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PRIMITIVE CHARACTERS OF SUBGROUPS OF M–GROUPS

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ABSTRACT. Let G be an M–group, let S be a subnormal subgroup of G, and let H be a Hall subgroup of S. If the character $\gamma \in \mathrm{Irr}(H)$ is primitive, then $\gamma(1)$ is a power of 2. Furthermore, if $|G : S|$ is odd, then $\gamma(1) = 1$.

1. Introduction

Combining Theorems A of [5] and [13], Isaacs has proved that if G is an M–group and S is a subnormal subgroup of G, then the primitive characters of S all have degrees that are powers of 2, and if S has odd index in G, then the primitive characters of G are all linear. Using the techniques found in [12] and [13], we have found the following improvement of this result.

Theorem A. *Let G be an M–group, let S be a subnormal subgroup of G, and let H be a Hall subgroup of S. If the character $\gamma \in \mathrm{Irr}(H)$ is primitive, then $\gamma(1)$ is a power of 2. Furthermore, if $|G : S|$ is odd, then $\gamma(1) = 1$.*

In fact, we are able to use these techniques to prove even more. If G is an M–group, then by Taketa's theorem (Corollary 5.13 of [3]), we know that G is solvable. Thus, if S is a subnormal subgroup of G, then there exist subgroups S_i so that

$$S = S_0 \subseteq S_1 \subseteq ... \subseteq S_n = G$$

and S_i has prime index in S_{i+1}. If S is any subgroup of G for which such a chain exists, then we will say that S is **reachable by primes** in G. When G is a solvable group, subnormality is a special case of reachable by primes. Therefore, Theorem A is a consequence of the following result.

Theorem B. *Let G be an M–group, let S be a subgroup of G that is reachable by primes, and let H be a Hall subgroup of S. If the character $\gamma \in \mathrm{Irr}(H)$ is primitive, then $\gamma(1)$ is a power of 2. Furthermore, if $|G : S|$ is odd, then $\gamma(1) = 1$.*

We can generalize this result further by removing the hypothesis that H be a Hall subgroup. As such, this next result also generalizes Theorem B of [16].

Theorem C. *Let G be an M–group, let S be a subgroup of G that is reachable by primes, let H be a subgroup of S, and let π be the set of prime divisors of $|S : H|$. If the character $\gamma \in \mathrm{Irr}(H)$ is primitive, then γ has $\pi \cup \{2\}$–degree. Furthermore, if $|G : S|$ is odd, then γ has π–degree.*

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We can use Theorem C to prove Theorem B.

Proof of Theorem B. Let π be the set of prime divisors of $|S:H|$. Thus, H is a Hall π'–subgroup of S, and so, γ has π'–degree. By Theorem C, we know that γ has $\pi \cup \{2\}$–degree. The only way that this can occur is if γ has 2–power degree. If $|G:S|$ is odd, then γ has π–degree, and so, γ is linear. $\square$

We can generalize Theorem C still further. In [7], given a π–separable group G for a set of primes π with $2 \notin \pi$, Isaacs defined a class of characters for G that we call $\mathrm{D}_\pi(G)$. In [13], for an odd prime p, Isaacs said that a group G was a $\mathbf{D}_p M$**–group** if all the elements of $\mathrm{D}_p(G)$ are monomial. If G is an M–group, then G is solvable and is a $\mathrm{D}_p M$–group for every odd prime p. Thus, the following theorem is a generalization of Theorem C.

Theorem D. *Let G be a solvable group, and assume that G is a $\mathrm{D}_p M$–group for every odd prime p. Let S be a subgroup of G that is reachable by primes, let H be a subgroup of S, and let π be the prime divisors of $|S:H|$. If the character $\gamma \in \mathrm{Irr}(H)$ is primitive, then γ has $\pi \cup \{2\}$–degree.*

The proof of Theorem D is broken into two main parts. We believe that these two results are interesting themselves. In the first result, we will consider a solvable group G.

Theorem E. *Let p be an odd prime, let G be a solvable group, and assume that G is a $\mathrm{D}_p M$–group. If S is a subgroup of G that is reachable by primes, then S is a $\mathrm{D}_p M$–group.*

For the final result, we assume that G is a p–solvable group for some odd prime p. Thus, we may use the idea of a π–special character that was defined for π–separable groups in [1]. In Corollary 7.2 of [7], Isaacs proved that the π–special characters of G are exactly the characters in $\mathrm{D}_\pi(G)$ that have π–degree.

