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SYMMETRY OF DICHROMATIC LINKS

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ABSTRACT. Let L be a 1-trivial dichromatic link in S^3 and $\widetilde{L}$ its covering link. A relationship between the dichromatic link polynomials, defined by Hoste and Przytycki, of L and $\widetilde{L}$ is given. As an application, it is shown that the link 7_1^2 has no symmetries with fixed point set is either of the components.

1. INTRODUCTION

A 1-trivial dichromatic link in S^3 is a link having at least two components, one of which is unknotted and labeled, or colored, "1", while all other components are colored "2". If L is a 1-trivial dichromatic link, then we may isotope L until the 1-component, that is the component colored "1", is the z-axis union the point at infinity. If we now project the link into the x-y plane, we obtain a diagram of the 2-sublink in the punctured plane $R^2 - \{0\}$. We may obviously use such punctured diagrams to represent 1-trivial dichromatic links. Generalizing the Jones polynomial [J], Hoste and Przytycki [HP] defined a unique polynomial invariant in $Z[A^{\pm 1}, h]$ of unoriented 1-trivial dichromatic links as follows:

$$d_L(A, h) = (-A^3)^{-\operatorname{sw}(D)}\langle D\rangle,$$

where D is any punctured diagram of the link L, and $\langle D\rangle$ is the invariant of D determined by the following properties:

(1) $$\langle \cdot\bigcirc \rangle = 1,$$

(2) $$\langle \odot \rangle = h,$$

(3) $$\langle \times \rangle = A\langle)(\rangle + A^{-1}\langle \asymp \rangle,$$

(4) $$\langle \cdot \bigcirc K\rangle = -(A^2 + A^{-2})\langle \cdot K\rangle, \ \ K \neq \varnothing,$$

(5) $$\langle \odot K\rangle = -(A^2 + A^{-2})h\langle \cdot K\rangle, \ \ K \neq \varnothing,$$

and $\operatorname{sw}(D)$ is the self writhe of D, that is, the sum of the signs of those crossings between strands belonging to the same component. Here we follow Kauffman's notation [K] with the additional convention of marking the puncture with a dot. If L is a 1-trivial dichromatic link, we denote the 1-component by L_1 and the

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2-sublink by L_2. Let $N(L_1)$ be a tubular neighborhood of L_1. Let $\mathrm{wr}(L)$ be the wrapping number of L_2 in the solid torus $V = S^3 - \overset{\circ}{N}(L_1)$, that is, the minimal geometric intersection number of L_2 with any meridian disk of V. In [HP] they show

$$\deg_h d_L \leq \mathrm{wr}(L), \tag{6}$$

where $\deg_h d_L$ is the highest degree of h appearing in d_L. We consider the p-fold cyclic branched cover $q_p\colon S^3 \to S^3$ branched over L_1. Then the upstairs branch set $\widetilde{L_1}$ and the preimage $\widetilde{L_2} = q_p^{-1}(L_2)$ again constitute a 1-trivial dichromatic link $\widetilde{L} = \widetilde{L_1} \cup \widetilde{L_2}$. Then we have the following:

Theorem 1. *Let L be a 1-trivial dichromatic link. Suppose $\mathrm{wr}(L) = m$. Let $f(A)$ be the coefficient of h^m in d_L. Then the coefficient of h^m in $d_{\widetilde{L}}$ equals $A^{6r}\delta^{-(p-1)(m-1)}\{f(A)\}^p$, where $r \in Z$, $\delta = -(A^2 + A^{-2})$, and $f(A) \in Z[A^{\pm 1}]$. In particular, if L_2 and $\widetilde{L_2}$ are knots, then $r = 0$.*

Concerning the wrapping numbers of L and $\widetilde{L}$, we have:

Proposition 1. *Let L be a 1-trivial dichromatic link. Then the wrapping number of $\widetilde{L_2}$ equals the wrapping number of L_2.*

Combining Theorem 1, Proposition 1, and (6), we obtain:

Theorem 2. *Let L be a 1-trivial dichromatic link. Then $\deg_h d_L = \mathrm{wr}(L)$ if and only if $\deg_h d_{\widetilde{L}} = \mathrm{wr}(\widetilde{L})$.*

If L is a 1-trivial dichromatic link, we say that L admits a Z_p-action fixing L_1 if there exists a Z_p-action of S^3 that preserves L and has fixed point set L_1.

