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MINIMAL DEGREES OF FAITHFUL CHARACTERS OF FINITE GROUPS WITH A T.I. SYLOW p-SUBGROUP

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ABSTRACT. Using the classification of the finite simple groups we show in this article that a faithful complex character χ of a finite group G with a nonnormal T.I. Sylow p-subgroup P has degree $\chi(1) > \sqrt{|P|} - 1$. This result verifies a conjecture of H. S. Leonard [**10**].

Introduction. Let p be a fixed prime, and let G be a finite group with a T.I. Sylow p-subgroup P. That is, two different conjugates of P have only the identity element in common. In [**10**] H. S. Leonard conjectured that if G has a faithful complex character χ with degree $\chi(1) \leqslant \sqrt{|P|} - 1$, then P is normal in G. Using the classification of the finite simple groups we prove Leonard's conjecture in this note (Theorem 3.2).

In §1 this theorem is first proved for p-solvable groups G (Proposition 1.3). Then we determine the composition series of a minimal counterexample G to Leonard's conjecture (Proposition 1.4). Since by Sibley's theorem [**12**] the main result of this article is known if P is cyclic, we give in §2 a complete list of all finite simple groups G having a noncyclic T.I. Sylow p-subgroup for some prime p (Proposition 2.3). Here for odd p we use Gorenstein and Lyons' theorem [**4**] classifying all finite groups G with $O_{p'}(G) = 1$, p-rank $m_p(G) > 1$, and containing a strongly p-embedded subgroup. If $p = 2$, then Proposition 2.3 is only a restatement of Suzuki's theorem [**13**]. After these preparations Leonard's conjecture is proved in §3. In Remark 3.3 we show that the bound of Theorem 3.2 cannot be replaced by $\frac{1}{2}(|P| - 1)$, which is the bound of Sibley's theorem [**12**].

For notation and terminology we refer to the books by Feit [**1**], Gorenstein [**2, 3**], Huppert [**5**], Huppert and Blackburn [**6**], and Landrock [**9**]. All character tables of finite simple groups used here are contained in the CAS-system [**11**] of J. Neubüser, H. Pahlings, and W. Plesken (TH. Aachen, Federal Republic of Germany).

1. Reduction to almost simple groups. In this section we determine the structure of a finite group G of minimal order among the groups H without a normal Sylow p-subgroup, but satisfying the hypothesis of Leonard's conjecture.

The following lemma due to Feit [**1**, p. 123] is our basic tool.

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LEMMA 1.1. *Let S be a splitting field of characteristic zero for the finite group G with a T.I. Sylow p-subgroup P. Let χ be a character of SG such that $\chi(1)^2 \leqslant |P|$. Let H be a subgroup of G containing $N_G(P)$. Then $(\chi, \chi)_G = (\chi, \chi)_H$.*

For a short proof of this result we refer to [**9**, p. 129].

LEMMA 1.2. *Let G be a finite group with a T.I. Sylow p-subgroup P. Then the following assertions hold.*

(a) *Every subgroup U of G with $p \mid |U|$ has a T.I. Sylow p-subgroup.*

(b) *G/N has a T.I. Sylow p-subgroup for every normal subgroup N of G with $(p, |N|) = 1$.*

(c) *$C_G(x)$ has a normal Sylow p-subgroup for every $1 \neq x \in P$.*

PROOF. See Suzuki [**13**, p. 59].

PROPOSITION 1.3. *Let G be a p-solvable group with T.I. Sylow p-subgroup P and a faithful complex character χ such that $\chi(1)^2 \leqslant |P|$. Then P is a normal subgroup of G.*

PROOF. Let G be a minimal counterexample and let $(F = R/\pi, R, S = \operatorname{quot}(R))$ be a splitting p-modular system for G (see [**9**, p. 47]). Then $O_p(G) = 1$, and so $Q = O_{p'}(G) \neq 1$. Let $H = QN_G(P)$. If $H \neq G$, then $P \lhd H$ by induction. Hence

$$O_{p',p}(H) = O_{p'}(H) \times P \geqslant O_{p',p}(G) \underset{\neq}{>} Q,$$

because G is p-solvable. This forces $O_p(G) \neq 1$, a contradiction. Therefore, $G = H$. But $G = O^{p'}(G)$ by minimality, whence $G = QP$. In particular, G is p-nilpotent, and every p-block B of G contains only one modular character by Theorem 14.9 of [**6**]. Since P is a T.I. Sylow p-subgroup, and $\chi(1) < |P|$ it follows from Theorem 14.8 of [**6**] that (the possibly reducible) χ contains an irreducible constituent μ belonging to a nonprincipal p-block B with defect group $\delta(B) =_G P$, because χ is faithful. As G is p-nilpotent, by Theorem 2.1 of [**1**, p. 419], we also may assume that μ remains irreducible under restriction modulo π. Let $\bar{\mu}$ be a module over F affording μ modulo π.

