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# CARDINALITIES OF $\eta_1$-ORDERED FIELDS

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ABSTRACT. We show, using GCH, that a cardinal $\kappa$ is the cardinality of an $\eta_1$-ordered field iff $\kappa^\omega = \kappa$. We also show, without using GCH, that a cardinal $\kappa$ is the cardinality of a maximally valued $\eta_1$-ordered field only if $\kappa^\omega = \kappa$.

## INTRODUCTION

Pierce, Comfort, Hager, and Koppelberg have shown that $|B|^\omega = |B|$ for classes of Boolean algebras satisfying progressively weaker completeness conditions. We will investigate these issues in the setting of ordered fields and obtain the Comfort-Hager type condition for the maximally valued ordered fields of Kaplansky. Let us fix terminology first.

Let $\mathscr{G} = (G, +, 0, -)$ be a totally ordered, divisible Abelian group. The absolute value of an element $a \in G$ is defined as $|a| = \max\{a, -a\}$. An $o$-subgroup $\mathscr{H}$ of $\mathscr{G}$ is convex if for every $b \in H$ and $a \in G$, $|a| \le |b| \Rightarrow a \in H$. The principal convex subgroup generated by $a \in G$ is $V(a) = \{b \in G|\ |b| \le n|a|$ for some $n \in \omega\}$. $\mathscr{G}$ is Archimedean if all the principal convex subgroups generated by nonzero elements coincide. The maximal convex subgroup not containing $a$ is $V^-(a) = \{b \in G|\ n|b| < |a|$ for every $n \in \omega\}$. $V^-(a)$ is a subgroup of $V(a)$ and $V(a)/V^-(a)$ is an Archimedean group. If $\{\mathscr{G}_\pi | \pi \in \Pi\}$ is a family of ordered groups indexed by a totally ordered set $\Pi$, then $\Gamma\{\mathscr{G}_\pi | \pi \in \Pi\}$ is the ordered subgroup of their antilexicographically ordered direct product consisting of those elements whose sets of nonzero coordinates are antiwell ordered in $\Pi$. For an element $\pi \in \Pi$ the operation $C_\pi$ assigning to each element $a \in \Gamma\mathscr{G}_\pi$ the element $b$ such that $b(\sigma) = a(\sigma)$ for $\sigma > \pi$ and $0$ for $\sigma \le \pi$ is the $\pi$th head operation. The set $\Pi = \{V(a) | a \in G\}$ totally ordered by inclusion will be called the skeleton of $\mathscr{G}$. For each $\pi \in \Pi$, let $D_\pi$ be a fixed subgroup of the additive group of reals isomorphic to $\pi/\pi^-$, and let $W(\mathscr{G}) = \Gamma\{D_\pi | \pi \in \Pi\}$. By the Hahn embedding theorem there exists an $o$-embedding of $\mathscr{G}$ into $W(\mathscr{G})$.

Let $\mathscr{F} = (F, +, \cdot, 0, 1, -, ^{-1}, \le)$ be a totally ordered field. We will use the Hahn embedding theorem for its additive group. We also define the addition in the skeleton $\Pi$ of its additive group as $V(a) + V(b) = V(ab)$. The ordered

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group $(\Pi, +, V(1), \leq)$ is called the value group of the field $\mathcal{F}$. For every $a \in F$, $V(a)/V(a)^- \cong V(1)/V(1)^-$ is called the residue field of $\mathcal{F}$. For $a \in F$ we denote by $\underline{a}$ its image under Hahn's embedding.

## 1. $\eta_1$-ORDERED FIELDS

Let $\alpha$ be an ordinal. An ordered field $\mathcal{F}$ is $\eta_\alpha$-ordered if $(F, \leq)$ is an $\eta_\alpha$-set, i.e., for every two subsets $H, K \subset F$ such that $|H| + |K| < \aleph_\alpha$ and $H < K$, there exists $a \in F$ such that $H < a < K$. We will now recall the definition of $\alpha$-maximal fields introduced by Kaplansky.