As a corollary of Theorems 1 and 2, we have:

Corollary 1. *Let L be a 1-trivial dichromatic link satisfying $\deg_h d_L = \mathrm{wr}(L)$. Let p be an integer that is more than 1. If L admits a Z_p-action fixing L_1, there exist an integer r and a Laurent polynomial $g(A)$ such that the coefficient of $h^{\mathrm{wr}(L)}$ in d_L is $A^{6r}\delta^{-(p-1)(\mathrm{wr}(L)-1)}\{g(A))\}^p$.*

If a 1-trivial dichromatic link L admits a Z_p-action fixing L_1, then it is shown in [HP] that

$$d_L(A, h) \equiv d_L(A^{-1}, h) \bmod (A^{4p} - 1, p)$$

for L, where p is prime. If L is a link that is both 1-trivial and 2-trivial, then we can also consider a Z_p-action on S^3 with fixed point set L_2. Using this, they show that the link $L = 7_6^2$ [R, Appendix C] as shown in Figure 1 admits no Z_p-actions fixing L_1 for $p > 2$ or fixing L_2 for $p > 3$. Using Corollary 1, we can show that 7_6^2 admits no Z_p-action for any integer $p \geq 2$ such that the fixed point set is either L_1 or L_2. Other examples are also given in §4. In §2 we prove Theorem 1. In §3 we prove Proposition 1.

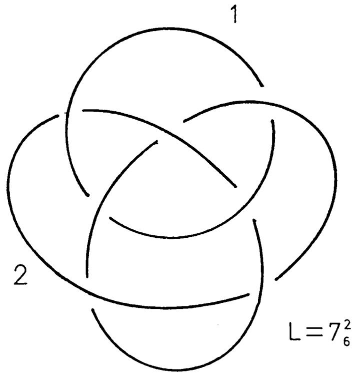


FIGURE 1

2. PROOF OF THEOREM 1

We now recall "state models" by Kauffman [K]. Let D be a diagram of a link L. If U is the underlying planar graph for D, then a state of U is a choice of splitting marker for every vertex of U. Let S be a state for a diagram D and $D(S)$ the diagram obtained from D by splitting the state S.

If we consider the diagrams of links, we can define the wrapping number of them as follows: Let D be a punctured diagram of L. We define $\mathrm{wr}(D)$, the wrapping number of D, to be the minimal intersection number between D and any ray emanating from the puncture and extending to infinity.

Lemma 1. *Let D be a punctured diagram of a 1-trivial dichromatic link L with* $\mathrm{wr}(D) = m$. *Then there exists a state S of D satisfying* $\deg_h\langle D(S)\rangle = m$.

Proof. Since $\mathrm{wr}(D) = m$ we may picture D as in Figure 2. We induct on m. If $m = 0$ or 1, then any state S of D suffices. If $m > 1$ then consider the

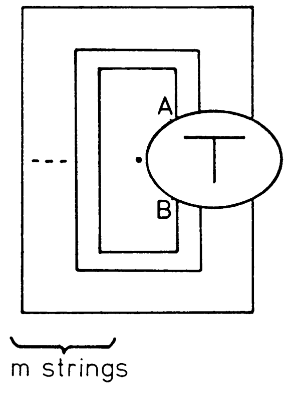


FIGURE 2

points labelled A and B in Figure 2. It is possible to smooth some subset of the crossings of D so that the string entering T at A then exits at B, has no crossings, and is part of an innermost circle of nontrivial ones in the resulting diagram D', where a nontrivial circle is a simple closed curve that is not null-homotopic in $R^2 - \{0\}$. To see this, simply enter T at A and smooth each crossing as it is encountered so as to always turn to the right. If this process does not force one to exit at B then $\mathrm{wr}(D) < m$. Now $D' = D_1' \cup D_2'$ where D_1' is the innermost circle mentioned above and D_2' has wrapping number $m-1$. By induction and property 5 of $\langle\ \rangle$, D' has a state S' such that $\deg_h \langle D'(S')\rangle = m$. Now let S be the extension of S' to D obtained by incorporating the original smoothings used to produce D' from D. □