Theorem F. *Let p be an odd prime, let G be a p–solvable group, and assume that G is a $\mathrm{D}_p M$–group. Let H be a subgroup of G such that p does not divide $|G:H|$. If the character $\theta \in \mathrm{Irr}(H)$ is p–special and primitive, then θ is linear.*

2. Preliminaries

We begin this section by proving a result regarding the structure of subgroups having prime index.

Lemma 2.1. *Let G be a p–solvable group and suppose $H \subseteq G$ such that $|G:H| = p$ for some prime p. If $\mathrm{core}_G(H) = 1$, then H is a cyclic group with order dividing $p-1$.*

Proof. Let K be a minimal normal subgroup of G, and observe that $G = HK$. Since $G = HK$, it follows that p divides $|K|$, and because G is p–solvable, this implies that K is a p–group. Thus, $H \cap K$ is normal in both H and K, and so, we conclude that $H \cap K$ is trivial. In particular, we know that $|K| = p$. Let $C = \mathbf{C}_G(K)$, and note that $K \subseteq C$. Since K is normal in G, it follows that C is normal in G. Thus, $\mathbf{C}_H(K) = H \cap C$ is normal in H. On the other hand, K centralizes C, and so, K centralizes $C \cap H$. Therefore, $G = HK$ normalizes $C \cap H$. Since $\mathrm{core}_G(H) = 1$, we have $C \cap H = 1$, and we see that $C = (C \cap H)K = K$ because $K \subseteq C$. By the N/C-Theorem (see Problem 3.09 of [10]), G/C is isomorphically contained in the automorphism group of K. We know that $\mathrm{Aut}(K)$ is cyclic of order $p-1$ because K is cyclic of order p. Thus, G/K is cyclic of order dividing $p-1$ and since H is isomorphic to G/K we have proved the lemma. □

We now review some results from Isaacs' π–theory. For a more detailed understanding of π–theory, the reader may wish to consult the expository papers [8], [9], and [12].

We fix a set of primes π and a π–separable group G. The π–**partial characters** of G are the restrictions of the ordinary characters to the set G^0 of π–elements of G. (Given an ordinary character $\chi \in \mathrm{Char}(G)$, we write χ^0 for the restriction of χ to G^0.) A π–partial character is **irreducible** if it cannot be written as a sum of two nonzero π–partial characters, and we write $\mathrm{I}_\pi(G)$ for the set of all irreducible π–partial characters of G. Given a π–partial character ψ, we say that the ordinary character χ is a **lift** for ψ if $\chi^0 = \psi$. If H is a subgroup of G and $\phi \in \mathrm{I}_\pi(H)$, then we write $\mathrm{I}_\pi(G|\phi)$ for those partial characters $\psi \in \mathrm{I}_\pi(G)$ such that ϕ is a constituent of ψ_H. We now assume that $2 \notin \pi$. In Theorem B of [13], Isaacs proved that $\mathrm{D}_\pi(G)$ is a subset of $\mathrm{Irr}(G)$ with the property that restriction to G^0 is bijection between $\mathrm{D}_\pi(G)$ and $\mathrm{I}_\pi(G)$. In [13], given an odd prime p and a p–solvable group G, Isaacs proved another connection between $\mathrm{D}_p(G)$ and $\mathrm{I}_p(G)$. Given a character $\chi \in \mathrm{D}_p(G)$, he proved, in Theorem F of that paper, that χ is monomial if and only if χ^0 is monomial. Using this fact, we have the next result.

Lemma 2.2. *Let G be a p–solvable group for some odd prime p, and let K be a subnormal subgroup of G. Consider the partial character $\psi \in \mathrm{I}_p(G)$. If ψ is monomial, then every irreducible constituent of ψ_K is monomial.*

Proof. Let χ be the lift for ψ that lies in $\mathrm{D}_p(G)$. By Theorem F of [13], ψ monomial implies that χ is monomial. Let $\mu \in \mathrm{I}_p(K)$ be an irreducible constituent of ψ_K, and let ν be the lift of μ that lies in $\mathrm{D}_p(K)$. By Theorem 7.10 of [7], we know that every constituent of χ_K lies in $\mathrm{D}_p(K)$. Since μ is a constituent of ψ_K, it follows that some constituent of χ_K lifts μ. This implies that ν is a constituent of χ_K. By Theorem 5.2 of [13], we know that ν is monomial, and by Theorem F of [13], we conclude that μ is monomial. □

3. Theorem E

In this section, we prove Theorem E of the Introduction. Most of the work is found in this next theorem. We have used the hypothesis that G is a solvable group in Theorem 3.1 in order to keep the statement simple. In fact, the proof that we include works if we assume that G is a p–solvable and q–solvable group, where p and q are the two primes mentioned in the theorem.

