PROCEEDINGS OF THE
AMERICAN MATHEMATICAL SOCIETY
Volume 102, Number 1, January 1988

ATRIODIC HOMOGENEOUS NONDEGENERATE CONTINUA ARE ONE-DIMENSIONAL

JAMES T. ROGERS, JR.

(Communicated by Doug W. Curtis)

ABSTRACT. C. L. Hagopian [**3**] has shown that atriodic, homogeneous, nondegenerate continua are one-dimensional. This answered a question of Maćkowiak and Tymchatyn [**4**]. We use a decomposition theorem to get a quick proof of this.

A *continuum* is a compact, connected, nonvoid metric space. A space is *homogeneous* if its homeomorphism group acts transitively on it.

A metric space X has the *Effros property* if given $\varepsilon > 0$, there exists $\delta > 0$ such that whenever y and z are points of X satisfying $d(y,z) < \delta$, there exists a homeomorphism $h: X \to X$ such that $h(y) = z$ and $d(x, h(x)) < \varepsilon$ for all x in X. Effros [**2**] has shown that each homogeneous continuum has the Effros property.

A continuum is *decomposable* if it is the union of two of its proper subcontinua. Otherwise it is *indecomposable*. A continuum is *hereditarily indecomposable* if it contains no decomposable subcontinuum.

A *partition* of the continuum X is a collection of disjoint sets whose union is X. The homeomorphism group $H(X)$ *respects the partition* $\mathbf{G}$ if each homeomorphism permutes the elements of $\mathbf{G}$.

A subcontinuum Z of the continuum X is *terminal* in X if each subcontinuum Y of X that intersects Z satisfies either $Y \subset Z$ or $Z \subset Y$. A partition of X is *terminal* if each element of the partition is a terminal subcontinuum of X.

A continuum is a *triod* if it is the union of three continua such that the common part of all three of them is both a proper subcontinuum of each of them and the common part of every two of them. A continuum is *atriodic* if it contains no triod.

In addition to the Effros property, we use four propositions as tools in our proof. The first was proved by Maćkowiak and Tymchatyn [**4**, p. 31] in their investigation of atriodic, homogeneous continua, the second is due to Bing [**1**], and the last two were proved by the author in [**5**].

PROPOSITION 1. *Each indecomposable subcontinuum of an atriodic, homogeneous continuum X is terminal in X.*

PROPOSITION 2. *If the continuum X does not contain a nondegenerate, hereditarily indecomposable subcontinuum, then* $\dim X \leq 1$.

Received by the editors August 21, 1986 and, in revised form, October 22, 1986.

1980 *Mathematics Subject Classification* (1985 *Revision*). Primary 54F20; Secondary 54F50.

Key words and phrases. Continuum, homogeneous, atriodic, terminal continuum, hereditarily indecomposable.

This research was partially supported by National Science Foundation Grant No. DMS-8300569-02.

PROPOSITION 3. *If X is a hereditarily indecomposable, homogeneous continuum, then* $\dim X \leq 1$.

PROPOSITION 4. *If* $\mathbf{G}$ *is a partition of the homogeneous continuum* X *into nondegenerate, proper, terminal subcontinua and if the homeomorphism group* $H(X)$ *of* X *respects* $\mathbf{G}$, *then each element of* $\mathbf{G}$ *is a homogeneous continuum of the same dimension as* X.

THEOREM 1. *If X is an atriodic, homogeneous continuum, then each point of X is contained in a unique maximal hereditarily indecomposable subcontinuum of X.*

PROOF. For $x \in X$, let $M(x)$ be the union of all the hereditarily indecomposable subcontinua of X containing x. We show that every proper subcontinuum Z of $\overline{M(x)}$ is indecomposable, and therefore terminal in X. It follows that $\overline{M(x)}$ itself is also indecomposable.

By the Effros property, there exists a homeomorphism $h: X \to X$ such that $h(Z) \cap M(x) \neq \varnothing \neq M(x) \backslash h(Z)$. Choose hereditarily indecomposable subcontinua M, N of X containing x, with $h(Z) \cap M \neq \varnothing \neq N \backslash h(Z)$. Since M is terminal, either $h(Z) \subset M$ or $M \subset h(Z)$, In the former case, $Z \approx h(Z)$ is indecomposable. In the latter case, since N is terminal and since $x \in M \cap N \subset h(Z) \cap N$, $h(Z) \subset N$. Thus in either case, $Z \approx h(Z)$ is indecomposable.

Hence $\overline{M(x)}$ is hereditarily indecomposable. Clearly, $\overline{M(x)}$ is the unique maximal hereditarily indecomposable subcontinuum of X containing x. The proof of the theorem is complete. Note that $M(x) = \overline{M(x)}$.

THEOREM 2. *If X is an atriodic, homogeneous continuum, then* $\dim X \leq 1$.

PROOF. Let $\mathbf{G} = \{M(x): x \in X\}$ be the collection of maximal hereditarily indecomposable subcontinua of X. The collection $\mathbf{G}$ is a partition of X into terminal subcontinua, and the homeomorphism group of X respects $\mathbf{G}$.

If $M(x) = X$, then X is hereditarily indecomposable and $\dim X \leq 1$. If $M(x)$ is a point, then X does not contain a nondegenerate, hereditarily indecomposable subcontinuum, so $\dim X \leq 1$. Otherwise the elements of $\mathbf{G}$ are proper, nondegenerate subcontinua of X and thus homogeneous continua of the same dimension as X. Since $M(x)$ is homogeneous and hereditarily indecomposable, $\dim M(x) \leq 1$. Therefore $\dim X \leq 1$.

REFERENCES

1. R. H. Bing, *Higher-dimensional hereditarily indecomposable continua*, Trans. Amer. Math. Soc. **21** (1951), 267–273.
2. E. G. Effros, *Transformation group and C^*-algebras*, Ann. of Math. (2) **81** (1965), 38–55.
3. C. L. Hagopian, *Atriodic homogeneous continua*, Pacific J. Math. **113** (1984), 333–347.
4. T. Maćkowiak and E. D. Tymchatyn, *Continuous mappings on continua*. II, Dissertationes Math. **225** (1983).
5. J. T. Rogers, Jr., *Orbits of higher-dimensional hereditarily indecomposable continua*, Proc. Amer. Math. Soc. **95** (1985), 483–485.

DEPARTMENT OF MATHEMATICS, TULANE UNIVERSITY, NEW ORLEANS, LOUISIANA 70118