Let D be a diagram of L as shown in Figure 2. Let U be the underlying planar graph for D and U_T the subgraph of U for a tangle T. We may regard a state S for D as a state for T, that is, a choice of splitting marker for every vertex of U_T is S. Let $T(S)$ be the tangle obtained from T by splitting the state S. Then we easily obtain:

Lemma 2. *Let L be a 1-trivial dichromatic link with $\mathrm{wr}(L) = m \geq 1$ and D a punctured diagram of L as shown in Figure 2. Let S be a state for D. Then the following two conditions are equivalent:*

(1) $\deg_h \langle D(S)\rangle = m$;
(2) *$T'(S)$ is a trivial m-braid,*

where $T'(S)$ is the tangle obtained from $T(S)$ by removing all the trivial components contained entirely in $T(S)$.

Remark. Lemma 2 really is just a statement about the bracket polynomial of a diagram with no crossings. Namely the following lemma:

Lemma 2′. *Let $D_{t,n}$ be a punctured diagram having no crossings, t trivial circles, and n nontrivial circles. Then $\langle D_{t,n}\rangle = \delta^{t+n-1}h^n$.*

This lemma is essentially the same thing as Lemma 2.

We define $\langle D|S\rangle$ for a diagram D and a state S by the formula

$$\langle D|S\rangle = A^{n-2i},$$

where n is the number of crossings of D and i is the number of state markers corresponding to splittings which join the regions labelled A^{-1} in Figure 3.

Lemma 3. *Let L be a 1-trivial dichromatic link. If $\mathrm{wr}(L) = m$, then the coefficient of h^m in d_L has the form $\delta^{m-1}f(A)$, where $f(A) \in Z[A^{\pm 1}]$.*

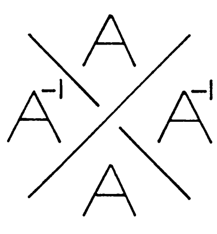


FIGURE 3

Proof. Since $\mathrm{wr}(L) = m$, a diagram D of L can be drawn as in Figure 2. Then

$$\langle D\rangle = \sum_{S} \langle D|S\rangle \langle D(S)\rangle ,$$

where this summation is over all states of the diagram. Now we compute the coefficient of h^m in d_L. We need only consider the states

$$\mathcal{S} = \{S| \deg_h \langle D(S)\rangle = m\} ,$$

which by Lemma 1 is nonempty. Let $|D(S)|$ be the number of the trivial components in $D(S)$ (recall Lemma 2′). Let $D'(S)$ be the diagram obtained from $D(S)$ by removing all the trivial components contained entirely in $T(S)$. If $m = 0$, then $\langle D(S)\rangle = \delta^{|D(S)|-1}$. If $m \geq 1$, then

$$\langle D(S)\rangle = \delta^{|D(S)|} \langle D'(S)\rangle = \delta^{|D(S)|+(m-1)} h^m.$$

So the coefficient of h^m in $\langle D\rangle$ is given by

$$\sum_{S\in\mathcal{S}} \langle D|S\rangle \delta^{|D(S)|+(m-1)} = \delta^{m-1} \sum_{S\in\mathcal{S}} \langle D|S\rangle \delta^{|D(S)|}.$$

Hence the coefficient of h^m in d_L is

$$(-A^3)^{-\,\mathrm{sw}(D)} \delta^{m-1} \sum_{S\in\mathcal{S}} \langle D|S\rangle \delta^{|D(S)|} = \delta^{m-1} \left\{ (-A^3)^{-\,\mathrm{sw}(D)} \sum_{S\in\mathcal{S}} \langle D|S\rangle \delta^{|D(S)|} \right\}.$$

Putting $f(A) = (-A^3)^{-\,\mathrm{sw}(D)} \sum_{S\in\mathcal{S}} \langle D|S\rangle \delta^{|D(S)|}$, we obtain the desired formula. □