Theorem 3.1. *Let p be an odd prime, q be any prime, and G be a solvable group. Suppose H is a subgroup of G such that $|G : H| = q$. Consider a p–partial character $\gamma \in \mathrm{I}_p(H)$, and assume that all the p–partial characters lying in $\mathrm{I}_p(G|\gamma)$ are monomial. Then γ is monomial.*

Proof. We assume that G, H, and γ form a counterexample with $|G|$ as small as possible. Let $L = \mathrm{core}_G(H)$, and let the p–partial character $\alpha \in \mathrm{I}_p(L)$ be a constituent of γ_L.

Step 1. α is invariant in H.

Proof of Step 1. Write S for the stabilizer of α in H, and assume that $S < H$. In particular, we know that $L < H$, and so, H is not normal in G. Let K be a normal subgroup of G containing L such that K/L is a G–chief factor. It is easy to see that $G = HK$ and $H \cap K = L$. This implies that $SK < G$ and that $|SK : S| = |K : L| = |G : H| = q$. By Proposition 3.2 of [6] (Clifford's theorem for partial characters), we know that there is a p–partial character $\sigma \in \mathrm{I}_p(S)$ so that $\sigma^H = \gamma$, and note that γ is the only partial character lying in $\mathrm{I}_p(H|\sigma)$. By part b of that proposition, we know that σ is a constituent of γ_S and that the multiplicities of α in γ_L and σ_L are equal.

Consider partial characters $\mu \in \mathrm{I}_p(SK|\sigma)$ and $\nu \in \mathrm{I}_p(G|\mu)$. We observe that σ is a constituent of ν_S, and so, some constituent of ν_H lies over σ. It follows that γ is a constituent of ν_H, and so, $\nu \in \mathrm{I}_p(G|\gamma)$. Hence, by hypothesis, we have that ν is monomial. On the other hand, we know by Lemma 2.1 that H/L is cyclic, and so, G/K is cyclic. This implies that SK is normal in G, and by Lemma 2.2, we now have that μ is monomial. Thus, every partial character lying in $\mathrm{I}_p(SK|\sigma)$ is monomial, and so, SK, S, and σ satisfy the hypotheses of the theorem. Since $|SK| < |G|$, we have that σ is monomial. This implies that γ is monomial, but this is a contradiction. $\square$

Step 2. α is invariant in G.

Proof of Step 2. Let T be the stabilizer of α in G, and assume that $T < G$. By Step 1, we know that $H \subseteq T$. Since H has prime index in G, this implies that $H = T$ or $G = T$. Because $T < G$, we see that $T = H$. Using Proposition 3.2 of [6], we have that $\chi = \gamma^G$ is irreducible and that γ is a constituent of χ_H. This implies that $\chi \in \mathrm{I}_p(G|\gamma)$, and by hypothesis, χ is monomial. Applying Theorem F of [13], we see that every character inducing χ is monomial, and since γ induces χ, we conclude that γ is monomial. This contradicts the choice of counterexample. $\square$

Write θ for the character in $\mathrm{D}_p(H)$ such that $\theta^0 = \gamma$.

Step 3. θ does not extend to G.

Proof of Step 3. Assume that $\chi \in \mathrm{Irr}(G)$ is an extension of θ, and we write $\psi = \chi^0$. It is easy to see that

$$\psi_H = (\chi^0)_H = (\chi_H)^0 = \theta^0 = \gamma,$$

and so, we have that ψ extends γ and lies in $\mathrm{I}_p(G|\gamma)$. Therefore, ψ is monomial, and since ψ extends γ, we conclude that γ is monomial. $\square$

Step 4. Final contradiction:

Proof of Step 4. Let the character $\phi \in \mathrm{D}_p(L)$ be such that $\phi^0 = \alpha$. Since α is a constituent of $(\theta_L)^0$, since ϕ is the unique D_p–lift of α, and since all the constituents of θ_L lie in $\mathrm{D}_p(L)$, it follows that ϕ is a constituent of θ_L. We know that ϕ is G–invariant because α is G–invariant. Suppose that H is normal in G, so that $\phi = \theta$. Since H has prime index, it follows that θ extends to G and by Step 3, this is a contradiction. Thus, H is not normal in G.

