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GROTHENDIECK OPERATORS ON TENSOR PRODUCTS

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ABSTRACT. We prove that for Banach spaces E, F, G, H and operators $T \in \mathcal{L}(E, G)$, $S \in \mathcal{L}(F, H)$ the tensor product $T \otimes S : E \otimes_\varepsilon F \to G \otimes_\varepsilon H$ is a Grothendieck operator, provided T is a Grothendieck operator and S is compact.

1. INTRODUCTION

J. Diestel and B. Faires proved in '76 that for Banach spaces E, F, G, H, for $T \in \mathcal{A}(E, G)$ and compact $S \in \mathcal{L}(F, H)$ the tensor product of T and S defined as $T \otimes S : E \otimes_\varepsilon F \to G \otimes_\varepsilon H$ belongs again to the operator ideal $\mathcal{A}$, provided $\mathcal{A}$ is closed and injective [DF]. For the ideal of weakly compact operators E. Saksman and H. O. Tylli [ST] have obtained similar results for both the projective and injective tensor product.

The mentioned results open a natural, and interesting in itself, question on stability of non-injective operator ideals with respect to injective tensor products. We solve this problem for the non-injective, closed ideal of Grothendieck operators. We are interested in exactly that ideal because the corresponding problem of tensor stability turns out to be closely related to the question of existence of complemented copies of c_0 in injective tensor products even for Fréchet spaces. In fact, it was shown [R, p. 98] that for a large class of Banach spaces E (containing all $E = C(K)$) we have that E is a Grothendieck space (that is, weak* and weak sequential convergence coincide on equicontinuous subsets) if and only if E contains no complemented copy of c_0.

On the other hand, by a surprising result of Freniche [Fr1] (compare [C]), each completed injective tensor product $E \otimes_\varepsilon F$ of a Fréchet space E containing a copy of c_0 and a Fréchet space F satisfying the Josefson-Nissenzweig type theorem (that is, weak* and strong convergence do not coincide for sequences in the dual) contains always a complemented copy of c_0. A fortiori, such a tensor product cannot be a Grothendieck space. All infinite dimensional Banach spaces satisfy the Josefson-Nissenzweig theorem and for Fréchet spaces it was proved by Bonet, Lindström and Valdivia [BLV] that this property exactly characterizes the non-Montel spaces.

This development leads to two natural questions: Let E be a Fréchet space and F a Fréchet-Montel space. When exactly does $E \otimes_\varepsilon F$ contain a complemented copy of c_0 and when exactly is it a Grothendieck space? Both problems can also be interpreted in terms of tensor stability.

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In case of $E = C(K)$ it follows immediately from results of Freniche [Fr2] (compare [DL, Cor. 3.7]) that $C(K, F)$ is a Grothendieck space if and only if $C(K)$ is a Grothendieck space which automatically implies that $C(K, F)$ contains a complemented copy of c_0 if and only if $C(K)$ contains a complemented copy of c_0.

For general injective tensor products, the known results are contained in [DL, Th. 2.3, Th. 3.6]:

(i) if F has the approximation property, then $E \otimes_\varepsilon F$ contains a complemented copy of c_0 if and only if E contains a complemented copy of c_0.
(ii) if either F or E'' has the approximation property, then $E \otimes_\varepsilon F$ is a Grothendieck space if and only if E is a Grothendieck space.

Using our injective tensor stability result for the ideal of Grothendieck operators we are able to remove the approximation property assumption from the second result when E is a Banach space and F is a Schwartz space.

Let us now fix some notations and definitions. E, F, G, H are Banach spaces. $B(E)$ stands for the unit ball, while E^* denotes the topological dual. By an operator T from E into F we mean a bounded linear map. Let us call an operator $T \in \mathcal{L}(E, F)$ *approximable* if there exists a sequence of finite rank operators $(v_n) \subset \mathcal{L}(E, F)$ such that $\|T - v_n\| \overset{n\to\infty}{\to} 0$ (cf. [Jh]). We refer to [Pi], [DU] and [DFl] for background information on operator ideals, measure theory and tensor products, respectively.

Definition 1.1. Let E, F be Banach spaces. An operator $T \in \mathcal{L}(E, F)$ is called a Grothendieck operator if every w^*-null sequence (y_n^*) is mapped by the adjoint T^* into a weak null sequence $(T^*(y_n^*)) \subset E^*$.

