

EXPLICIT FORMULAS FOR THE SZEGŐ KERNEL ON CERTAIN WEAKLY PSEUDOCONVEX DOMAINS

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ABSTRACT. The objective of this paper is to determine the Szegő kernel of the domain $\mathcal{D} = \{(z, \zeta, w) \in \mathbb{C}^{n+m+1}; \Im w > \|z\|^2 + \|\zeta\|^{2p}\}$ explicitly in closed form.

The purpose of this paper is to determine the Szegő kernel explicitly for a new class of weakly pseudoconvex domains. This class of domains is given by

$$\mathcal{D} = \{(z, \zeta, w) \in \mathbb{C}^{n+m+1}; \Im w > \|z\|^2 + \|\zeta\|^{2p}\}$$

in the complex space \mathbb{C}^{n+m+1} . Here $z \in \mathbb{C}^n$, $\zeta \in \mathbb{C}^m$, $w \in \mathbb{C}$, n and m are integers, $n \geq 0$, $m \geq 1$, and p is any positive number. We use the notation $\|z\|^2 = z \cdot \bar{z} = \sum_{j=1}^n |z_j|^2$ and $\|\zeta\|^{2p} = (\zeta \cdot \bar{\zeta})^p = (\sum_{l=1}^m |\zeta_l|^2)^p$. The unbounded domain \mathcal{D} is weakly pseudoconvex with degenerate Levi form whenever $p > 1$. Furthermore the domain \mathcal{D} is not “decoupled” when $m > 1$. The definition of decoupled domains can be found, for example, in [Mc]. Decoupled domains were considered recently, for example, by [DT], [Mc].

There are relatively few classes of domains with smooth boundary where the Szegő kernel is explicitly known in closed form. We consider here a class of weakly pseudoconvex domains with smooth boundary. In particular, we do not assume that the defining function is decoupled. The authors believe that this is one of the few examples of such domains where the Szegő kernel is computed explicitly. See also the work of Christ [C].

Recent interest in explicit formulas for the Szegő and Bergman kernels is motivated by the surprising effectiveness of these formulas. We illustrate this point by two cases. The striking discovery of Christ and Geller [CG] that the Szegő kernel of certain weakly pseudoconvex domains is not analytic off the diagonal was derived from Nagel’s [N] explicit formula. In [M2], an explicit formula allowed Machedon to test if the Szegő kernel is a singular integral with respect to a certain nonisotropic metric.

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We recall that a holomorphic function f on the domain \mathcal{D} belongs to the Hardy space $\mathcal{H}^2(\mathcal{D})$, by definition, if and only if

$$\sup_{\rho > 0} \int_{\mathbb{R}^{2n+2m+1}} |f(z, \zeta, t + i\|z\|^2 + i\|\zeta\|^{2p} + i\rho)|^2 dV(z, \zeta) dt < \infty.$$

The subspace $\mathcal{H}^2(\partial\mathcal{D})$ of $L^2(\partial\mathcal{D})$, consisting of boundary values of holomorphic functions $f \in \mathcal{H}^2(\mathcal{D})$, is a closed subspace of $L^2(\partial\mathcal{D})$. The Szegő projection is the orthogonal projection $S : L^2(\partial\mathcal{D}) \rightarrow \mathcal{H}^2(\partial\mathcal{D})$ and the Szegő kernel $S(z, \zeta, t; z', \zeta', t')$ is the distribution kernel on $\partial\mathcal{D} \times \partial\mathcal{D}$ given by

$$Sf(z, \zeta, t) = \int_{\partial\mathcal{D}} S(z, \zeta, t; z', \zeta', t') f(z', \zeta', t') dV(z', \zeta', t').$$

We identify the boundary of \mathcal{D} , $\partial\mathcal{D}$ with $\mathbb{C}^n \times \mathbb{C}^m \times \mathbb{R}$, where the coordinates are (z, ζ, t) .