If D is a diagram of L as shown in Figure 2, then the diagram $\widetilde{D}$ given in Figure 4 is that of $\widetilde{L}$, where each T_i, $1 \leq i \leq p$, is a copy of T and $\widetilde{T}$ is the sum of $T_1, T_2, \ldots, T_p$. Let U be the underlying planar graph for $\widetilde{D}$ and U_{T_i} subgraph of U for T_i. In Figure 4 let $S_1, S_2, \ldots, S_p$ be the states for the tangles $T_1, T_2, \ldots, T_p$, respectively. The union of $S_1, S_2, \ldots, S_p$ defines a state S for the diagram $\widetilde{D}$, which we denote by $S = (S_1, S_2, \ldots, S_p)$. We may also regard S as a state for $\widetilde{T}$. Let $T_i'(S_i)$ (resp. $\widetilde{T}'(S)$) be the tangle obtained from $T_i(S_i)$ (resp. $\widetilde{T}(S)$) by removing all the trivial components contained entirely in $T_i(S_i)$ (resp. $\widetilde{T}(S)$). Then we have:

Lemma 4. *Let L be a 1-trivial dichromatic link with $\mathrm{wr}(L) = m \geq 1$ and $\widetilde{D}$ a punctured diagram of $\widetilde{L}$ as shown in Figure 4. Let S be a state for $\widetilde{D}$. Then the following three conditions are equivalent*:

(1) *For each i, $1 \leq i \leq p$, $T_i'(S_i)$ is a trivial m-braid*;
(2) *$\widetilde{T}'(S)$ is a trivial m-braid*;
(3) $\deg_h \langle \widetilde{D}(S)\rangle = m$.

Let L be a 1-trivial dichromatic link. Let D be a diagram of L as shown in Figure 2 and $\widetilde{D}$ the diagram of $\widetilde{L}$ as shown in Figure 4. Suppose L_2 is an oriented link having t components: $L_2 = K_1 \cup K_2 \cup \cdots \cup K_t$. Let $\widetilde{K}_u$ be the preimage of K_u, $1 \leq u \leq t$. We also denote the components of $\widetilde{K}_u$, $1 \leq u \leq t$, by $K_{u,1}, K_{u,2}, \ldots, K_{u,n_u}$, so $\widetilde{L_2} = K_{1,1} \cup \cdots \cup K_{1,n_1} \cup \cdots \cup K_{t,1} \cup \cdots \cup K_{t,n_t}$, where $K_{u,i}$ is oriented so that it induces the same orientation on K_u downstairs. Putting $r = \sum_{u=1}^{t} \sum_{i\neq j} lk(K_{u,i}, K_{u,j})$, we easily obtain:

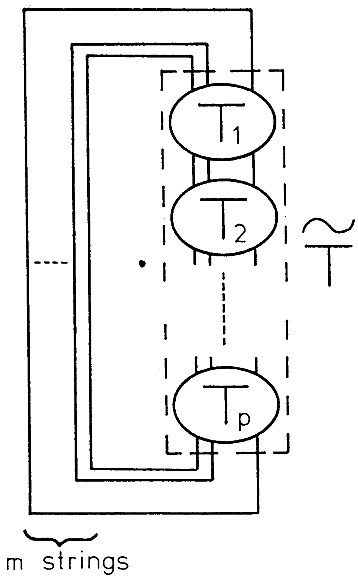


FIGURE 4

Lemma 5. $\mathrm{sw}(\widetilde{D}) = \mathrm{psw}(D) - 2r$.

Proof of Theorem 1. Let D be a diagram as shown in Figure 2 and $\widetilde{D}$ the diagram as shown in Figure 4. In Figure 4 let $S_1, S_2, \dots, S_p$ be the states for the tangles $T_1, T_2, \dots, T_p$, respectively. Let $S = (S_1, S_2, \dots, S_p)$. By Lemma 1, there exists a state $S = (S_1, S_2, \dots, S_p)$ satisfying $\deg_h\langle\widetilde{D}(S)\rangle = m$. In case $m = 0$ it is easy. Suppose $m \geq 1$. By Lemmas 2 and 4, in order to compute the coefficient of h^m in $d_{\widetilde{L}}$, we may only consider the states

$$\widetilde{\mathscr{S}} = \{S \mid \deg_h\langle\widetilde{D}(S)\rangle = m\} = \{(S_1, S_2, \dots, S_p) \mid \deg_h\langle D(S_i)\rangle = m,\ 1 \leq i \leq p\},$$

where we regard S_i, $1 \leq i \leq p$, as a state for D. Moreover by Lemma 4, we have

$$|\widetilde{D}(S)| = \sum_{i=1}^{p} |D(S_i)|, \qquad \langle\widetilde{D}|S\rangle = \prod_{i=1}^{p}\langle D|S_i\rangle.$$