The ideal of Grothendieck operators $\mathcal{GR}(E, F)$ is not injective, since the inclusion map $\iota : c_0 \hookrightarrow \ell_\infty$ is Grothendieck (note that ℓ_∞ is a Grothendieck space, since w^*-null sequences are weakly null in the dual of ℓ_∞). But the identity $id : c_0 \to c_0$ is *not* Grothendieck, since otherwise it would be weakly compact.

Definition 1.2. A subset $K \subset E$ is called a Grothendieck set if for all $T \in \mathcal{L}(E, c_0)$ the set $T(K)$ is relatively weakly compact in c_0.

We have immediately the following result (cf. [DU, p. 179]).

Lemma 1.3. *Let E, F be Banach spaces, $T \in \mathcal{L}(E, F)$. Then the following conditions are equivalent*:

(a) $T \in \mathcal{GR}(E, F)$,
(b) $\forall S \in \mathcal{L}(F, c_0) : S \circ T$ *is weakly compact,*
(c) $\forall A \subset E$ *bounded*: $T(A)$ *is a Grothendieck set.*

It follows that the ideal $\mathcal{GR}$ is surjective and closed.

2. Main results

Let $\mathcal{A}$ be a closed operator ideal and α be a tensor norm. We define the class $\mathcal{A}_\alpha$ of all operators $S : E \to F$ such that for any pair of Banach spaces E_1, F_1 and any operator $T \in \mathcal{A}(E_1, F_1)$ the map $T \otimes S : E_1 \otimes_\alpha E \to F_1 \otimes_\alpha F$ belongs to $\mathcal{A}$ as well. J. Diestel and B. Faires (see [DF, Th. 1 and Th. 2]) proved that $\mathcal{A}_\alpha$ is always a closed operator ideal which is injective whenever $\mathcal{A}$ and α are injective. Analogously, it is easily seen that if $\mathcal{A}$ is surjective and α is projective, then $\mathcal{A}_\alpha$ is surjective. Thus we obtain immediately:

Proposition 2.1. *Let E, F, G, H be Banach spaces, $\mathcal{A}$ be a closed operator ideal, α be a tensor norm and $T \in \mathcal{A}(E, G)$.*

(a) *If $S \in \mathcal{L}(F, H)$ is approximable, then $T \otimes S : E \otimes_\alpha F \to G \otimes_\alpha H$ is again in $\mathcal{A}$.*
(b) *If α and $\mathcal{A}$ are injective, then* (a) *holds even for compact S.*
(c) *If α is projective and $\mathcal{A}$ is surjective, then* (a) *holds even for compact S.*

Proof. The ideal of approximable operators is the smallest closed operator ideal. Similarly, the ideal of compact operators is the smallest surjective (injective) closed operator ideal. □

Since $\mathcal{GR}$ is surjective, we can state the following result.

Corollary 2.2. *Let E, F, G, H be Banach spaces. Then $T \otimes S : E \otimes_\pi F \to G \otimes_\pi H$ belongs to $\mathcal{GR}$ whenever T is Grothendieck and S is compact.*

Remark 2.3. If we apply Theorem 2.2 and Remark 2.9 of [S] the following criterion of weak compactness in the dual space $(E \otimes_\pi F)^* = \mathcal{L}(E, F^*)$ can be obtained: Let $(T_n) \subset \mathcal{L}(E, F^*)$ be a bounded sequence. Then $T_n \overset{n\to\infty}{\to} 0$ weakly if and only if $\{(\langle T_n(x), y^{**}\rangle)_{n\in\mathbb{N}}; x \in B(E), y^{**} \in B(F^{**})\} \subset c_0$ is relatively weakly compact. This result or Theorem 1 in [K] can be used to give a direct proof of the above corollary. It also follows from the proof that $E \otimes_\pi F$ is a Grothendieck space if E is a Grothendieck space, F is reflexive and every operator from E into F^* is compact.

At this stage we mention that from P. Enflo's famous example [E] it is an easy consequence that there is a Banach space E for which there is a non-approximable but compact operator from E into itself. In [A] F. A. Alexander obtained a similar result for a closed subspace E of l^p when $2 < p < \infty$.

The ideal of Grothendieck operators is not injective. Thus our main aim is to improve 2.1 in that case and to obtain injective tensor stability with compact operators. First we reduce the problem to reflexive F and H.