Let b be the block of $U = N_G(P)$ associated with B by the Brauer correspondence. Since $\mu(1)^2 < |P|$, Lemma 1.1 asserts that $\mu|_U$ is irreducible. Because P is a T.I. set, Green's correspondence theorem implies that $\bar{\mu}|_U$ is an indecomposable module in b. Notice that b contains only one modular irreducible character, so that all composition factors of $\bar{\mu}|_U$ are isomorphic. In particular, if $\bar{\mu}|_U$ is not irreducible, it has a nonzero nilpotent FU-endomorphism τ, namely any nonzero map from the head to the socle of $\bar{\mu}|_U$. As P is a T.I. set, Corollary 5.8 of [**9**, p. 122], implies that τ is a projective endomorphism of $\bar{\mu}|_U$. Therefore, $\mu(1)^2 > |P|$ by Corollary 6.11 of [**9**, p. 128], a contradiction. We now know that $\bar{\mu}|_U$ is an irreducible FU-module of b and that P acts trivially on $\bar{\mu}|_U$. Let $A = \ker\bar{\mu}$ in G. Now $A \neq G$ since B is not the principal block. We have

$$P \leqslant \ker\bar{\mu} = A < G.$$

Therefore, $Q \nleqslant A$ as $G = QP$.

By Lemma 1.2 P is a T.I. Sylow p-subgroup of A, and χ_A is a faithful complex character of A with $\chi_A(1)^2 < |P|$. So $P \lhd A$ by induction, which implies $P \lhd G$. This contradiction completes the proof.

The center of the group G is denoted by $Z(G)$.

PROPOSITION 1.4. *Let G be a minimal counterexample to Leonard's conjecture. Let P be a T.I. Sylow p-subgroup of G, and let χ be a faithful complex character of degree $\chi(1) \leqslant \sqrt{|P|} - 1$. Then*:

(a) $Z(G) = O_{p'}(G) < O^p(G) = H$, $G = O^{p'}(G)$.

(b) *$H/Z(G)$ is a nonabelian simple group with a T.I. Sylow p-subgroup.*

(c) *χ may be assumed to be irreducible.*

PROOF. As G is a minimal counterexample, $G = O^{p'}(G)$. Let $H = O^p(G)$, and let $H/N \neq 1$ be a chief factor of G.

Suppose that H/N is a p'-group. By Proposition 1.3 G is not p-solvable. Thus $P_0 = P \cap N \neq 1$, and $L = N_G(P_0) < G$. Since $P \cap N \lhd P$, $P \leqslant L$. So by induction $P \lhd L$. As $G = O^{p'}(G)$, and as $G = NL$ by the Frattini argument, we obtain $G = NP$, and so $N = H$, a contradiction.

Therefore H/N is a direct product of isomorphic nonabelian simple groups A with $p \mid |A|$. Hence $NN_G(P) < G$, which implies $P \lhd NN_G(P)$ by induction. Since P is a T.I. set in G, we now get $O_p(N) = N \cap P = 1$. So N is a p'-group commuting with P. Hence

$$C_G(N) \geqslant \langle P^g \mid g \in G \rangle = O^{p'}(G) = G,$$

and so $Z(G) = N \leqslant O_{p'}(G)$.

As $(p, |N|) = 1$, Lemma 1.2 asserts that H/N has a T.I. Sylow p-subgroup. Thus by Lemma 1.2(c) H/N is simple. Since $O_{p'}(G) \subset O^p(G) = H$, it follows that $N = O_{p'}(G)$.

Finally, we may replace χ by an irreducible constituent, which does not have H in its kernel. This completes the proof.

2. Simple groups with a noncyclic T.I. Sylow p-subgroup. In this section we list the simple groups with a noncyclic T.I. Sylow p-subgroup. In [**13**] Suzuki classified the simple groups with such a Sylow 2-subgroup. For odd primes p our subsidiary result follows from Gorenstein and Lyons' classification [**4**] of the finite groups G with $O_{p'}(G) = 1$, p-rank $m_p(G) > 1$, and containing a strongly p-embedded subgroup.