Theorem 1. *The Szegő kernel of the domain $\mathcal{D} = \{(z, \zeta, w) \in \mathbb{C}^{n+m+1}; \Im m w > \|z\|^2 + \|\zeta\|^{2p}\}$ is*

$$S(z, \zeta, t; z', \zeta', t') = \sum_{k=1}^{n+1} c_k \frac{(A - z \cdot \bar{z}')^{\frac{k}{p} - n - 1}}{[(A - z \cdot \bar{z}')^{\frac{1}{p}} - \zeta \cdot \bar{\zeta}']^{m+k}}.$$

The function A is defined as

$$(1) \quad A = \frac{1}{2} [\|z\|^2 + \|z'\|^2 + \|\zeta\|^{2p} + \|\zeta'\|^{2p} - i(t - t')].$$

The constants $c_k = c_k(n, m, p)$ can be computed explicitly by an elementary recursive calculation. Moreover, c_{n+1} is never zero, and the $c_k, k = 1, \dots, n$, vanish when $p = 1$.

Corollary 1. *The Bergman kernel of the domain \mathcal{D} is*

$$B(z, \zeta, w; z', \zeta', w') = \sum_{k=1}^{n+2} d_k \frac{(A - z \cdot \bar{z}')^{\frac{k}{p} - n - 2}}{[(A - z \cdot \bar{z}')^{\frac{1}{p}} - \zeta \cdot \bar{\zeta}']^{m+k}}.$$

Here, the function A is given by $A = \frac{1}{2}(\bar{w}' - w)$. The constants d_k can be computed explicitly from the constants c_1, \dots, c_{n+1} .

We mention that Corollary 1 can also be obtained from the work of D'Angelo on bounded domains [D'A2].

Remark 1. Notice that the extreme case, $n = 0$, is included in our method. The Szegő kernel of the domain $\mathcal{D} = \{(\zeta, w) \in \mathbb{C}^{m+1}; \Im m w > \|\zeta\|^{2p}\}$ is

$$S(\zeta, t; \zeta', t') = \frac{m!}{4\pi^{m+1}} \frac{A^{\frac{1}{p} - 1}}{(A^{\frac{1}{p}} - \zeta \cdot \bar{\zeta}')^{m+1}}.$$

A further special case $n = 0, m = 1$ coincides with the well-known example of Greiner and Stein [GS]. Moreover, in the very special case $n = 1, m = 1$, i.e. the domain $\{(z, \zeta, w) \in \mathbb{C}^3; \Im m w > |z|^2 + |\zeta|^2\}$ gives exactly the same formula that appears in [M1, p. 30]. The method used in [M1] is different from

ours. Another special case is the Siegel upper half plane, that is $p = 1$. Since the coefficients $c_k, k = 1, \dots, n$, vanish when $p = 1$, our result is consistent with the classical formula

$$S(z, w; z', w') = \frac{j^{n+1} 2^{n-1} n!}{\pi^{n+1} (w - \bar{w}' - 2iz \cdot \bar{z}')^{n+1}}.$$

The origin of our method goes back to the papers [D'A1], [GS]. This method can be applied to more general domains than \mathcal{D} . Let us describe the results without stating them in full generality. Let m_1, \dots, m_l be positive integers, $\underline{m} = (m_1, \dots, m_l)$ and let p_1, \dots, p_l be positive numbers. We use $\alpha_{(j)} = (\alpha_{j,1}, \dots, \alpha_{j,m_j}), j = 1, \dots, l$, for multi-indices. Consider the domain

$$\mathcal{D}_\phi = \{(z, w) \in \mathbb{C}^{|\underline{m}|+1}; \Im w > \phi(z)\},$$

where

$$\phi(z) = \phi(z_{(1)}, \dots, z_{(l)}) = \|z_{(1)}\|^{2p_1} + \dots + \|z_{(l)}\|^{2p_l}$$

with $z_{(j)} = (z_{j,1}, \dots, z_{j,m_j}) \in \mathbb{C}^{m_j}, j = 1, \dots, l$.

