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NON-EXISTENCE AND UNIQUENESS RESULTS FOR BOUNDARY VALUE PROBLEMS FOR YANG-MILLS CONNECTIONS

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Abstract. We show uniqueness results for the Dirichlet problem for Yang-Mills connections defined in n-dimensional ($n \geq 4$) star-shaped domains with flat boundary values. This result also shows the non-existence result for the Dirichlet problem in dimension 4, since in 4-dimension, there exist countably many connected components of connections with prescribed Dirichlet boundary value. We also show non-existence results for the Neumann problem. Examples of non-minimal Yang-Mills connections for the Dirichlet and the Neumann problems are also given.

1. Introduction

Let M be a Riemannian manifold with boundary and G a compact Lie group. Let A_0 be a given smooth connection defined on a principal G-bundle $P_0 \to \partial M$. We denote by $\mathcal{A}(A_0)$ the space of smooth connections defined in principal G-bundles over M with Dirichlet boundary value A_0 at ∂M. That is,

$$\mathcal{A}(A_0) = \{A : A \text{ is a smooth connection defined in some principal } G\text{-bundle over } M \text{ with } i^*A \sim A_0 \text{ over } \partial M\},$$

where $i^*A \sim A_0$ means that i^*A is gauge equivalent to A_0 over ∂M and $i : \partial M \hookrightarrow M$ is the inclusion map.

By definition, a connection A is a solution to the Dirichlet problem for Yang-Mills equations defined in M with boundary value A_0 at ∂M if $A \in \mathcal{A}(A_0)$ with finite energy and A is Yang-Mills, that is, $D_A^* F_A = 0$ in M. Here D_A^* is the formal adjoint of the covariant exterior derivative $D_A = d + [A,\ ]$ with respect to the L^2-metric on $\Lambda^2 T^*M \otimes Ad(P)$ induced from the Riemannian metric on M and adjoint invariant metric on $\mathfrak{g}$, the Lie algebra of G.

We also recall the definition of the Neumann problem. A connection A is a solution to the Neumann problem for Yang-Mills equations if and only if A is Yang-Mills in M with finite energy and $i^*(*F_A) = 0$ on ∂M. Here $* : \Lambda^2 T^*M \otimes \mathrm{Ad}(P) \to \Lambda^2 T^*M \otimes \mathrm{Ad}(P)$ is the Hodge star operator.

Dirichlet and Neumann problems for Yang-Mills connections were first defined and studied by Marini [7]. In [7], Marini showed the existence and regularity of

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absolute minimum solutions for the Dirichlet problem, and Neumann problem with prescribed class $\eta \in H^2(M;\pi_1(G))$, when $\dim M = 4$.

In [4], Isobe and Marini studied the existence of "topologically distinct solutions" to the Dirichlet problem when $M = B^4 = \{x \in \mathbb{R}^4 : |x| \leq 1\}$ and $G = SU(2)$ using the fact that in this case $\mathcal{A}(A_0)$ is the disjoint union of infinitely many connected components $\mathcal{A}_k$ indexed by $k \in \mathbb{Z}$ (specifically, for some fixed $B \in \mathcal{A}(A_0)$, $\mathcal{A}_k = \{A : C(B) - C(A) = k\}$, where $C(A) = \frac{1}{8\pi^2}\int_M \mathrm{tr}(F_A \wedge F_A) * 1)$. It is shown that for "generic boundary values", there exist infinitely many topologically distinct solutions, and for any non-flat boundary values, there exist at least two topologically distinct solutions. These solutions all minimize the action on their component. In [4] we also showed that for some boundary values A_0 the action attains its infimum on only finitely many components, and hence does not attain its infimum on infinitely many others.

But the following question remained for the Dirichlet problem: Is there a Yang-Mills connection (not necessarily minimizing) in each component of $\mathcal{A}(A_0)$?