Here $m_p(G)$ denotes the maximum rank of an elementary abelian subgroup of a Sylow p-subgroup P of the finite group G.

DEFINITION [**3**]. Let P be a Sylow p-subgroup of the finite group G, and let k be a positive integer. The *k-generated p-core* of G is $\Gamma_{P,k}(G) = \langle N_G(Q) \mid Q \leqslant P, m_p(Q) \geqslant k \rangle$.

The proper subgroup M of G is called *strongly p-embedded* in G if $\Gamma_{P,1}(G) \leqslant M$.

REMARK 2.1. If the finite group G contains a nonnormal T.I. Sylow p-subgroup P, then $M = N_G(P)$ is strongly p-embedded in G, as is easily seen.

A finite group G is *quasi-simple* if $G = G'$ and $G/Z(G)$ is simple. The *layer* $L(G)$ of G is the product of all subnormal quasi-simple subgroups of G, where $L(G) = 1$ if no such subnormal subgroup exists. The *generalized Fitting* subgroup of the finite group G is defined as $F^*(G) = F(G)L(G)$, where $F(G)$ denotes the Fitting subgroup of G (see [**3**, p. 44]).

In view of the classification theorem of the finite simple groups we now can restate Theorems (24.1), (24.2), and (24.9) of Gorenstein and Lyons [**4**, pp. 307, 311, and 318, respectively], as

PROPOSITION 2.2. *Let p be an odd prime, M a strongly p-embedded subgroup of the finite group G with $O_{p'}(G) = 1$ and $m_p(G) > 1$. Let $V = O^{p'}(G)$ and let P be a Sylow p-subgroup of M. Then $F^*(G) = L(V)$ is simple and one of the following holds.*

(1) $V \cong \mathrm{PSL}_2(p^n)$ *or* $\mathrm{PSU}_3(p^n)$, *and* $M = N_G(P)$.

(2) $V \cong \mathfrak{A}_{2p}$ *and* $F^*(M) \cong \mathfrak{A}_p \times \mathfrak{A}_p$.

(3) $p = 3$, $V \cong {}^2G_2(3^{2m+1})$, *and* $M = N_G(P)$, *where* $m \geqslant 0$.

(4) $p = 3$, $V \cong M_{11}$ *or* $\mathrm{PSL}_3(4)$, *and* $M = N_G(P)$.

(5) $p = 5$, $V \cong M(22)$, *and* $V \cap M \cong \mathrm{Aut}(D_4(2))$.

(6) $p = 5$, $V \cong {}^2F_4(2)'$, $\mathrm{Aut}({}^2B_2(2^5))$ *or Mc, and* $M = N_G(P)$.

(7) $p = 11$, $V \cong J_4$, *and* $M = N_G(P)$.

PROOF. By hypothesis, $\Gamma_{P,1}(G) \leqslant M \neq G$ and $P \leqslant V \cap M$. Thus $O_p(G) = 1 = F(G)$, because otherwise $G = N_G(O_p(G)) \leqslant \Gamma_{P,1}(G) \leqslant M \neq G$, a contradiction.

Let K be a normal subgroup of G. As $O_{p'}(G) = 1$, $P_0 = P \cap K \neq 1$. The Frattini argument asserts that $G = N_G(P_0)K$. Hence $K \nleqslant \Gamma_{P,1}(G)$. It follows that every quasi-simple subnormal subgroup L of G is simple and $L \neq \Gamma_{P_1,1}(L)$, where $P_1 \in \mathrm{Syl}_p(L)$.

Thus $F^*(G) = L(G) = L(V)$ is a direct product of simple groups E_i, $1 \leqslant i \leqslant k$, each of which contains a strongly p-embedded subgroup.

Let $E \in \{E_i \mid 1 \leqslant i \leqslant k\}$, $P^* = P \cap L(V)$, and $X = EP^*$. Then $O_{p'}(X) = 1 = O_p(X)$ and $\Gamma_{P^*,1}(X) \neq X$, because $P^* \subset \Gamma_{P,1}(G)$, but $E \nleqslant \Gamma_{P,1}(G)$. Applying now Theorem (24.9)(4) of Gorenstein and Lyons [**4**, p. 318], we obtain that $\Omega_1(P^*) \leqslant E$ or $E \in \{G_2(3)', {}^2B_2(2^5)\}$. Since $P^* \in \mathrm{Syl}_p(L(V))$ it follows that $P^* \leqslant E$. As $O_{p'}(L(V)) = 1$ we get $F^*(G) = L(G) = L(V) = E$. Hence $F^*(G)$ is simple. Now Theorems (24.1) and (24.2) of Gorenstein and Lyons [**4**, pp. 307, 311] complete the proof.