Theorem 2. *The Szegő kernel of the domain \mathcal{D}_ϕ has the series representation*

$$(2) \quad S(z, t; z', t') = \frac{p_1 \cdots p_l}{4\pi^{|\underline{m}|+1}} A^{-1 - \sum_{j=1}^l \frac{m_j}{p_j}} \sum_{\alpha_{(1)}, \dots, \alpha_{(l)}} \frac{\Gamma(1 + \sum_{j=1}^l \frac{|\alpha_{(j)}| + m_j}{p_j})}{\prod_{j=1}^l \Gamma(\frac{|\alpha_{(j)}| + m_j}{p_j}) \alpha_{(j)}!} \cdot \prod_{j=1}^l \Gamma(|\alpha_{(j)}| + m_j) \prod_{j=1}^l \left(A^{-1/p_j} z_{(j)} \bar{z}'_{(j)} \right)^{\alpha_{(j)}}.$$

The function A is given by $A = \frac{1}{2}[\phi(z) + \phi(z') - i(t - t')]$. Also note that by definition $z_{(j)} \bar{z}'_{(j)} = (z_{j,1} \bar{z}'_{j,1}, \dots, z_{j,m_j} \bar{z}'_{j,m_j})$.

We will prove only Theorem 1, since similar techniques provide the proof of Theorem 2. The relation of the Bergman and Szegő kernels associated to the domain \mathcal{D}_ϕ and the hypergeometric functions in several variables will be explored in [FH].

Proof of Theorem 1. We start by recalling a few identities from [D'A2]. Let S_+^{m-1} be the subset of S^{m-1} containing all the points with positive coordinates. Then for any multi-index $\beta = (\beta_1, \dots, \beta_m)$

$$(3) \quad \int_{S_+^{m-1}} \omega^{2\beta+1} d\sigma(\omega) = \frac{\beta!}{2^{m-1} \Gamma(|\beta| + m)}.$$

For any positive number s we have

$$(4) \quad \sum_{\alpha} \frac{\Gamma(|\alpha| + s)}{\alpha!} x^\alpha = \frac{\Gamma(s)}{(1 - \sum_{j=1}^n x_j)^s}$$

for all $x = (x_1, \dots, x_n) \in \mathbb{C}^n$ such that $|\sum_{j=1}^n x_j| < 1$.

For any polynomial of one variable, q_d , of degree at most d and for any positive number s there are constants $c_k = c_k(m, s, d)$ satisfying the identity

$$(5) \quad \sum_{\beta} \frac{\Gamma(|\beta| + s)}{\beta!} q_d(|\beta|) y^{\beta} = \sum_{k=0}^d c_k \frac{1}{(1 - \sum_{l=1}^m y_l)^{s+k}}$$

for all $y = (y_1, \dots, y_m) \in \mathbb{C}^m$ such that $|\sum_{l=1}^m y_l| < 1$. One can compute the constants c_k by an elementary recursive calculation. \square

We introduce a Bargmann-type Hilbert space of entire functions, $H_{\lambda}(\mathbb{C}^{n+m})$. This space consists of entire holomorphic functions in the variables $(z, \zeta) \in \mathbb{C}^{n+m}$ satisfying the growth condition

$$\|g\|_{H_{\lambda}(\mathbb{C}^{n+m})}^2 = \int_{\mathbb{R}^{2n+2m}} e^{-4\pi\lambda(\|z\|^2 + \|\zeta\|^{2p})} |g(z, \zeta)|^2 dV(z, \zeta) < \infty.$$

Proposition 1. *The functions*

$$(6) \quad \frac{z^{\alpha} \zeta^{\beta}}{\|z^{\alpha} \zeta^{\beta}\|_{H_{\lambda}(\mathbb{C}^{n+m})}}$$

form a complete orthonormal system in the space $H_{\lambda}(\mathbb{C}^{n+m})$. Moreover the norm of $z^{\alpha} \zeta^{\beta}$ can be evaluated explicitly for any multi-indices α, β as

$$(7) \quad \|z^{\alpha} \zeta^{\beta}\|_{H_{\lambda}(\mathbb{C}^{n+m})}^2 = \frac{\pi^{n+m} \Gamma(\frac{|\beta|+m}{p}) \alpha! \beta!}{p(4\pi\lambda)^{\frac{|\beta|+m}{p} + |\alpha|+n} \Gamma(|\beta| + m)}.$$