In this paper, we show that for a flat boundary value A_0, a flat connection is the only solution for the Dirichlet problem with boundary value A_0 when the base manifold is star-shaped (see §2 for the definition) and structure group G is an arbitrary compact Lie group. Thus we cannot, in general, expect the existence of Yang-Mills connections in each connected component of $\mathcal{A}(A_0)$.

We also show that this uniqueness result does not hold for general M. We give an example of M (annulus) such that there exists a non-flat Yang-Mills connection on some principal bundle over M which is flat at ∂M. This connection is necessarily a non-minimal Yang-Mills connection. The construction of this connection comes from the one given in [9], where Parker constructed non-minimal Yang-Mills connections over S^4 or $S^3 \times S^1$. However, in our case, the argument is simplified by using a direct variational method.

As for the Neumann problem, if $\eta \in H^2(M;\pi_1(G))$ is trivial, the solution obtained by Marini [7] is a flat connection. This raises the following problem: Does there exist a non-flat connection for the Neumann problem if $\eta \in H^2(M;\pi_1(G))$ vanishes?

We show in this paper, when $\dim M = 4$, for star-shaped domains, a flat connection is the only solution for the Neumann problem. We also give an example of a non-flat Yang-Mills connection on some principal $SU(2)$-bundle P with $\eta(P) = 0$ which satisfies the Neumann condition.

See also [14] for a uniqueness result to the Dirichlet problem for (anti-)self-dual connections. Note that we do not restrict ourselves to (anti-)self dual connections.

Our result also shows similarities between our results and other non-existence (or uniqueness) results related to Yamabe equations ([10]), harmonic mappings ([6]), constant mean curvature equations ([17]), etc...

2. Proof of the main results

Our main results are the following. The first result is concerned with the Dirichlet problem:

Theorem 2.1. (1) *Let $n \geq 5$, M a C^2-star-shaped bounded domain in $\mathbb{R}^n$ with flat metric, and G a compact Lie group. Let A_0 be a flat connection on some principal G-bundle $P_0 \to \partial M$. Assume $A \in \mathcal{A}(A_0)$ is a solution to the Dirichlet problem* *for Yang-Mills connections. Then* A *is a flat connection, that is, the curvature* $F_A = dA + A \wedge A$ *of* A *vanishes.*

(2) *Let* $n = 4$ *and* G *be as in* (1). *Assume* M *is a* C^2*-strictly star-shaped bounded domain in* $\mathbb{R}^4$ *with flat metric. Then the same conclusion as in* (1) *holds.*

Our next result is concerned with the Neumann problem:

Theorem 2.2. *Let* $n = 4$ *and* G *a compact Lie group. Let* M *be a* C^2*-strictly star-shaped bounded domain in* $\mathbb{R}^4$ *with flat metric. Assume* A *is a solution to the Neumann problem for Yang-Mills connections. Then* A *is a flat connection.*

Before we give the proofs of the above theorems, we give here the definitions of *star-shaped* and *strictly star-shaped* domains.

Definition 2.3. (1) A domain $M \subset \mathbb{R}^n$ is called *star-shaped* if there exists a point $x_0 \in M$ such that the line segment $\overline{xx_0}$ is contained in M for all $x \in M$. In this case, we have $\langle x - x_0, \nu(x)\rangle \geq 0$ for any point $x \in \partial M$, where $\nu(x)$ is the outer normal at $x \in \partial M$ and $\langle \cdot, \cdot \rangle$ is the inner product in $\mathbb{R}^n$.

(2) A domain $M \subset \mathbb{R}^n$ is called *strictly star-shaped* if M is star-shaped and $\langle x - x_0, \nu(x)\rangle > 0$ for any $x \in \partial M$, where x_0 is as in (1).

Note that the star-shaped domain is contractible. Therefore for such base manifold M and any compact Lie group G, $\eta \in H^2(M; \pi_1(G))$ is always trivial.