Let K be a normal subgroup of G containing L so that K/L is a G–chief factor. It is easy to see that $G = HK$ and $H \cap K = L$. Since L has prime index in K, we know that ϕ extends to K. By Corollary 2.2 of [14], we know that ϕ has an H–invariant extension $\epsilon \in \mathrm{Irr}(K)$. We may now use Lemma 10.5 of [2], to see that θ extends to G. Since this contradicts Step 3, we have completed the proof. $\square$

Observe that the following result is a strong form of Theorem E in the Introduction.

Corollary 3.2. *Let p be an odd prime, let G be a solvable group, and let the subgroup S of G be reachable by primes. Consider the character $\theta \in \mathrm{D}_p(S)$. If all the characters lying in $\mathrm{D}_p(G|\theta)$ are monomial, then θ is monomial.*

Proof. Work by induction on $|G|$. If $S = G$, then we are done because we know that θ lies in $\mathrm{D}_p(G)$, and by hypothesis, this implies that θ is monomial. Now assume that $S < G$. Then, there is a subgroup S_1 such that $S < S_1 \subseteq G$ and S has prime index in S_1. Consider a character $\psi \in \mathrm{D}_p(S_1|\theta)$. Observe that $\mathrm{D}_p(G|\psi) \subseteq \mathrm{D}_p(G|\theta)$, and so, all the characters lying in $\mathrm{D}_p(G|\psi)$ are monomial. By the inductive hypothesis, we conclude that ψ is monomial. Thus, every character lying in $\mathrm{D}_p(S_1|\theta)$ is monomial. Since $\theta \in \mathrm{D}_p(S)$, we know that θ^0 lies in $\mathrm{I}_p(S)$. We apply the fact that all the characters in $\mathrm{D}_p(S_1|\theta)$ are monomial to see that all the p–partial characters in $\mathrm{I}_p(S_1|\theta^0)$ are monomial. By Theorem 3.1, we know that θ^0 is monomial, and using Theorem F of [13], we conclude that θ is monomial. □

4. Theorem F

We begin this section by proving that Theorem D implies Theorem C. In order to do this we need Theorem 4.7 of [16]. In that theorem Navarro proved that if G is an M–group, π is a set of primes, and J is a subgroup of G having π'–index, then the π–special primitive characters of J must be linear.

Proof of Theorem C. It is obvious that Theorem D implies all of Theorem C except for the last sentence, and so, we may assume that $|G : S|$ is odd. In particular, Theorem D implies that θ has $\pi \cup \{2\}$–degree. If $|S : H|$ is not odd, then $\pi = \pi \cup \{2\}$, and there is nothing to prove. Hence, we assume that $|S : H|$ is odd, and so, $|G : H|$ is odd. Let ν be the prime divisors of $|G : H|$, and note that $2 \notin \nu$. Since G is solvable, we have, by Corollary 2.7 of [4], that $\theta = \theta_\nu \theta \nu'$, where θ_ν and $\theta_{\nu'}$ are primitive ν–special and ν'–special characters, respectively. We know that $(\theta_{\nu'})^0$ is irreducible as a ν'–partial character, and by Lemma 4.4 of [15], it is primitive. By Theorem 4.7 of [16], $\theta_{\nu'}$ is linear, and so, θ has ν–degree. Since $2 \notin \nu$, this implies that $\theta(1)$ is odd, and the result follows. □

In order to prove the remaining theorems we will need a stronger version of Theorem 4.7 of [16]. We believe that the proof Navarro employed in that paper proves the following theorem, but the result presented there is weaker than the result found here. In particular, we do not need to assume that G is an M–group. Consider the situation that G is a π–separable group for some set of primes π, J is a subgroup having π'–index in G, and θ is a π–special primitive character of J. In order to prove that θ is linear, we only need to assume that the characters lying in $\mathrm{D}_\pi(G|\theta)$ are monomial. Thus, for all the π–special primitive characters of J to be linear, it suffices to assume that all the characters lying in $\mathrm{D}_\pi(G)$ are monomial. If $\pi = \{p\}$, then this implies that G is a $\mathrm{D}_p M$–group.