Lemma 2.4. *Let E, F, G, H be Banach spaces, $T \in \mathcal{L}(E, G)$ and $S \in \mathcal{L}(F, H)$ is compact. Then there exist reflexive Banach spaces G_1, H_1 and operators $S_1 \in \mathcal{L}(E, G_1), S_2 \in \mathcal{L}(G_1, H_1), S_2$ compact, $S_3 \in \mathcal{L}(H_1, H)$, such that*

$$T \otimes S = (id_G \otimes S_3) \circ (T \otimes S_2) \circ (id_E \otimes S_1).$$

Proof. Every compact $S \in \mathcal{L}(F, H)$ admits a compact factorization through a reflexive Banach space according to a result of T. Figel and W. Johnson [Fi, Jo] (see also [DU, p. 260]). Then the proof is straightforward. □

We write $Bo(B(E^*))$ for the Borel sets on $B(E^*)$ w.r.t. the w^*-topology. If $m : Bo(B(X^*)) \to F$ is a vector measure of bounded variation, then $\|m\|$ is the variation norm. Let us recall the representation of the dual of $E \otimes_\varepsilon F$, provided F is reflexive.

Definition and Lemma 2.5. *Let E, F be Banach spaces with F reflexive. $\mathcal{PI}(E, F) \subset \mathcal{L}(E, F)$ are the Pietsch-integral operators, defined as*:

$$T \in \mathcal{PI}(E, F) \Leftrightarrow \exists m : Bo(B(E^*)) \to F \textit{ vector measure of bounded variation}$$

$$\forall x \in E : T(x) = \int_{B(E^*)} x(x^*)\, dm(x^*).$$

We equip $\mathcal{PI}(E,F)$ with the integral norm, i.e. $\|T\|_{PI} := \inf\{\|m\|; \forall x \in E : T(x) = \int_{B(E^*)} x(x^*)\, dm(x^*)\}$ (*cf.* [DFl, p. 522]). *Then $\mathcal{PI}(E,F^*)$ is isometric isomorphic to $(E \otimes_\varepsilon F)^*$ by the identity* $T(x \otimes y) = \langle y, \int_{B(E^*)} x(x^*)\, dm(x^*)\rangle$ (*cf.* [DFl, p. 522]).

Notation. Let E, F be Banach spaces with F reflexive, and let $(z_n^*) \subset B((E \otimes_\varepsilon F)^*)$. According to 2.5 for all $n \in \mathbb{N}$ we choose a vector measure $m_n := m_n(z_n^*) : Bo(B(X^*)) \to F$ of bounded variation, satisfying:

i) $\lim_{n\to\infty} |\, \|m_n\| - \|z_n^*\|\, | = 0$,
ii) $\forall e \in E, f \in F, n \in \mathbb{N} : z_n^*(e \otimes f) = \langle f, \int_{B(E^*)} e(e^*)\, dm_n(e^*)\rangle$.

Furthermore we define a finite scalar-valued measure $\mu(\cdot) := \mu((z_n^*))(\cdot) := \sum_{n\in\mathbb{N}} 2^{-n} var(m_n(z_n^*), \cdot)$, where var denotes the variation of the corresponding measure. Then m_n is absolutely continuous w.r.t. μ for all $n \in \mathbb{N}$.

We write

$$L_{H_1} := \left\{ f \in L_1(\mu, H_1^*) : \forall e \in E : \int_{B(E^*)} e(e^*) f(e^*)\, d\mu(e^*) = 0 \right\}$$

for a subspace $H_1 \subset H$. For a Banach space H we denote by $q_H : L_1(\mu, H^*) \to L_1(\mu, H^*)/L_H$ the canonical quotient map. If $\mu = \mu(z_n^*)$ and $H_1 \subset H$, then let $r_{H_1} : L_1(\mu, H_1^*)/L_{H_1} \to (E \otimes_\varepsilon H_1)^*$ be the canonical injection.

Theorem 2.6. *Let E, F, G, H be Banach spaces. If $T \in \mathcal{GR}(E,G)$ and $S \in \mathcal{L}(F,H)$ is compact, then $T \otimes S : E \otimes_\varepsilon F \to G \otimes_\varepsilon H$ is Grothendieck.*

Proof. By Lemma 2.4 we assume that F, H are reflexive. W.l.o.g. let $\|T\|, \|S\| \le 1$. Let $(x_n^*) \subset B((G \otimes_\varepsilon H)^*)$ be w^*-converging to 0. First we consider the map $T \otimes id : E \otimes_\varepsilon H \to G \otimes_\varepsilon H$. For a finite dimensional subspace $H_1 \subset H$, according to 2.1, we have that for $(z_n^*) := (T \otimes id)^*(x_n^*)$ the restriction

$$((z_n^*)|_{E \otimes H_1}) \xrightarrow{n\to\infty} 0 \text{ weakly.} \tag{1}$$