Combining this result with Suzuki's theorem [**13**] we obtain

PROPOSITION 2.3. *Let G be a nonabelian simple group with a noncyclic T.I. Sylow p-subgroup P. Then G is isomorphic to one of the following groups.*

(a) $\mathrm{PSL}_2(q)$ *or* $\mathrm{PSU}_3(q)$, *where* $q = p^n$ *and* $n \geqslant 2$ *or* $n \geqslant 1$, *respectively.*

(b) $p = 2$ *and* $G \cong {}^2B_2(2^{2m+1})$.

(c) $p = 3$ *and* $G \cong {}^2G_2(3^{2m+1})$, *where* $m \geqslant 1$.

(d) $p = 3$ *and* $G \cong \mathrm{PSL}_3(4)$ *or* M_{11}.

(e) $p = 5$ *and* $G \cong {}^2F_4(2)'$ *or Mc.*

(f) $p = 11$ *and* $G \cong J_4$.

PROOF. If $p = 2$, then (a) and (b) follow from Theorem 1 of [**13**].

Let p be odd. By Remark 2.1 G can only be one of the simple $L(V)$ occurring in the list of Proposition 2.2. Since $\mathfrak{A}_p \times \mathfrak{A}_p$ is a subgroup of $\mathfrak{A}_{2p}$, Lemma 1.2 asserts that $G \ncong \mathfrak{A}_{2p}$. A group H with a T.I. Sylow p-subgroup has only p-blocks of defect zero and of highest defect. By the character table system CAS [**11**] $M(22)$ has a 5-block of defect one. Thus $G \ncong M(22)$. Since $\mathrm{Aut}({}^2B_2(2^5))$ is not simple, $G \ncong \mathrm{Aut}({}^2B_2(2^5))$.

Now $\mathrm{PSL}_2(p^n)$ and $\mathrm{PSU}_3(p^n)$ have T.I. Sylow p-subgroups (see [**5**, pp. 191, 242]). By Ward [**14**] ${}^2G_2(3^{2m+1})$ has a T.I. Sylow 3-subgroup. As can be seen from the character table of $\mathrm{PSL}_3(4)$ the Sylow 3-subgroup P equals $C_G(x)$ for every $1 \neq x \in P$. Hence P is a T.I. set.

It is well known and easy to check that the Sylow 3-subgroups of M_{11} and the Sylow 5-subgroups of the Tits group ${}^2F_4(2)'$ and the McLaughlin group Mc are T.I. By Propositions 22 and 26 of Janko [**7**] the Sylow 11-subgroups of J_4 are T.I. This completes the proof.

3. Proof of the main result. In this section, Leonard's conjecture is proved by means of the results mentioned above.

Let G be a finite group with a Sylow p-subgroup P. If $C_G(P) = C_G(x)$ for every $1 \neq x \in P$, then P is called weakly self-centralizing. The following lemma is well known.

LEMMA 3.1. *Let G be a finite group with a cyclic Sylow p-subgroup P. Then P is a T.I. set if and only if P is weakly self-centralizing.*

THEOREM 3.2. *Let G be a finite group with a T.I. Sylow p-subgroup P. If G has a faithful complex character χ with degree $\chi(1) \leqslant \sqrt{|P|} - 1$, then P is a normal subgroup of G.*

PROOF. If P is cyclic, then P is weakly self-centralizing. As $\sqrt{|P|} - 1 < \frac{1}{2}(|P| - 1)$ for every prime $p > 0$, it follows from Sibley's theorem [**12**] that $P \lhd G$.

Now let G be a counterexample of minimal order. Then P is not cyclic, $G = O^{p'}(G)$, and by Proposition 1.4 $Z = Z(G) = O_{p'}(G) < O^p(G) = H$. Furthermore, H/Z is a nonabelian simple group with a T.I. Sylow p-subgroup, and we may assume that χ is irreducible. We also can assume that G does not have a proper abelian direct factor.