**Definition.** Let $\mathcal{F}$ be an ordered field with the valuation $V$ and let $\mathcal{G}$ be its value group. Also let $\alpha$ be an ordinal. For a sequence $\{a_\beta | \beta < \alpha\}$ we will say that it is pseudoconvergent if for every $\delta < \gamma < \beta < \alpha$ we have $V(a_\beta - a_\gamma) < V(a_\gamma - a_\delta)$. We will also say for an element $a \in F$ that it is a pseudolimit of that pseudoconvergent sequence if $V(a - a_\alpha) = V(a_{\beta+1} - a_\beta)$.

**Definition.** Let $\aleph_\alpha$ be the $\alpha$th infinite cardinal. We will say that $\mathcal{F}$ is $\alpha$-maximal if every pseudoconvergent sequence of length less than $\aleph_\alpha$ has a pseudolimit. $\mathcal{F}$ is maximal if it is $\alpha$-maximal for $|F| = \aleph_\alpha$.

Since we are interested in $\eta_1$-fields, the following theorem due to Alling is of great interest to us. That, and some unclear details in the proof given in (2), were the reasons to include the proof of that theorem here.

**Theorem 1** (Alling). *Let $\mathcal{F}$ be an ordered field with the valuation $V$ and the value group $\mathcal{G}$. The following three conditions are necessary and sufficient for $\mathcal{F}$ to be $\eta_\alpha$-ordered:*

(i) *Its residue field is isomorphic to $\mathcal{R}$.*
(ii) *Its value group $\mathcal{G}$ is an $\eta_\alpha$-ordered group.*
(iii) *It is $\alpha$-maximal.*

*Proof.* (i) The residue field is Archimedean and could be embedded into $\mathcal{R}$. Let $g = V(1)$. Let for a field $\mathcal{H}$, $Q(\mathcal{H})$ denote its set of rationals. Finally, let $(A, B)$ be a cut in $Q(g/g^-)$. Then there exist $C, D \subset Q(\mathcal{F})$ such that $\forall x \in A \exists q \in C (x = q + g^-)$ & $\forall x \in B \exists q \in D (x = q + g^-)$. Obviously $C < D$. Since $\mathcal{F}$ is an $\eta_1$-set there exists $a \in F$ such that $C < a < D$, so $C + g^- \leq a + g^- \leq D + g^-$, i.e., $A \leq a + g^- \leq B$. Since $g/g^-$ is Dedekind complete, it is isomorphic to $\mathcal{R}$.

(ii) Let $A, B \subset G$ so that $A < B$ and $|A| + |B| < \aleph_\alpha$. For $g \in G$ let $e_g$ denote a fixed representative of $g/g^-$, and let $A_1 = \{ne_g | g \in A, n \in \omega\}$ and $B_1 = \{e_g/n | n \in \omega, g \in B\}$. Now we have in $\mathcal{F}$ that $A_1 < B_1$ and $|A_1| + |B_1| < \aleph_\alpha$. Since $\mathcal{F}$ is $\eta_\alpha$-ordered there exists an element $a \in F$ such that $A_1 < a < B_1$. In this case we also have $V(A_1) < V(a) < V(B_1)$, i.e., $A < V(a) < B$.