Proof of the Theorem 2.1 and Theorem 2.2. Both theorems follow from the following first variation formula for Yang-Mills fields (see [11]):

Lemma 2.4. *Let* $\{e_1, \ldots, e_n\}$ *be an orthonormal tangent frame for* TM. *Let* A *be a solution to the Dirichlet or the Neumann problem for Yang-Mills equations. Then the following holds for any vector field* X *in* M *with compact support in* M°:

$$\int_M |F_A|^2 \; divX - 4(F_A(\nabla_{e_i} X, e_j), F_A(e_i, e_j)) = 0. \tag{2.1}$$

Here $(\cdot, \cdot)$ *in the above equation is the adjoint invariant inner product of* $\mathfrak{g}$.

In [11] this result is stated for Yang-Mills connections without boundary conditions, however, this is also true for Yang-Mills connections with boundary value conditions, since the variation used in [11] does not change boundary values.

We first prove Theorem 2.1.

Without loss of generality we may assume that $0 \in M$ and M is (strictly) star-shaped with respect to the point $x_0 = 0$ (see Definition 2.3).

For $\delta > 0$, define

$$\mathcal{O}_\delta(\partial M) := \{x \in M : d(x, \partial M) < \delta\}.$$

Since ∂M is C^2, there exists $\delta > 0$ such that the map $\pi : \mathcal{O}_\delta(\partial M) \to \mathbb{R}$ defined by $\pi(x) = d(x, \partial M)$ is C^1.

For such δ define the map $\Pi : M \to \mathbb{R}$ as

$$\Pi(x) := \begin{cases} \pi(x) & \text{if } x \in \mathcal{O}_\delta(\partial M), \\ \delta & \text{if } x \in M \setminus \mathcal{O}_\delta(\partial M). \end{cases}$$

Π is a Lipschitz function defined in M.

Next define the map $\rho_\epsilon : \mathbb{R} \to \mathbb{R}$ for $\epsilon > 0$ as

$$\rho_\epsilon := \begin{cases} 0 & \text{if } x \le \epsilon, \\ 1 & \text{if } x \ge 2\epsilon, \\ \frac{x}{\epsilon} - 1 & \text{if } \epsilon \le x \le 2\epsilon. \end{cases}$$

For $\epsilon > 0$ with $2\epsilon < \delta$, define the vector field X_ϵ in M by

$$X_\epsilon = \rho_\epsilon(\Pi(x)) \sum_{i=1}^{n} x_i \frac{\partial}{\partial x_i}. \tag{2.2}$$

Note that the vector field X_ϵ is only Lipschitzian, and the formula (2.1) holds for Lipschitzian vector fields by a density argument. Also note that $\mathrm{supp}(X_\epsilon) \subset \{x \in M : \Pi(x) \ge \epsilon\} \subset M^\circ$.

We insert this vector field X_ϵ in the first variational formula (2.1), taking $e_i = \partial/\partial x_i$. A short calculation gives

$$\begin{aligned} 0 =&(n-4) \int_M \rho_\epsilon(\Pi(x)) |F_A|^2 \, dx + \int_M \dot{\rho}_\epsilon(\Pi(x)) \langle \nabla \Pi(x), x \rangle |F_A|^2 \, dx \\ &- 4 \int_M \dot{\rho}_\epsilon(\Pi(x)) \Big(F_A\Big(x_k \frac{\partial}{\partial x_k}, \frac{\partial}{\partial x_j}\Big), F_A\Big(\nabla \Pi(x), \frac{\partial}{\partial x_j}\Big)\Big) \, dx. \end{aligned} \tag{2.3}$$

Here and in the following, we use the summation convention.