Consider now $id \otimes S : E \otimes_\varepsilon F \to E \otimes_\varepsilon H$. For $n \in \mathbb{N}$ let $h_n \in L_1(\mu, H^*)$ be the density of m_n with respect to $\mu := \mu(z_n^*)$. To show that $(id \otimes S)^*((z_n^*))$ is weakly null (then we are done), we have to show that for all $\overline{g} \in B((E \otimes_\varepsilon F)^{**})$:

$$\begin{aligned} \langle \overline{g}(id \otimes S)^*(z_n^*)\rangle &= \int_{B(E^*)} \langle q_F^* \circ r_F^*(\overline{g}), S^* \circ h_n\rangle\, d\mu \\ &= \int_{B(E^*)} \langle S \circ q_F^* \circ r_F^*(\overline{g}), h_n\rangle\, d\mu \to 0. \end{aligned} \tag{2}$$

The following arguments are devoted to proving this. We define $g := S \circ q_F^* \circ r_F^*(\overline{g})$. Then $g \in L_\infty(\mu, H)$, since H is reflexive. Further, g has relatively compact range, since S is compact. We assume that (2) is not true. Then

$$\exists (h_{n_k}) \text{ subsequence } \exists \varepsilon > 0 : \inf_{k\in\mathbb{N}} \left| \int_{B(E^*)} \langle g, h_{n_k}\rangle\, d\mu \right| > \varepsilon. \tag{3}$$

For the sake of simplicity assume that (h_n) satisfies (3). Since g has relatively compact range, there is an increasing sequence of finite $Bo(B(E^*))$-partitions (π_k), such that

$$\|\mathbb{E}_{\pi_k}(g) - g\|_\infty \to 0 \text{ and } \forall n \in \mathbb{N} : \|\mathbb{E}_{\pi_k}(h_n) - h_n\|_1 \to 0. \tag{4}$$

We define $\Sigma_0 := \sigma(\bigcup_{k\in\mathbb{N}} \pi_k)$. Since H is reflexive, for all $k \in \mathbb{N}$ the sequence $(\mathbb{E}_{\pi_k}(h_n))$ is relatively weakly compact in $L_1(\mu, H^*)$. Hence, for all $k \in \mathbb{N}$ there is an $m_k \in L_1(\mu, H^*)$, so that $\mathbb{E}_{\pi_k}(h_n) \to m_k$ weakly (for at least going to a subsequence by a diagonalization argument). (π_k) is increasing, thus, (m_k, π_k) is a bounded martingale, which converges in the $L_1(\mu, H^*)$-norm to an $M \in L_1(\mu, H^*)$ (note that the (h_n) are bounded and H^* has the RNP as a reflexive space). We show now that for all $G \in L_\infty(\mu|_{\sum_0}, H)$ with relatively compact range:

$$\text{(5)} \qquad \begin{aligned} &\exists \text{ subsequence } (h_{n_j}) \text{ such that} \\ &\qquad \forall\delta > 0\ \exists N \in \mathbb{N}\ \forall j \geq N : |\langle G, M\rangle - \langle G, h_{n_j}\rangle| < \delta. \end{aligned}$$

Proof of (5). G has relatively compact range, thus there exists an increasing sequence of finite $Bo(B(E^*), w^*)$-partitions $(\pi_k(G))$ such that $\pi_k \subset \sigma(\pi_k(G))$ for $k \in \mathbb{N}$ and $\|G - \mathbb{E}_{\pi_k(G)}(G)\| \to 0$ (cf. [DU, p. 67, Lemma 1]). Let (h_{n_j}) be a subsequence with $\mathbb{E}_{\pi_k(G)}(h_{n_j}) \to m_k(G) \in L_1(\mu, H^*)$ weakly for all $k \in \mathbb{N}$ (subsequence argument like above). Thus, since $\pi_k \subset \sigma(\pi_k(G))$, it follows $\mathbb{E}_{\pi_k}(\mathbb{E}_{\pi_k(G)}(h_{n_j})) \to m_k$ (m_k as above). Again $(m_k(G))$ is a martingale. Hence, as above, there exists an $M(G) \in L_1(\mu, H^*)$, such that $m_k(G) \to M(G)$. We have

$$M = M(G).$$

Let $A \in \bigcup_{k\in\mathbb{N}} \pi_k$. Then

$$\int_A M(G)\,d\mu - \int_A M\,d\mu = \lim_{k\to\infty} \int_A m_k(G) - m_k\,d\mu = 0,$$