(iii) Let $\beta < \aleph_\alpha$ be an ordinal and $\{a_\gamma | \gamma < \beta\}$ a pseudoconvergent sequence of distinct elements. Let for $\gamma < \beta$, $d_\gamma = a_{\gamma+1} - a_\gamma$, $b_\gamma = a_\gamma + 2|d_\gamma|$, $c_\gamma = a_\gamma - 2|d_\gamma|$. We have $c_\gamma < a_\gamma < b_\gamma$ and $V(b_\gamma - c_\gamma) = V(d_\gamma)$. On the other hand, for $\gamma < \delta < \beta$ we have $b_\gamma - b_\delta = a_\gamma - a_\delta + 2|d_\gamma| - 2|d_\delta|$, and since $|a_\delta - a_\gamma| \leq |a_{\gamma+1} - a_\gamma| = d_\gamma$, we have $b_\gamma - b_\delta \geq -|d_\gamma| + 2|d_\gamma| - 2|d_\delta| = |d_\gamma| - 2|d_\delta| > 0$. So we conclude that the sequence $B = \{b_\gamma | \gamma < \beta\}$ is strictly decreasing, and in a similar way we can obtain that $C = \{c_\gamma | \gamma < \beta\}$ is strictly increasing. Together with the fact that for each $\gamma < \beta$ we have $c_\gamma < b_\gamma$, this gives $C < B$. Since $\mathscr{F}$ is $\eta_\alpha$-ordered, there exists an element $a \in F$ such that $C < a < B$. It is a pseudolimit of the initial pseudoconvergent sequence. Indeed for every $\gamma < \beta$ we have $V(a - a_\gamma) \leq V(b_\gamma - c_\gamma) = d_\gamma = V(a_{\gamma+1} - a_\gamma)$.

**Theorem 2** (GCH). *Let $\kappa$ be a cardinal. There exists an $\eta_1$-ordered field of cardinality $\kappa$ iff $\kappa^\omega = \kappa$.*

*Proof.* One direction involves the usual ultraproduct construction. So suppose that $\kappa^\omega = \kappa$, and let $\mathscr{F}$ be an ordered field of cardinality $\kappa$. Such a field exists by the Löwenheim-Skolem-Tarski theorem. Also let $\mathscr{J}$ be an $\omega_1$-incomplete ultrafilter over $\omega$. The ultraproduct $\mathscr{F}^\omega/\mathscr{J}$ is an $\omega_1$-saturated ordered field, hence it is $\eta_1$-ordered. Its cardinality is between $\kappa$ and $\kappa^\omega$, and since we assumed that they are equal, it is just $\kappa$.

Let us prove the other direction. Let $\mathscr{F}$ be an $\eta_1$-ordered field of cardinality $\kappa$, $\mathscr{G}$ its value group, and $V$ its valuation. We will prove that $|G|^\omega < |F|$. We will first define a 1-1 mapping $\theta\colon D \to F$, from the set of decreasing sequences in $G$, into $F$. Let $g = \{g_n | n \in \omega\}$ be a decreasing sequence in $\mathscr{G}$, and for $a \in G$, let $e_a$ be the element of $F$ such that $e_a$ has the $a$th coordinate 1 and all others 0. Also for $n \in \omega$ let $a_n(g) = \sum\{e_{g_k} | k < n\}$. $\{a_n(g) | n \in \omega\}$ is obviously a pseudoconvergent sequence. Since $\mathscr{F}$ is 1-maximal, $\{a_n(g) | n \in \omega\}$ has a pseudolimit $a(g)$. Let $\theta(g) = a(g)$. We want to show that $\theta$ is a 1-1 mapping. Suppose that $h = \{h_n | n \in \omega\}$ is another sequence in $D$ and that $m$ is the minimal index where $g$ and $h$ differ from each other, and without loss of generality suppose that $g_m < h_m$. In that case it is easy to see that $V(a(g) - a_m(h)) = h_m$ and $V(a(h) - a_m(h)) = h_{m+1}$. Since $h_{m+1} < h_m$, we have that $a(g) - a_m(h) \neq a(h) - a_m(h)$ and also $a(g) \neq a(h)$. Hence $\theta$ is a 1-1 mapping.