The proof that we present is inspired by the simplification of the arguments employed by Navarro that are found in Theorem 7.1 of [12]. As in that paper, we set the following notation. Let π be a set of primes, and let G be a π–separable group. Fix the π–partial character $\phi \in \mathrm{I}_\pi(G)$. A pair (U, μ) is called a π–**inducing**

pair for ϕ if the following conditions are satisfied: (i) $U \subseteq G$, (ii) $\mu \in \mathrm{I}_\pi(U)$, (iii) $\mu^G = \phi$, and (iv) $\mu(1)$ is a π–number. We construct the undirected graph $\mathcal{G} = \mathcal{G}(\phi)$ whose vertex set is the collection of all π–inducing pairs for ϕ. We join two distinct pairs (U, μ) and (V, ν) in $\mathcal{G}$ if either $U \subseteq V$ and $\mu^V = \nu$ or $V \subseteq U$ and $\nu^U = \mu$. In Theorem 6.2 of [12], Isaacs proved that the connected components of $\mathcal{G}(\phi)$ are all conjugate in G.

Theorem 4.1. *Let π be a set of primes and G be a π–separable group. Suppose that J is a subgroup of G such that $|G : J|$ is a π'–number, and consider a primitive π–partial character $\theta \in \mathrm{I}_\pi(H)$. If every π–partial character lying in $\mathrm{I}_\pi(G|\theta)$ is monomial, then θ is linear.*

Proof. By Theorem 4.2 of [16], there is a subgroup U containing J and a quasi-primitive π–partial character $\mu \in \mathrm{I}_\pi(U)$ such that $\mu_J = \theta$. If we write $\chi = \mu^G$, then $\chi \in \mathrm{I}_\pi(G)$. Since μ extends the primitive character θ, it follows that μ is primitive. By Proposition 3.4 of [6], since μ is primitive, μ must have π–degree. In particular, (U, μ) is a π–inducing pair for χ.

Suppose that some pair (V, ν) is joined to (U, μ) in $\mathcal{G}(\chi)$. There are two possibilities: either $U \subseteq V$ and $\nu = \mu^V$ or $V \subseteq U$ and $\mu = \nu^U$. Since the two pairs (U, μ) and (V, ν) are distinct, it follows that $U \neq V$.

In the first case, where $U < V$, we know that $|V : U|$ divides $\nu(1)$, and so, this index is a nontrivial π–number. On the other hand, since U has π'–index in G, we have that U contains a full Hall π–subgroup of G. Thus, we have a contradiction. In the other case, $V < U$ and $\mu = \nu^U$, and this contradicts the fact that μ is primitive. Therefore, no pair is joined to (U, μ) in $\mathcal{G}(\chi)$.

We conclude that the connected component of $\mathcal{G}(\chi)$ containing (U, μ) consists of just that one node. By Theorem 6.2 of [12], every π–inducing pair (V, ν) for χ is conjugate to (U, μ) via G. In particular, it follows that $\nu(1) = \mu(1)$ whenever $\nu^G = \chi$ and $\nu(1)$ is a π–number.

By Lemma 3.3 of [11], μ is a constituent of χ_U. Thus, we know that θ is a constituent of χ_J, and so, by hypothesis, that χ is monomial. Consequently, there is some subgroup $W \subseteq G$ and linear π–partial character $\lambda \in \mathrm{I}_\pi(W)$ such that $\lambda^G = \chi$. Now, $\lambda(1) = 1$ is a π–number, and so, (W, λ) is a vertex in $\mathcal{G}(\chi)$, and thus, $\theta(1) = \mu(1) = \lambda(1) = 1$, as required. □

Let π be a set of primes, let G be a π–separable group, and let H be a subgroup of G. Consider a character $\theta \in \mathrm{D}_\pi(H)$. We know that $\theta^0 \in \mathrm{I}_\pi(H)$. Thus, we may consider the set $\mathrm{I}_\pi(G|\theta^0)$. We define the set $\mathrm{D}_\pi(G|\theta) = \{\chi \in \mathrm{D}_\pi(G) | \chi^0 \in \mathrm{I}_\pi(G|\theta^0)\}$. Observe that the following result contains Theorem F of the Introduction.