Letting $\epsilon \downarrow 0$ in (2.3), we obtain

$$\begin{aligned} 0 =&(n-4) \int_M |F_A|^2 \, dx - \int_{\partial M} \langle x, \nu(x) \rangle |F_A|^2 \\ &+ 4 \int_{\partial M} \Big(F_A\Big(x_k \frac{\partial}{\partial x_k}, \frac{\partial}{\partial x_j}\Big), F_A\Big(\frac{\partial}{\partial \nu}, \frac{\partial}{\partial x_j}\Big)\Big). \end{aligned} \tag{2.4}$$

We rewrite (2.4) using the new tangent frame $\{\nu, \tau_1, \dots, \tau_{n-1}\}$ at ∂M. Here $\{\tau_1, \dots, \tau_{n-1}\}$ is an orthonormal tangent frame of ∂M. Then $\sum_{k=1}^{n} x_k \frac{\partial}{\partial x_k} = \langle x, \nu \rangle \frac{\partial}{\partial \nu} + \sum_{k=1}^{n-1} \langle x, \tau_k \rangle \frac{\partial}{\partial \tau_k}$, and (2.4) becomes

$$\begin{aligned} 0 =&(n-4) \int_M |F_A|^2 \, dx - \int_{\partial M} \langle x, \nu \rangle |F_A|^2 + 4 \int_{\partial M} \langle x, \nu \rangle |F_A(\nu, \tau_k)|^2 \\ &+ 4 \int_{\partial M} \langle x, \tau_k \rangle (F_A(\tau_k, \tau_l), F_A(\nu, \tau_l)). \end{aligned} \tag{2.5}$$

By the Dirichlet boundary condition $i^* A \sim 0$, we have $F_A\Big(\frac{\partial}{\partial \tau_k}, \frac{\partial}{\partial \tau_l}\Big) = 0$ for all $1 \le k, l \le n-1$. Thus from (2.5), we get

$$(n-4) \int_M |F_A|^2 \, dx - \int_{\partial M} \langle x, \nu \rangle |F_A|^2 + 4 \int_{\partial M} \langle x, \nu \rangle |i^*(*F_A)|^2 = 0. \tag{2.6}$$

Using again the condition $i^* A \sim 0$, we reduce (2.6) to the following form:

$$(n-4) \int_M |F_A|^2 \, dx + 3 \int_{\partial M} \langle x, \nu \rangle |i^*(*F_A)|^2 = 0. \tag{2.7}$$

When $n > 4$ and M is star-shaped, (2.7) implies $F_A = 0$. This is the assertion of Theorem 2.1 (1).

To prove Theorem 2.1. (2), we work more.

Under the assumption of Theorem 2.1 (2) we have, by (2.7), $i^*(*F_A) = 0$ on ∂M. Combining this with the Dirichlet boundary condition, we conclude that all components of F_A vanish on ∂M.

Let $\rho > 0$ be such that the nearest point retraction $r : \mathcal{O}_\rho(M) \to \overline{M}$ defined by $r(x) = x$ if $x \in M$ and $r(x) = y_x$ if $x \in \mathcal{O}_\rho(M) \setminus M$, where $y_x \in \partial M$ is the unique point satisfying $d(x, M) = d(x, y_x)$, is well defined. Here $\mathcal{O}_\rho(M) := \{x \in \mathbb{R}^n : d(x, M) < \rho\}$.

We define the connection $\tilde{A}$ defined in $\mathcal{O}_\rho(M)$ by $\tilde{A} = r^*A$. Note that $F_{\tilde{A}} = 0$ on $\mathcal{O}_\rho(M) \setminus M$.

We need the following lemma.

Lemma 2.5. *$\tilde{A}$ is a (weak) Yang-Mills connection in $\mathcal{O}_\rho(M)$.*

Proof. First note that

$$F_{\tilde{A}} \in L^2(\Lambda^2 T^*\mathcal{O}_\rho(M) \otimes Ad(r^*P)) \quad \text{and} \quad \tilde{A} \in L^2_1(T^*\mathcal{O}_\rho(M) \otimes Ad(r^*P)).$$

We need to show the following:

$$\int_{\mathcal{O}_\rho(M)} (F_{\tilde{A}}, D_{\tilde{A}}\varphi) = 0 \quad \text{for all } \varphi \in C^\infty(T^*\mathcal{O}_\rho(M) \otimes Ad(r^*P)) \\ \text{with } \operatorname{supp}(\varphi) \subset \mathcal{O}_\rho(M).$$