On the other hand, the mapping $\phi\colon G^\omega \to D$, defined by $\Phi(\{g_n | n \in \omega\}) = \{f_n | n \in \omega\}$ where $f_n = \sum_{k<n} |g_k|$, is "at most $2^\omega$ to 1," since $g_n = (-1)^{\chi(P)} f_n$ where $P = \{n \in \omega | g_n > 0\}$. So we have $|G|^\omega \leq |D| \cdot 2^\omega = |D| \leq |F|$. If $|F| = |G|^\omega$ we are done. If $|G|^\omega < |F|$ then by the Hahn representation theorem we have

$$|F| \leq |\Gamma\{\mathscr{R} | g \in G\}| \leq \prod\{2^\omega | g \in G\} = (2^\omega)^{|G|} = |G|^+ .$$

Since $|G|^\omega < F$, we have $|F| = |G|^+$. Hence $|F|^\omega = |F|$.

Actually a more general theorem is true. It was suggested to me by Prikry.

**Theorem 3** (GCH). *Let $(L, \leq)$ be an $\eta_1$-ordered set such that each two intervals or half-intervals are o-isomorphic or o-anti-isomorphic. $|L|^\omega = |L|$.*

*Proof.* Let $L = \{a_\alpha | \alpha < \kappa\}$. Since in the presence of GCH $\kappa^\omega = \kappa$ iff $\mathrm{cf}(\kappa) > \omega$, we have to prove that $\mathrm{cf}(\kappa) > \omega$. So suppose, to the contrary, that $\mathrm{cf}(\kappa) = \omega$. Let $\kappa = \sum \kappa_n$ where $\{\kappa_n | n \in \omega\}$ is a strictly increasing sequence of cardinals. We will define a sequence of colorings $\{\theta_n | n \in \omega\}$ on $\kappa$ in the following way: for $\alpha < \beta < \kappa$, $\theta_n(\{\alpha, \beta\}) = 1$ if $a_\alpha < a_\beta$ and 0 otherwise. By the Erdös-Rado partition theorem (GCH version) we have $(\kappa_n)^+ \to (\kappa_n)^2_2$, and by the monotonicity property, we have $\kappa \to (\kappa_n)^2_2$. Hence we can suppose without loss of generality that for each $n \in \omega$, there exists $A_n \subset \kappa$ such that $|A_n| = \kappa_n$ and $\theta_0[[A_n]^2] = 1$, i.e., the sequence $\{a_\alpha | \alpha \in A_n\}$ is increasing. Since $\{(a_\alpha, a_{\alpha+1}) | \alpha \in A_n\}$ is a family of disjoint intervals, we can conclude that for each $n \in \omega$ we can find a family of $\kappa_n$ disjoint intervals of $L$. Since $(L, \leq)$ is homogeneous, we conclude that each interval of $(L, \leq)$ has that property. Let $K = \prod\{\kappa_n | n \in \omega\}$, the direct product of the sets $\kappa_n$, and for each $n \in \omega$ let $K_n = \{a \restriction n | a \in K\}$. Let $\mathscr{J}$ be the family of all intervals of $(L, \leq)$. We will define a mapping $\phi\colon \mathbb{U}\{K_n | n \in \omega\} \to \mathscr{J}$ so that

(i) $m < n \to \phi(a \restriction n) \subset \phi(a \restriction m)$,
(ii) $a \restriction n \neq b \restriction n \Rightarrow \phi(a \restriction n) \cap \phi(b \restriction n) = \varnothing$.

First let us define $\phi$ on $K_0$. The only member of $K_0$ is the empty word $(\,)$, and we set $\phi((\,)) = L$. Let $n \in \omega$, and suppose that $\phi$ was already defined on $\mathbb{U}\{K_m | m < n\}$ in the prescribed manner. For each $a \in K$, let $\mathscr{J}_{a \restriction n-1}$ be a family of disjoint intervals in $\phi(a \restriction n-1)$ indexed as a $\kappa_n$-sequence. We will define $\phi(a \restriction n)$ as the $a(n-1)$th element of that sequence. Obviously $\phi$ has the prescribed properties.