Corollary 4.2. *Let p be an odd prime, and let G be a p–solvable group. Consider a subgroup H of G such that $|G : H|$ is p'. Assume that the character $\theta \in \mathrm{Irr}(H)$ is primitive and p–special. If all the characters lying in $\mathrm{D}_p(G|\theta)$ are monomial, then θ is linear.*

Proof. Since θ is p–special, we know that $\theta^0 \in \mathrm{I}_p(H)$. By Lemma 4.4 of [15], it follows that θ^0 is primitive. Consider a partial character $\gamma \in \mathrm{I}_p(G|\theta^0)$, and write χ for the character in $\mathrm{D}_p(G|\theta)$ such that $\chi^0 = \gamma$. By Theorem F of [13], γ is monomial. Therefore, every partial character lying in $\mathrm{I}_p(G|\theta^0)$ is monomial, and so, we may apply Theorem 4.1 to see that θ^0 is linear. This implies that θ is linear. □

5. Theorem D

We will finish by proving Theorem D, but first, we prove a combination of Corollaries 3.2 and 4.2.

Theorem 5.1. *Let p be an odd prime, and let G be a solvable group. Suppose that the subgroup S of G is reachable by primes. Consider a subgroup H of S such that $|S:H|$ is a p'–number. Let the p–special character $\theta \in \mathrm{Irr}(H)$ be primitive. If all the characters lying in $\mathrm{D}_p(G|\theta)$ are monomial, then θ is linear.*

Proof. Consider a character $\gamma \in \mathrm{D}_p(S|\theta)$. It is easy to see that $\mathrm{D}_p(G|\gamma) \subseteq \mathrm{D}_p(G|\theta)$. By Corollary 3.2, we know that γ is monomial. Therefore, every character lying in $\mathrm{D}_p(S|\theta)$ is monomial. Since θ is primitive and $|S:H|$ is p', we may use Theorem 4.2 to conclude that θ must be linear. □

Proof of Theorem D. By [4], we know that θ can be fully factored as a product of primitive p_i–special characters for distinct primes p_i. By Theorem 5.1, we see that each of these p_i–special primitive characters is linear if p_i is an odd prime not in π. Therefore, θ has $\pi \cup \{2\}$–degree. □

References

1. D. Gajendragadkar, *A Characteristic Class of Characters of Finite π–Separable Groups*, J. Algebra **59** (1979), 237-259. MR **82b:**20012
2. I. M. Isaacs, *Characters of Solvable and Symplectic Groups*, Amer. J. Math. **95** (1973), 594-635. MR **48:**11270
3. I. M. Isaacs, *Character Theory of Finite Groups*, Academic Press, New York, 1976. MR **57:**417
4. I. M. Isaacs, *Primitive Characters, Normal Subgroups, and M–Groups*, Math. Z. **177** (1981), 267-284. MR **82f:**20026
5. I. M. Isaacs, *Characters of Subnormal Subgroups of M–Groups*, Arch. Math. **42** (1984), 509-515. MR **86a:**20009
6. I. M. Isaacs, *Fong Characters in π–Separable Groups*, J. Algebra **99** (1986), 89-107. MR **87e:**20015
7. I. M. Isaacs, *Induction and Restriction of π–Special Characters*, Can. J. Math. **38** (1986), 576-604. MR **87g:**20014
8. I. M. Isaacs, *Partial Characters of π–Separable Groups*, Prog. Math. **95** (1991), 273-287. MR **93c:**20015
9. I. M. Isaacs, *The π–Character Theory of Solvable Groups*, J. Austral. Math. Soc. (Series A) **57** (1994), 81-102. MR **95c:**20011
10. I. M. Isaacs, *Algebra: A Graduate Course*, Brook/Coles Publishing Company, Pacific Grove, CA, 1994. MR **95k:**00003
11. I. M. Isaacs and G. Navarro, *Weights and Vertices for Characters of π–Separable Groups*, J. Algebra **177** (1995), 339-366. CMP 96:02
12. I. M. Isaacs, *Characters and Sets of Primes for Solvable Groups, in* Finite and Locally Finite Groups, NATO ASI Series, Kluwer Academic Publishers, Dordrecht, The Netherlands, 1995, pp. 347-376. CMP 96:04
13. I. M. Isaacs, *Induction and Restriction of π–Partial Characters and Their Lifts*, Submitted to *Can. J. Math.*
14. M. L. Lewis, *Nonabelian Fully-Ramified Sections*, To Appear in *Can. J. Math.*
15. M. L. Lewis, *Characters of Maximal Subgroups of M–Groups*, To Appear in *J. Algebra*.
16. G. Navarro, *Primitive Characters of Subgroups of M–Groups*, Math. Z. **218** (1995), 439-445. MR **96b:**20011

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