Fix such $\varphi \in C^\infty(T^*\mathcal{O}_\rho(M) \otimes Ad(r^*P))$. We have

$$\begin{aligned} \int_{\mathcal{O}_\rho(M)} (F_{\tilde{A}}, D_{\tilde{A}}\varphi) &= \int_{\mathcal{O}_\rho(M)\setminus\overline{M}} (F_{\tilde{A}}, D_{\tilde{A}}\varphi) + \int_M (F_{\tilde{A}}, D_{\tilde{A}}\varphi) \\ &= \int_M (F_{\tilde{A}}, D_{\tilde{A}}\varphi), \end{aligned} \tag{2.8}$$

since $F_{\tilde{A}} = 0$ in $\mathcal{O}_\rho(M) \setminus \overline{M}$. On the other hand, since $D^*_{\tilde{A}} F_{\tilde{A}} = D^*_{\tilde{A}} F_A = 0$ in M and all components of $F_{\tilde{A}}$ vanish on ∂M, by integration by parts, we have

$$\int_M (F_{\tilde{A}}, D_{\tilde{A}}\varphi) = 0. \tag{2.9}$$

Combining (2.8) and (2.9), we complete the proof. □

We continue the proof of Theorem 2.1 (2).

By the regularity theory for weak Yang-Mills connections over 4-manifolds [15], there exists a gauge $g \in L^2_2(\mathcal{O}_\rho(M); G)$ such that $\hat{A} := g^*A \in C^\infty$. Since $\hat{A}$ is a Yang-Mills connection in $\mathcal{O}_\rho(M)$, and by the Bianchi identity, it satisfies $(D_{\hat{A}} D^*_{\hat{A}} + D^*_{\hat{A}} D_{\hat{A}}) F_{\hat{A}} = 0$.

On the other hand, since $F_{\tilde{A}} = 0$ in $\mathcal{O}_\rho(M) \setminus \overline{M}$, by the unique continuation theorem applied to the 2nd order elliptic partial differential operator $D_{\hat{A}} D^*_{\hat{A}} + D^*_{\hat{A}} D_{\hat{A}}$ [1], we have $F_{\tilde{A}} = 0$ in $\mathcal{O}_\rho(M)$. Thus $F_A = 0$ in M and A is a flat connection. This completes the proof of Theorem 2.1 (2).

Next we prove Theorem 2.2.

By (2.5) and the Neumann condition $i^*(*F_A) = 0$ on ∂M, we have

$$(n-4) \int_M |F_A|^2 \, dx - \int_{\partial M} \langle x, \nu \rangle |F_A|^2 = 0. \tag{2.10}$$

$n = 4$, $\langle x, \nu \rangle > 0$ on ∂M and (2.10) imply $F_A = 0$ on ∂M. The same argument in the case of Theorem 2.1 (2) implies $F_A = 0$ in this case. This completes the proof. □

3. Examples

In this section, we give an example of a principal $SU(2)$-bundle $P \to M$ such that the Dirichlet problem

$$\text{(D)} \qquad \begin{cases} D_A^* F_A & = 0 \quad \text{in} \quad M, \\ i^* A & \sim 0 \quad \text{on} \quad \partial M \end{cases}$$

has a non-flat solution. We also construct an example of a non-flat Yang-Mills connection for the Neumann problem when $\eta(P) \in H^2(M; \pi_1(G))$ vanishes. Our example is constructed for $M =$ cylindrical domain in dimension 4 or, by conformal invariance, for $M =$ annular domain $\subset \mathbb{R}^4$. The construction is based on the work of Parker [9]. See also [5], [16].