Since $(L, \leq)$ is an $\eta_1$-ordering, each decreasing countable sequence of intervals has nonempty intersection; namely, it contains the point that separates the set of left end points from the set of right end points. The mapping $\psi\colon K \to L$, defined by $\psi(a) = \bigcap\{\psi(a \restriction n) | n \in \omega\}$, is 1-1. Since $\kappa^\omega = \prod\{\kappa_n | n \in \omega\}$, we conclude $\kappa^\omega = \prod\{\kappa_n | n \in \omega\} = |K| \leq |L| = \kappa$.

This theorem is a generalization of the previous one, since each field has the above-mentioned homogeneity property.

## 2. Maximally valued ordered fields

**Theorem 1.** *Let $\mathscr{G}$ be an $\eta_1$-ordered Abelian group and $W$ the set of all well-ordered sequences in $\mathscr{G}$. Then $|W|^\omega = |W|$.*

*Proof.* Let $\nu$ be the cofinality of $\mathscr{G}$ and $\{g_\alpha | \alpha < \nu\}$ a strictly increasing sequence of positive elements that are not Archimedean equivalent, cofinal in $\mathscr{G}$. Let $G_\alpha = [g_\alpha, g_{\alpha+1})$, $W_\alpha$ the set of well-ordered sequences of $G_\alpha$, and $\nu_\alpha = |W_\alpha|$. We will prove first that $\nu_{\alpha+1} \geq \nu_\alpha^\omega$. Let $\{\theta_n | n \in \omega\}$ be a sequence of well-ordered sequences in $G_\alpha$. Let $\widetilde{\theta}_n$ be the image of $\theta_n$ under the translation isomorphism of $[0, g_\alpha]$ onto $[(n-1)g_\alpha, ng_\alpha)$. $\mathbb{U}\{\widetilde{\theta}_n | n \in \omega\}$ is a well-ordering since $\omega$, the index set, and each $\widetilde{\theta}_n$ are well ordered. Obviously the mapping $\{\theta_n | n \in \omega\} \to \mathbb{U}\{\widetilde{\theta}_n | n \in \omega\}$ is a 1-1 mapping. Since $g_\alpha$ and $g_{\alpha+1}$ are not Archimedean equivalent, it is a mapping into $W_{\alpha+1}$.

Therefore $ng_\alpha < g_{\alpha+1}$, and $\mathbb{U}[g_\alpha, g_{\alpha+1}) \subset [0, g_{\alpha+1})$. The existence of this 1-1 mapping proves the claim. $\nu_{\alpha+1} \geq \nu_\alpha^\omega$.

Now for a well-ordered sequence $\theta$ in $\mathscr{G}^+$ let $\theta_\alpha = \theta \cap [g_\alpha, g_{\alpha+1})$. If we denote by $W^+$ the set of all well-ordered sequences in $\mathscr{G}^+$, the mapping $\theta \to \{\theta_\alpha | \alpha < \nu\}$ is a 1-1 mapping of $W^+$ onto $\prod\{W_\alpha | \alpha < \nu\}$.

$$\begin{aligned} |W^+|^\omega &= \left|\prod\{W_\alpha | \alpha < \nu\}\right|^\omega = \prod\{|W_\alpha|^\omega | \alpha < \nu\} \\ &\leq \prod\{|W_{\alpha+1}| \,|\alpha < \nu\} = \prod\{|W_\alpha| |\alpha < \nu\} = |W^+|. \end{aligned}$$

Now for $g \in G$, let $W_g$ be the set of all well-ordered sequences in the half-interval $(g, \to)$. Since each half-interval $(g, \to)$ is $o$-isomorphic to $(0, \to)$, we have $|W_g| = |W^+|$. Also since each well-ordered sequence has a least element, $W = \mathbb{U}\{W_g | g \in G\}$. Finally, we have

$$\begin{aligned}|W^+| \leq |W| &< |\mathbb{U}\{W_g | g \in G\}| \\ &\leq \sum\{|W_g|\,|g \in G\} = |W^+| \cdot |G| \leq |W^+| \cdot |W^+| = |W^+|,\end{aligned}$$

and we conclude that $|W| = |W^+|$, so $|W|^\omega = |W|$.