Proof. We begin by evaluating the norm of $z^{\alpha} \zeta^{\beta}$ by using polar coordinates in each variable $z_j = r_j e^{i\theta_j}$, $\zeta_l = \rho_l e^{i\phi_l}$

$$(8) \quad \begin{aligned} \|z^{\alpha} \zeta^{\beta}\|_{H_{\lambda}}^2 &= (2\pi)^{n+m} \int_{\mathbb{R}_+^{n+m}} r^{2\alpha+1} \rho^{2\beta+1} e^{-4\pi\lambda(\|r\|^2 + \|\rho\|^{2p})} dr d\rho \\ &= (2\pi)^{n+m} \left(\int_{\mathbb{R}_+^n} r^{2\alpha+1} e^{-4\pi\lambda\|r\|^2} dr \right) \left(\int_{\mathbb{R}_+^m} \rho^{2\beta+1} e^{-4\pi\lambda\|\rho\|^{2p}} d\rho \right). \end{aligned}$$

In the first integral we use the substitution $t = 4\pi\lambda r_j^2$ in the definition of the gamma function $\Gamma(x) = \int_0^{\infty} t^{x-1} e^{-t} dt$ to obtain

$$2 \int_0^{\infty} r_j^{2\alpha_j+1} e^{-4\pi\lambda r_j^2} dr_j = \frac{\Gamma(\alpha_j + 1)}{(4\pi\lambda)^{\alpha_j+1}}$$

and

$$2^n \int_{\mathbb{R}_+^n} r^{2\alpha+1} e^{-4\pi\lambda\|r\|^2} dr = \frac{\alpha!}{(4\pi\lambda)^{|\alpha|+n}}.$$

In the second integral of (8) we use spherical coordinates $\rho = (\rho_1, \dots, \rho_m) = \|\rho\| \omega$ with $\omega \in S_+^{m-1}$. Then

$$(9) \quad \begin{aligned} &2^m \int_0^{\infty} \int_{S_+^{m-1}} \|\rho\|^{2|\beta|+m} \|\rho\|^{m-1} \omega^{2\beta+1} e^{-4\pi\lambda\|\rho\|^{2p}} d\|\rho\| d\sigma(\omega) \\ &= 2^m \left(\int_0^{\infty} \|\rho\|^{2|\beta|+2m-1} e^{-4\pi\lambda\|\rho\|^{2p}} d\|\rho\| \right) \left(\int_{S_+^{m-1}} \omega^{2\beta+1} d\sigma(\omega) \right). \end{aligned}$$

Here substituting $t = 4\pi\lambda\|\rho\|^{2p}$ into the integral defining the gamma function gives

$$\frac{\Gamma(\frac{|\beta|+m}{p})}{2p(4\pi\lambda)^{\frac{|\beta|+m}{p}}}$$

for the first factor of (9). Combining this with (3) we get

$$2^m \int_{\mathbb{R}_+^n} \rho^{2\beta+1} e^{-4\pi\lambda\|\rho\|^{2p}} d\rho = \frac{\Gamma(\frac{|\beta|+m}{p})\beta!}{p(4\pi\lambda)^{\frac{|\beta|+m}{p}}\Gamma(|\beta|+m)},$$

which proves (7).

The next step in the proof is a version of the very important representation theorem for Hardy spaces.