The coefficient of h^m in $\langle\widetilde{D}\rangle$ is given by

$$\begin{aligned}\sum_{S\in\widetilde{\mathscr{S}}}\langle\widetilde{D}|S\rangle\delta^{|\widetilde{D}(S)|+(m-1)} &= \delta^{m-1}\sum_{(S_1, S_2, \dots, S_p)\in\widetilde{\mathscr{S}}}\left(\prod_{i=1}^{p}\langle D|S_i\rangle\right)\delta^{\sum_{i=1}^{p}|D(S_i)|}\\ &= \delta^{m-1}\left(\sum_{S_i\in\mathscr{S}_i}\langle D|S_i\rangle\delta^{|D(S_i)|}\right)^p,\end{aligned}$$

where $\mathscr{S}_i = \{S_i \mid \deg_h\langle D(S_i)\rangle = m\}$. Then by Lemma 5 the coefficient of h^m in

$d_{\widetilde{L}}$ is

$$(-A^3)^{-\operatorname{sw}(\widetilde{D})}\delta^{m-1}\left(\sum_{S_i\in\mathscr{S}_i}\langle D|S_i\rangle\delta^{|D(S_i)|}\right)^p$$
$$=(-A^3)^{-p\operatorname{sw}(D)+2r}\delta^{m-1}\left(\sum_{S_i\in\mathscr{S}_i}\langle D|S_i\rangle\delta^{|D(S_i)|}\right)^p$$
$$=A^{6r}\delta^{m-1}\left\{(-A^3)^{-\operatorname{sw}(D)}\sum_{S_i\in\mathscr{S}_i}\langle D|S_i\rangle\delta^{|D(S_i)|}\right\}^p.$$

Putting $g(A)=(-A^3)^{-\operatorname{sw}(D)}\sum_{S_i\in\mathscr{S}_i}\langle D|S_i\rangle\delta^{|D(S_i)|}$, we obtain the formula by Lemma 3. □

3. Proof of Proposition 1

In case $m=0$, it is obvious. Suppose $\operatorname{wr}(L)=m\geq 1$. We may assume that a disk D_0 giving the wrapping number of L_2 is a standard meridian disk of $V=S^3-\overset{\circ}{N}(L_1)$. We denote the preimage of D_0 in $\widetilde{V}=q_p^{-1}(V)$ by $\widetilde{D_1}, \widetilde{D_2}, \dots, \widetilde{D_p}$. Suppose $\widetilde{D_1}$ is not a disk that gives the wrapping number of $\widetilde{L_2}$. Let $\widetilde{C}$ be a circle obtained by moving $\partial\widetilde{D_1}$ slightly on $\partial\widetilde{V}$. Hence $\widetilde{C}\cap\partial\widetilde{D_1}=\varnothing$. We consider properly embedded disks $\mathscr{D}=\{D|\partial D=\widetilde{C}\}$ in $\widetilde{V}$ satisfying:

(1) D intersects $\widetilde{L_2}$ transversally in less than m points;
(2) D intersects $\widetilde{D_1}\cup\widetilde{D_2}\cup\cdots\cup\widetilde{D_p}$ transversally, and hence the intersection consists of disjoint circles; and
(3) $D\cap(\widetilde{D_1}\cup\widetilde{D_2}\cup\cdots\cup\widetilde{D_p})\cap\widetilde{L_2}=\varnothing$.