Obviously the same theorem is true for $W$, the set of all antiwell-ordered sequences in $\mathscr{G}$.

For an element $a \in F$, $\tilde{a}$ will denote the antiwell-ordered sequence of nonzero coordinates of $\underline{a}$.

**Lemma.** *Let $\mathscr{F}$ be a maximal $\eta_1$-ordered field, $\mathscr{G}$ its value group, $W$ the set of all antiwell-ordered sequences in $G$, and $W_0 = \{\tilde{a} | a \in F\}$. Then*

(i) $W_0 = W$;
(ii) *for every $\theta \in W$ and every sequence $f\colon G \to R$ such that $\{g \in G | f(g) = 0\} = G \backslash \theta$, there exists $a \in F$ such that $\underline{a} = f$, i.e., $\mathscr{F} = \Gamma\{R | g \in G\}$;*
(iii) $|F| = |W|$.

*Proof.* (i) Let $\theta = \{g_\alpha | \alpha < \nu\} \in W$. We will define the sequence $\{a_\alpha | \alpha < \nu\}$ in $F$ so that $\tilde{a}_\alpha = \{g_\beta | \beta < \alpha\}$ and $\underline{a_\alpha}(g_\beta) = 1$, $\beta < \alpha$. We will construct it recursively. From the proof of Hahn's embedding theorem it is easy to see that we can choose $a_0 = 0$, and $a_\alpha = a_\beta + e g_\alpha$ for $\alpha = \beta + 1$. It has the prescribed properties. Now suppose that $\alpha$ is a limit ordinal and that the sequence has been constructed for $\beta < \alpha$ in the prescribed manner. The sequence $\{a_\beta | \beta < \alpha\}$ is pseudoconvergent since for each $\delta < \gamma < \beta$ we have $V(a_\beta - a_\gamma) = g_\gamma < g_\delta = V(a_\gamma - a_\delta)$. Since $\mathscr{F}$ is maximal, it has a pseudolimit. Let $a$ be one of them, and let $f$ be the $g_\alpha$th head of $\underline{a}$. By the Hahn embedding theorem there exists an element $b \in F$ such that $\underline{b} = f$. It has the prescribed properties and, since $\tilde{a}_\nu = \theta$, $\theta \in W_0$. This proves $W = W_0$.

(ii) The proof of this part is similar to the proof of part (i). It suffices to set $a_\alpha = a_\beta + f(g_\alpha) \cdot e g_\alpha$, for $\alpha = \beta + 1$.

(iii) For $\theta \in W$ let $F_\theta = \{a \in F | \tilde{a} \subset \theta\}$. In this case we have $F = \mathbb{U}\{F_\theta | \theta \in W\}$. $|F_\theta| \leq |\Gamma\{D_g | g \in \theta\}| \leq \prod\{|D_g|\,|g \in \theta\} \leq \prod\{|R|\,|g \in \theta\} = c^{|\theta|} = 2^{|\theta|}$, since $|\theta| \geq \omega$, so we have

$$|F| = |\mathbb{U}\{F_\theta | \theta \in W\}| \leq \sum\{|F_\theta|\,|\theta \in W\} \leq \sum\{2^{|\theta|} | \theta \in W\}.$$

Since $P(\theta) \subset W$, $2^{|\theta|} \leq |W|$. Finally we get

$$|F| \leq \sum\{|W|\,|\theta \in W\} = |W| \cdot |W| = |W|.$$

On the other hand, since the mapping $a \to \tilde{a}$ is a mapping of $\mathscr{F}$ onto $W_0$ and, by (ii), $W = W_0$, we get $|W| \leq |F|$. This proves $|F| = |W|$.

Combining the two previous results and using Alling's theorem, we get the following results.

**Theorem 2.** *Let $\mathscr{F}$ be an $\eta_1$-ordered maximal field. Then $|F|^\omega = |F|$.*

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