since $(m_k(G))$ and (m_k) are martingales and there is a $k_0 \in \mathbb{N}$, such that $A \in \pi_{k_0} \subset \sigma(\pi_k(G))$. Hence, for all $B \in \Sigma_0 : \int_B M(G)\,d\mu = \int_B M\,d\mu$. Thus to prove (5) we first note that it suffices to demonstrate (5) for all $G = \mathbb{E}_{\pi_k(G)}G$ ($k \in \mathbb{N}$), since G has relatively compact range and M, h_n, $n \in \mathbb{N}$, are measurable w.r.t. Σ_0. But then (5) follows by:

$$\begin{aligned} \langle \mathbb{E}_{\pi_k(G)}G, M\rangle - \langle \mathbb{E}_{\pi_k(G)}G, h_{n_j}\rangle &= \langle \mathbb{E}_{\pi_k(G)}G, m_k(G)\rangle - \langle \mathbb{E}_{\pi_k(G)}G, \mathbb{E}_{\pi_k(G)}h_{n_j}\rangle \\ &= \langle \mathbb{E}_{\pi_k(G)}G, m_k(G) - \mathbb{E}_{\pi_k(G)}h_{n_j}\rangle. \end{aligned}$$

For a finite dimensional subspace $H_1 \subset H$ we consider the canonical restriction operator $\mathrm{rest}_{H_1} : L_1(\mu, H^*)/L_{H^*} \to L_1(\mu, H_1^*)/L_{H_1^*}$. Then according to (1) we have:

$$\text{(6)} \qquad \forall z^{**} \in (E \otimes_\varepsilon H_1)^{**} : \int_{B(E^*)} \langle q_H^* \circ \mathrm{rest}_{H_1}^* \circ r_{H_1}^*(z^{**}), h_n\rangle\,d\mu \to 0.$$

Since $q_H^* \circ \mathrm{rest}_{H_1}^* \circ r_{H_1}^*(z^{**})$ has relatively compact range for all $z^{**} \in (E \otimes_\varepsilon H_1)^{**}$ (H_1 is finite dimensional), (5) and (6) imply:

$$\forall z^{**} \in (E \otimes_\varepsilon H_1)^{**} : \langle z^{**}, r_{H_1} \circ \mathrm{rest}_{H_1} \circ q_H(M)\rangle = 0.$$

Note that since $M \in L_1(\mu|_{\Sigma_0}, H^*)$ we may assume that $q_H^* \circ \mathrm{rest}_{H_1}^* \circ r_{H_1}^*(z^{**})$ is measurable w.r.t. Σ_0. Thus

$$\text{(7)} \qquad \forall H_1 \subset H \text{ finite dimensional } r_{H_1} \circ \mathrm{rest}_{H_1} \circ q_H(M) = 0.$$

But (7) implies

$$\text{(8)} \qquad r_H \circ q_H(M) = 0.$$

Hence we compute

$$\begin{aligned}0 &\overset{(8)}{=} \langle (id\otimes S)^{**}(\overline{g}), r_H\circ q_H(M)\rangle = \langle \overline{g}, (id\otimes S)^{*}(r_H\circ q_H(M))\rangle\\ &= \int_{B(E^*)} \langle q_F^*\circ r_F^*(\overline{g}), S^*\circ M\rangle\, d\mu = \int_{B(E^*)} \langle S\circ q_F^*\circ r_F^*(\overline{g}), M\rangle\, d\mu\\ &= \int_{B(E^*)} \langle g, M\rangle\, d\mu.\end{aligned}$$

Thus, this contradicts (3) and (5), and we are done. □

We shall now apply Theorem 2.6 and an operator ideal approach to obtain the announced result avoiding the assumption of the approximation property.

Corollary 2.7. *Let E be a Schwartz space and F a Banach space with the Grothendieck property. Then $E\otimes_\varepsilon F$ is a Grothendieck space.*

Proof. By a well-known representation of ε-tensor products as projective limits $E\otimes_\varepsilon F = proj_{U\in\mathcal{U}_E} E_U\otimes_\varepsilon F$, where $\mathcal{U}_E$ is a 0-basis in E. A locally convex space X is Grothendieck if and only if every continuous linear map from X into c_0 maps bounded sets into relatively weakly compact ones. Now, each continuous linear map $T: E\otimes_\varepsilon F\to c_0$ factorizes through $E_U\otimes_\varepsilon F$ for some $U\in\mathcal{U}_E$. Since E is a Schwartz space we can apply our main theorem so that for every $U\in\mathcal{U}_E$ there exists a $V\in\mathcal{U}_E$ contained in U such that the canonical map $E_V\otimes_\varepsilon F\to E_U\otimes_\varepsilon F$ is a Grothendieck operator. The result follows immediately. □

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