Lemma 1. *Every function in the Hardy space $\mathcal{H}^2(\mathcal{D})$ has the representation*

$$(10) \quad f(z, \zeta, w) = \int_0^\infty e^{2\pi i\lambda w} \hat{f}(z, \zeta, \lambda) d\lambda.$$

The function $\hat{f}(z, \zeta, \lambda)$ belongs to the space $H_\lambda(\mathbb{C}^{n+m})$ and satisfies the condition

$$\int_0^\infty \int_{\mathbb{R}^{2n+2m}} e^{-4\pi\lambda(\|z\|^2 + \|\zeta\|^{2p})} |\hat{f}(z, \zeta, \lambda)|^2 dV(z, \zeta) d\lambda < \infty.$$

Moreover \hat{f} is determined by

$$(11) \quad \hat{f}(z, \zeta, \lambda) = \int_{-\infty}^\infty e^{-2\pi i\lambda w} f(z, \zeta, w) d(\Re w),$$

where $\Im w = \|z\|^2 + \|\zeta\|^{2p}$.

The proof of this lemma is standard. It goes back to Bochner’s representation result on tube domains. The reduction to the tube case is due to Stein [S]. See also Stein and Weiss [SW].

The orthonormal system (6) immediately provides a reproducing kernel

$$(12) \quad K_\lambda(z, \zeta; z', \zeta') = \sum_{\alpha, \beta} \frac{z^\alpha \zeta^\beta \bar{z}'^\alpha \bar{\zeta}'^\beta}{\|z^\alpha \zeta^\beta\|_{H_\lambda}^2}$$

in $H_\lambda(\mathbb{C}^{n+m})$, that is, for all $g \in H_\lambda(\mathbb{C}^{n+m})$

$$(13) \quad g(z, \zeta) = \int_{\mathbb{R}^{2n+2m}} K_\lambda(z, \zeta; z', \zeta') g(z', \zeta') e^{-4\pi\lambda(\|z'\|^2 + \|\zeta'\|^{2p})} dV(z', \zeta').$$

We express the Szegő kernel in terms of $K_\lambda(z, \zeta; z', \zeta')$. Indeed, applying (13) to the function \hat{f} and using (10) we have

$$f(z, \zeta, w) = \int_0^\infty e^{2\pi i\lambda w} \int_{\mathbb{R}^{2n+2m}} K_\lambda(z, \zeta; z', \zeta') \cdot \hat{f}(z', \zeta', \lambda) e^{-4\pi\lambda(\|z'\|^2 + \|\zeta'\|^{2p})} dV(z', \zeta') d\lambda.$$

Combining this formula with (11) we obtain

$$f(z, \zeta, w) = \int_{\mathbb{R}^{2n+2m}} \int_{-\infty}^{\infty} S(z, \zeta, w; z', \zeta', w') \cdot f(z', \zeta', w') d(\operatorname{Re} w') dV(z', \zeta').$$

The Szegő kernel is written as

$$(14) \quad \begin{aligned} S(z, \zeta, w; z', \zeta', w') &= \int_0^\infty e^{2\pi i \lambda(w-w')} K_\lambda(z, \zeta; z', \zeta') d\lambda \\ &= \int_0^\infty e^{-4\pi \lambda A} K_\lambda(z, \zeta; z', \zeta') d\lambda \end{aligned}$$

with the same A as in (1). From (7) and (12) we obtain the series

$$\frac{p}{\pi^{n+m}} \sum_{\alpha, \beta} \frac{\Gamma(|\beta| + m)(z \bar{z}')^\alpha (\zeta \bar{\zeta}')^\beta}{\Gamma(\frac{|\beta|+m}{p}) \alpha! \beta!} \int_0^\infty e^{-4\pi \lambda A} (4\pi \lambda)^{\frac{|\beta|+m}{p} + |\alpha| + n} d\lambda$$

for the Szegő kernel. Here the value of the integral is

$$\frac{\Gamma(\frac{|\beta|+m}{p} + |\alpha| + n + 1)}{4\pi} A^{-\frac{|\beta|+m}{p} - |\alpha| - n - 1}.$$

Therefore we have to evaluate the series

$$\begin{aligned} &\frac{p}{4\pi^{n+m+1}} A^{-\frac{m}{p} - n - 1} \sum_{\alpha, \beta} \frac{\Gamma(\frac{|\beta|+m}{p} + |\alpha| + n + 1) \Gamma(|\beta| + m)}{\Gamma(\frac{|\beta|+m}{p}) \alpha! \beta!} \\ &\quad \cdot (A^{-1} z \bar{z}')^\alpha (A^{-1/p} \zeta \bar{\zeta}')^\beta. \end{aligned}$$