Let $n(D)$ be the number of circles of (2). Let $\overline{D}$ be a disk such that $n(\overline{D})\leq n(D)$ for any $D\in\mathscr{D}$. Suppose $n(\overline{D})=0$. Then $\overline{D}$ lies in a fundamental region, which is one of the p regions divided by $\widetilde{D_1}, \widetilde{D_2}, \dots, \widetilde{D_p}$. Hence $q_p(\overline{D})$ is a disk having no singularities in V, and we have $\#\{q_p(\overline{D})\cap L_2\}=\#\{\overline{D}\cap\widetilde{L_2}\}<m=\#\{D_0\cap L_2\}$. This is a contradiction. Suppose $n(\overline{D})\neq 0$. Let C_0 be one of innermost circles in $\overline{D}$ and $C_0\subset\widetilde{D_k}$, and let D' be the disk in $\overline{D}$ with $\partial D'=C_0$. Hence D' is a disk in $\widetilde{V}$ and is embedded in a fundamental region W. Let $\widehat{D_k}$ be the disk in $\widetilde{D_k}$ with $\partial\widehat{D_k}=C_0$. See Figure 5. If $\#\{D'\cap\widetilde{L_2}\}<\#\{\widehat{D_k}\cap\widetilde{L_2}\}$, then for the disk $\widetilde{D}=(\widetilde{D_k}-\overset{\circ}{\widehat{D_k}})\cup D'$, we have the following inequality $\#\{\widetilde{D}\cap\widetilde{L_2}\}<\#\{\widetilde{D_k}\cap\widetilde{L_2}\}$. Hence $\#\{q_p(\widetilde{D})\cap L_2\}=\#\{\widetilde{D}\cap\widetilde{L_2}\}<\#\{\widetilde{D_k}\cap\widetilde{L_2}\}=\#\{D_0\cap L_2\}$, a contradiction. Suppose $\#\{D'\cap\widetilde{L_2}\}\geq\#\{\widehat{D_k}\cap\widetilde{L_2}\}$. Then $(\overline{D}-\overset{\circ}{D'})\cup\widehat{D_k}$ is an immersed disk in $\widetilde{V}$. We denote it by $\widetilde{D}$. Note that $\#\{\widetilde{D}\cap\widetilde{L_2}\}<m$. We consider the cases:

Case 1. $\widetilde{D}$ is a disk having no singularities.

Case 2. $\widetilde{D}$ has singularities.

In Case 1 we can push $\widetilde{D}$ slightly away from $\widetilde{D_k}$ near $\widehat{D_k}$. This shows that $n(\widetilde{D})<n(\overline{D})$, which is a contradiction.

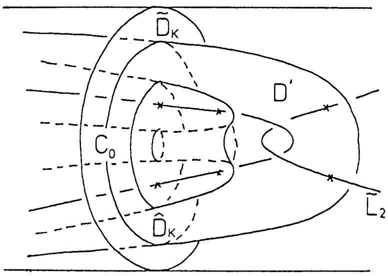


FIGURE 5

In Case 2 we note that singularities are circles $C_1, C_2, \ldots, C_s$ in $\widetilde{D_k}$ and consist of double points. Let $N(C_i)$ be a tubular neighborhood of C_i, $0 \le i \le s$, in $\widetilde{V}$. We consider $N(C_0)$, $N(C_1), \ldots, N(C_s)$ such that $N(C_i) \cap \widetilde{L_2} = \varnothing$, $0 \le i \le s$, and $N(C_i) \cap N(C_j) = \varnothing$, $i \neq j$. By a suitable cut and paste operation in $N(C_1)$ as Hempel's Lemma (see [H, Lemma 4.6]), we can obtain a new disk $\widetilde{D}'$ whose singularities are $C_2, \ldots, C_s$. Then the number of intersection of $\widetilde{D}'$ and $\widetilde{L_2}$ equals that of $\widetilde{D}$ and $\widetilde{L_2}$ since $N(C_1) \cap \widetilde{L_2} = \varnothing$. Using this repeatedly, we have a nonsingular disk $\widehat{D}$ with $\#\{\widehat{D} \cap \widetilde{L_2}\} = \#\{\widetilde{D} \cap \widetilde{L_2}\} < m$. We can push the part $\widehat{D} \cap \widehat{D_k}$ of $\widehat{D}$ slightly away from $\widetilde{D_k}$. This shows that $n(\widehat{D}) < n(\overline{D})$, which is a contradiction. It follows that $\widetilde{D_1}$ is a disk giving the wrapping number of $\widetilde{L_2}$.

4. APPLICATIONS

Using Corollary 1, we prove that the link 7_1^2 [R, Appendix C] as shown in Figure 6 admits no Z_p-action for any integer $p \ge 2$ such that the fixed point set is each of the trivial components, and that the link 8_9^3 [R, Appendix C] as shown in Figure 7 admits no Z_p-action for any integer $p \ge 3$ such that the fixed point set is the component colored "1."