Suppose that p is odd. Then $m_p(G) > 1$, because P is not cyclic. By Remark 2.1 $N_G(P)$ is strongly p-embedded in G. Therefore it follows from Propositions 2.2 and $p = 3$ and $G \cong {}^2G_2(3)$ or 2.3 that $H = O^p(G) = G$ except when $p = 3$ and $G \cong {}^2G_2(3)$ or $p = 5$ and $G/Z \cong \mathrm{Aut}({}^2B_2(2^5))$. Now remembering that $p \nmid |Z|$ and using Gorenstein's table [**3**, Table 4.1, p. 302] of the Schur multipliers of the finite simple groups, the structure of G can be described as in the following table.

prime p	G/Z	Z
$p \mid q$	$\mathrm{PSL}_2(q)$ or $\mathrm{PSU}_3(q)$	$\lvert Z\rvert \leqslant 2$ or $\lvert Z\rvert \leqslant 3$
$p = 3$	${}^2G_2(3^{2m+1})$	$Z = 1$
$p = 3$	$\mathrm{PSL}_3(4)$ or M_{11}	$\lvert Z\rvert \leqslant 4$ or $Z = 1$
$p = 5$	${}^2F_4(2)'$	$Z = 1$
$p = 5$	$\mathrm{Aut}({}^2B_2(2^5))$	$Z = 1$
$p = 5$	Mc	$\lvert Z\rvert \leqslant 3$
$p = 11$	J_4	$Z = 1$

Applying the theorem of Landazuri and Seitz [**8**] on the minimal degrees of the nontrivial complex projective representations π of $\mathrm{PSL}_2(q)$, $\mathrm{PSU}_3(q)$, or ${}^2G_2(3^{2m+1})$ we see that $\pi(1) \geqslant \frac{1}{2}(q-1)$, $\pi(1) \geqslant q(q-1)$, and $\pi(1) \geqslant 3^{2m+1}(3^{2m+1}-1)$, respectively. In any case $\pi(1) > \sqrt{|P|} - 1$, a contradiction. If $p = 3$ and $G/Z \in \{\mathrm{PSL}_3(4), M_{11}\}$, then $|P| = 9$. But another contradiction is obtained since the nontrivial irreducible projective characters of these simple groups have minimal degrees

$$\chi(1) = \begin{cases} 4, & \text{if } G/Z = \mathrm{PSL}_3(4), \\ 10, & \text{if } G = M_{11}. \end{cases}$$

If $p = 5$ and $G = {}^2F_4(2)'$ then every nontrivial irreducible character χ of G has degree $\chi(1) \geqslant 26$. However, $|P| = 25$, a contradiction.

If $p = 5$ and $G = \mathrm{Aut}({}^2B_2(2^5))$, then every faithful irreducible character χ of G has degree $\chi(1) \geqslant \pi(1)$, where π is a nontrivial irreducible character of the Suzuki group ${}^2B_2(q)$, $q = 2^5$, of minimal degree. Now by Landazuri and Seitz [**8**, p. 419], $\pi(1) = 4 \cdot 31 = 124$. Since $|P| = 125$, we obtain $\chi(1) > \sqrt{|P|} - 1$, a contradiction.

If $p = 5$ and $G/Z = Mc$, then every irreducible nontrivial character χ of G has degree $\chi(1) \geqslant 22$ by the character table of Mc (see [**11**]). As $|P| = 125$, G cannot be a minimal counterexample. If $G/Z \cong Mc$ and $|Z| = 3$, we again use the character table of G (see [**11**]) and find that the nontrivial projective irreducible character χ of minimal degree has degree $\chi(1) = 126 \geqslant \sqrt{125} - 1$, another contradiction.

If $p = 11$ and $G = J_4$, then $\chi(1) \geqslant 1333$ by [**11**]. Since $|P| = 11^3$, $\chi(1) > \sqrt{|P|} - 1$, which is impossible by hypothesis.

Therefore $p = 2$. Hence by Theorem 2 of Suzuki [**13**] and Proposition 1.4 we get $G/Z \in \{\mathrm{PSL}_2(q), \mathrm{PSU}_3(q), {}^2B_2(q)\}$, where q is a power of 2. Using the theorem of Landazuri and Seitz [**8**] as above, we obtain our final contradiction. This completes the proof.

REMARK 3.3. It is not possible to replace the bound $\sqrt{|P|} - 1$ by the bound $\frac{1}{2}(|P| - 1)$ of Sibley's theorem [**12**]. Let $G = Mc$, $p = 5$, and χ be the irreducible character of G with degree $\chi(1) = 22$. The Sylow 5-subgroup P of G is T.I. and has order $|P| = 5^3 = 125$. Hence $\chi(1) = 22 < 62 = \frac{1}{2}(|P| - 1)$. However, P is not normal. In particular, Sibley's condition that P be weakly self-centralizing cannot be weakened to P being a T.I. set.

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