Let $M = S^3 \times [0,1]$. Identify S^3 with $SU(2)$. With this identification, $SU(2)$ acts S^3 from right and left. This action extends, in the obvious way, to an action on the trivial $SU(2)$ bundle over M. It is not hard to show (see [9] for details) that any biinvariant connection on M is gauge equivalent to one of the form

$$A = \left(\frac{x(t)+1}{2} \right) \sum e_i \otimes e^i, \tag{3.1}$$

where $\{e^i\}$ and $\{e_i\}$ are dual left-invariant bases of T^*S^3 and TS^3 and $x(t)$ is a real-valued function on $[0,1]$. When this connection is restricted to the sphere $S^3 \times \{t\}$, its curvature is

$$F_{ij} = \left(\frac{x(t)^2 - 1}{4} \right) [e_i, e_j]. \tag{3.2}$$

We can then express the Yang-Mills action in terms of $x(t)$ (again, see [9]):

$$\mathcal{YM}(A) = \frac{3}{4} \int_0^1 \left| \frac{dx}{dt} \right|^2 + (x^2 - 1)^2 \, dt. \tag{3.3}$$

By Palais' symmetric criticality principle, a critical point of this functional is a Yang-Mills field (see [8], [9]).

We first construct an example for the Dirichlet problem. Note that the connection (3.1) is flat on ∂M if $x(0) = -1$ and $x(1) = 1$. Thus we seek critical points of $\mathcal{Y}M$ under these boundary conditions. But these are easily found: direct methods in the calculus of variations show that the functional (3.3) has a minimizer on the Hilbert space

$$H_b^1 = \{x \in L^2([0,1]) \ : \ x' \in L^2([0,1]), x(0) = -1, x(1) = 1\}.$$

This minimizer is a smooth solution of $x'' = x(x^2 - 1)$ with $x(0) = -1$ and $x(1) = 1$.

We claim that the connection A corresponding to this x is a non-flat connection with flat boundary value. To prove this, we only need to show that A is not a flat connection. Suppose A is flat; then $\frac{dx}{dt} = 0$ and $x^2 = 1$. But these contradict the boundary conditions $x(0) = -1$ and $x(1) = 1$. Thus we complete the proof of our claim.

Next we construct an example for the Neumann problem. Notations are the same for the Dirichlet case. First, note that $\eta(P) = 0$ since $H^2(M; \pi_1(SU(2))) = 0$. Since the curvature of A is given by $F_A = dt \wedge \frac{dA}{dt} + F_A^{S^3}$, where $F_A^{S^3}$ is the curvature of the connection $A(t)$ on $S^3 \times \{t\}$, the Neumann condition is equivalent to the condition $\frac{dx}{dt}(0) = \frac{dx}{dt}(1) = 0$. Thus we need to find a solution of the equation $\frac{d^2x}{dt^2} = x(x^2 - 1)$ with $\frac{dx}{dt}(0) = \frac{dx}{dt}(1) = 0$. This has a trivial solution $x \equiv 0$. The connection A corresponding to this trivial solution is a solution for the Neumann problem and this is not flat since we have $\mathcal{YM}(A) = 3/4 \neq 0$. This completes the construction for the Neumann problem.

Remark 3.1. (1) The above construction is related to the construction of non-self-dual Yang-Mills connections over $S^3 \times S^1$ or S^4. See [5], [9] and [16].

Our above examples give the first examples of non-minimal Yang-Mills connections for the Dirichlet problem and for the Neumann problem with prescribed class $\eta(P)$ (in our case $\eta(P) = 0$), since flat connection is the only minimal solution for the Dirichlet problem with flat boundary condition (see [4]) and for the Neumann problem with prescribed class $\eta(P) = 0$.

(2) It is obvious from (3.1) that the connections constructed above are irreducible connections.

From Theorem 2.1, Theorem 2.2 and these examples, we can conclude that the existence of non-flat Yang-Mills connections for the Dirichlet and Neumann problems depends on the geometry of M.

It would be of interest to investigate the effect of the geometry (or topology) of M for the existence of the solutions to boundary value problems of Yang-Mills connections. Such relations are established for the existence problem of (anti-)self-dual connections over closed 4-manifolds, see [12] and [13]. For similar problems related to Yamabe equation and other semi-linear elliptic equations, see [2] and [3].

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