Pacific J. Math. **101** (1982), 199–208.
18. Y. Tanaka and Zhou Hao-xuan, *Spaces dominated by metric subsets*, Topology Proc. **9** (1984), 149–163.
19. J. H. C. Whitehead, *Combinatorial homotopy*. I, Bull. Amer. Math. Soc. **55** (1949), 213–245.

DEPARTMENT OF MATHEMATICS, TOKYO GAKUGEI UNIVERSITY, 4-1-1, NUKUIKITA-MACHI, KOGANEI-SHI, TOKYO, JAPAN