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DEFEAT OF THE FP^2F CONJECTURE: HUCKABA'S EXAMPLE

CARL FAITH

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ABSTRACT. A commutative ring R is FP^2F (resp. FPF) provided that all finitely presented (resp. finitely generated) faithful modules generate the category mod-R of all R-modules. A conjecture of the author dating to the middle 1970s states that any FP^2F ring R has FP-injective classical quotient ring $Q = Q_{cl}(R)$. It was shown by the author (*Injective quotient rings.* II, Lecture Notes in Pure and Appl. Math., vol. 72, Dekker, New York, 1982, pp. 71–105) that FPF rings R have injective Q and by the author and P. Pillay (*Classification of commutative FPF rings*, Notas Math., vol. 4, Univ. de Murcia, Murcia, Spain, 1990) that CFP^2F local rings (defined below) have FP-injective Q.

The counterexample is a difficult example due to Huckaba of a strongly Prüfer ring without "Property A." (A ring with Property A was labelled a McCoy ring by the author.)

This counterexample is CFP^2F in the sense that every factor ring of R is FP^2F.

1. **Theorem.** (1) *A ring R is CFP^2F iff* (2) *R_M is a valuation ring* ($= VR$) *for each maximal ideal M.*

Proof. In [F1] (also [FP]) CFP^2F is characterized by the statement: R is locally a VR, i.e., R_P is a VR for all prime ideals P. However, a look at the proof in [F1, Corollary 5E, p. 176] establishes the equivalence of (1) and (2).

A ring R is said to be *McCoy* (see [F2]) or have Property A (see [H]) provided that every finitely generated dense (= faithful) ideal is regular, i.e., contains a regular element. This is equivalent to the statement that $Q = Q_{cl}(R)$ is McCoy, i.e., that Q is the only finitely generated dense ideal. A sufficient condition for R to be McCoy is for Q to be FP-injective.

(Every FP-injective ring Q has the property that finitely generated ideals are annihilator ideals.)

A ring R is *Prüfer* if every finitely generated regular ideal is invertible (see [H, p. 29]), equivalently projective. A ring R is *strongly Prüfer* iff every finitely generated dense ideal is locally principal [H, p. 115].

Huckaba's example. There exists a strongly Prüfer ring R with the properties that R is not McCoy; hence $Q = Q_{cl}(R)$ is not FP-injective, but R_M is a VR, in fact a domain (= VD), for all maximal ideals. (See [H, Example 17, p. 191].)

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This provides the counterexample to the FP^2F conjecture since R is CFP^2F by Theorem 1, yet Q is not FP-injective.

We also remark the following:

Theorem (Huckaba and Keller). *A reduced coherent ring R is McCoy iff Q is VNR (= von Neumann regular).*

Proof. See [H, Theorem 4.7, p. 20].

References

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Department of Mathematics, Rutgers University, New Brunswick, New Jersey 08903

Current address: 199 Longview Drive, Princeton, New Jersey 08540