Now we sum over α using equality (4) with $s = \frac{|\beta|+m}{p} + n + 1$. Notice that $|A^{-1} \sum_{j=1}^n z_j \bar{z}'_j| < 1$ whenever $(z, \zeta, t) \neq (z', \zeta', t')$. So we have

$$\begin{aligned} &\frac{p}{4\pi^{n+m+1}} A^{-\frac{m}{p} - n - 1} \sum_{\beta} \frac{\Gamma(|\beta| + m)}{\Gamma(\frac{|\beta|+m}{p}) \beta!} (A^{-1/p} \zeta \bar{\zeta}')^\beta \frac{\Gamma(\frac{|\beta|+m}{p} + n + 1)}{(1 - A^{-1} z \cdot \bar{z}')^{\frac{|\beta|+m}{p} + n + 1}} \\ &= \frac{p}{4\pi^{n+m+1}} \frac{1}{(A - z \cdot \bar{z}')^{\frac{m}{p} + n + 1}} \sum_{\beta} \frac{\Gamma(|\beta| + m) \Gamma(\frac{|\beta|+m}{p} + n + 1)}{\Gamma(\frac{|\beta|+m}{p}) \beta!} \\ &\quad \cdot [(A - z \cdot \bar{z}')^{-1/p} \zeta \bar{\zeta}']^\beta. \end{aligned}$$

Now we rewrite $\Gamma(\frac{|\beta|+m}{p} + n + 1)$ as

$$\Gamma\left(\frac{|\beta| + m}{p}\right) \prod_{j=0}^n \left(\frac{|\beta| + m}{p} + j\right) = \frac{\Gamma(\frac{|\beta|+m}{p})}{p^{n+1}} \prod_{j=0}^n (|\beta| + m + pj).$$

We denote the polynomial $\prod_{j=0}^n (|\beta| + m + pj)$ by $q_{n+1}(|\beta|)$. So q_{n+1} is a polynomial of degree $n + 1$ in the variable $|\beta|$. Therefore the Szegő kernel is

equal to

$$\frac{1}{4p^n \pi^{n+m+1}} \frac{1}{(A - z \cdot \bar{z}')^{\frac{m}{p} + n + 1}} \sum_{\beta} \frac{\Gamma(|\beta| + m)}{\beta!} q_{n+1}(|\beta|) [(A - z \cdot \bar{z}')^{-1/p} \zeta \bar{\zeta}']^{\beta}.$$

Since $|(A - z \cdot \bar{z}')^{-1/p} \sum_{l=1}^m \zeta_l \bar{\zeta}'_l| < 1$ when $(z, \zeta, t) \neq (z', \zeta', t')$, we can apply the identity (5) to obtain

$$S(z, \zeta, t; z', \zeta', t') = \frac{1}{4p^n \pi^{n+m+1} (A - z \cdot \bar{z}')^{\frac{m}{p} + n + 1}} \sum_{k=0}^{n+1} c_k \left(\frac{1}{1 - (A - z \cdot \bar{z}')^{-1/p} \zeta \cdot \bar{\zeta}'} \right)^{m+k}.$$

This concludes the proof of Theorem 1 after simplifying the fractions. \square

Proof of Corollary 1. Invoking the formula

$$(15) \quad B(z, \zeta, w; z', \zeta', w') = 2i \frac{\partial S}{\partial \bar{w}'}(z, \zeta, w; z', \zeta', w')$$

from [NRSW] we obtain the Bergman kernel of the domain \mathcal{D} by a simple differentiation. In (15) the kernel S is obtained by substituting $w = t + i\|z\|^2 + i\|\zeta\|^{2p}$ and $w' = t' + i\|z'\|^2 + i\|\zeta'\|^{2p}$. \square

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