Since $L = 7_1^2$ is a link that is both 1-trivial and 2-trivial, we may compute d_L relative to either component. Call these two invariants d_L^1 and d_L^2, respectively. But 7_1^2 is interchangeable, so $d_L^1 = d_L^2$. The highest degree of h in d_L^1 is 3. Thus

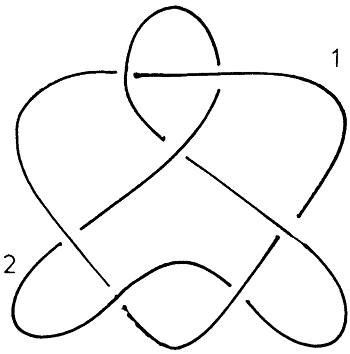
$$\deg_h d_L^1 = \operatorname{wr}(L^{(2)}) = 3,$$

where $\operatorname{wr}(L^{(2)})$ is the wrapping number of L_2 in the solid torus $S^3 - \overset{\circ}{N}(L_1)$. Suppose there is a Z_p-action with fixed point set L_1 for an integer $p \ge 2$. Then by Corollary 1 there exists a Laurent polynomial $f(A)$ such that the coefficient of h^3 in d_L^1 is equal to $\delta^{-2(p-1)}\{f(A)\}^p$. On the other hand the coefficient of h^3 in d_L^1 is $-A^4(A^2 + A^{-2})^2(A^4 - 1)$. Thus

$$\{f(A)/\delta^2\}^p = -A^4(A^4 - 1).$$

The right-hand term has the factor $A - 1$. Hence the left-hand term must have the factor $A - 1$, so it has the factor $(A - 1)^p$. But the right-hand term does not have this factor. This is a contradiction. Next we consider $L = 8_9^3$. The highest degree of h in d_L^1 is 2. Thus

$$\deg_h d_L^1 = \operatorname{wr}(L^{(2)}) = 2.$$



$L = 7^2_1$

FIGURE 6

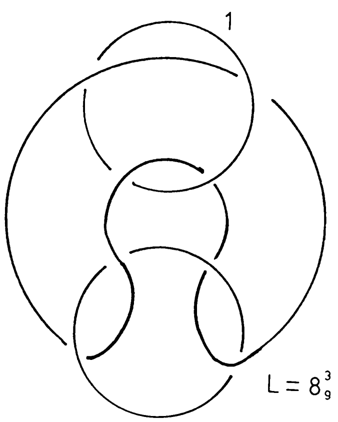


FIGURE 7

Suppose there is a Z_p-action with fixed point set L_1 for an integer $p \geq 2$. Since the coefficient of h^2 in d^1_L is $-(A^2 + A^{-2})(A^4 - 1)^2$, by Corollary 1

$$A^{6r}\delta^{-(p-1)}\{f(A)\}^p = -(A^2 + A^{-2})(A^4 - 1)^2,$$

for some $f(A) \in Z[A^{\pm 1}]$ and for some integer r. Thus

$$\{f(A)/\delta\}^p = A^{-6r}(A^4 - 1)^2.$$

This is impossible for $p \geq 3$. If $p = 2$, then 8^3_9 admits a Z_2-action with quotient the Whitehead's link.

Remark. $7^2_6 = L = L_1 \cup L_2$ admits no Z_p-action for any integer $p \geq 2$ such that the fixed point set is either L_1 or L_2. Suppose 7^2_6 admits a Z_p-action. Then there exists a factor link $\widehat{L} = \widehat{L_1} \cup \widehat{L_2}$ of L. Since L_2 is a knot and the linking number $\mathrm{lk}(L_1, L_2)$ of L_1 and L_2 is zero, $\widehat{L_2}$ becomes a knot and the linking number $\mathrm{lk}(\widehat{L_1}, \widehat{L_2})$ is equal to zero. This is a contradiction by the following fact: Suppose $L = K_1 \cup K_2$ is a two component link such that K_1 is a trivial knot and $\mathrm{lk}(K_1, K_2) = 0$. If we consider the n fold cyclic branched cover of S^3 branched over K_1, then for a covering link $\widetilde{L} = \widetilde{K_1} \cup \widetilde{K_2}$ of L, $\widetilde{K_2}$ is an n-component link.

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