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EXACT MULTIPLICITY FOR SOME NONLINEAR ELLIPTIC EQUATIONS IN BALLS

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ABSTRACT. We present the exact multiplicity results for some nonlinear elliptic equations in balls of radius R. We prove that there is a critical value R_0 such that, for $R < R_0$, the equation has no solution; when $R = R_0$, it has exactly one solution; when $R > R_0$, it has exactly two solutions. Our main tool is the bifurcation theorem due to Crandall and Rabinowitz.

1. INTRODUCTION

In this work, we shall study the exact multiplicity for the elliptic equation

$$\begin{cases} \Delta u + \lambda f(u) = 0 \text{ in } B_R, \\ u > 0, \text{ in } B_R, \ u = 0 \text{ on } \partial B_R, \end{cases} \tag{1.1}$$

where $B_R := \{x \in R^n : |x| < R\}, n \geq 1, \lambda > 0$ is a parameter and $f \in C^2[0, b]$ satisfies the following hypotheses:

(f1) $f(0) = f(a) = f(b) = 0, f(y) < 0$ on $(0, a), f(y) > 0$ on (a, b) and $\int_0^b f(s)ds > 0$.

(f2) $f'(0) < 0, f'(b) < 0$.

(f3) $(y - \phi)f'(y) < f(y)$ on (ϕ, b) where $\phi \in (a, b)$ satisfies $\int_0^\phi f(s)ds = 0$.

(f4) $f''(u)$ changes sign exactly once when $u > 0$ and $f''(u)$ has exactly one positive root.

Our phototype example is $f(u) = u(u - a)(1 - u)$ where $0 < a < \frac{1}{2}$. It is easy to see when a is not too small that $f(u)$ satisfies (f1)-(f4). Note that this example arises in the study of population genetics, see [1].

Problem (1.1) has been studied extensively by many authors. Gardner and Peletier [11] obtained the exact number of solutions of problem (1.1) under the assumptions (f1), (f2), (f3) and λ sufficiently large, while Dancer [4] obtained the same result for domains with symmetry and λ large. On the other hand, when $n = 1$, exact multiplicity results are proved by J. Smoller and A. Wasserman [15] and S.-H. Wang [16] using phase-plane analysis, and by Korman, Yi Li, and Ouyang [8], [9] using bifurcation analysis.

In this paper, we present the exact number of solutions for *all* $\lambda > 0$ and *all* $n \geq 1$. Namely, we shall prove the following

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Theorem 1.1. *Under the conditions (f1), (f2) , (f3) and (f4), there is a critical value $\lambda_0 > 0$ such that for all $\lambda < \lambda_0$ the problem* (1.1) *has no nontrivial solutions, it has exactly one nontrivial solution for $\lambda = \lambda_0$, and exactly two nontrivial solutions for $\lambda > \lambda_0$. Moreover, all solutions lie on a single solution curve, which for $\lambda > \lambda_0$ has two branches denoted by $u^-(x,\lambda) < u^+(x,\lambda)$ with $u^+(x,\lambda)$ strictly monotone increasing in $\lambda, u^-(x,\lambda)$ strictly monotone decreasing in λ and $\lim_{\lambda\to\infty} u^+(x,\lambda) = b, \lim_{\lambda\to\infty} u^-(x,\lambda) = 0$ for $x \in B_R$ while $u^-(0,\lambda) > \phi$ for all $\lambda > \lambda_0$.*

As in [8] and [9], our main tool is the following bifurcation theorem due to Crandall-Rabinowitz [2].

Theorem A. *Let X and Y be Banach spaces. Let $(\overline{\lambda}, \overline{x}) \in R \times X$ and let F be a continuously differentiable mapping of an open neighborhood of $(\overline{\lambda}, \overline{x})$ into Y. Let the null space $N(F_x(\overline{\lambda}, \overline{x})) = span\{x_0\}$ be one-dimensional, and let* $\operatorname{codim}(R(F_x(\overline{\lambda}, \overline{x}))) = 1$. *Let $F_\lambda(\overline{\lambda}, \overline{x}) \notin R(F_x(\overline{\lambda}, \overline{x}))$. If Z is a complement of span x_0 in X, then the solutions of $F(\lambda, x) = F(\overline{\lambda}, \overline{x})$ near $(\overline{\lambda}, \overline{x})$ form a curve $(\lambda(s), x(s)) = (\overline{\lambda} + \tau(s), \overline{x} + sx_0 + z(s))$ where $s \longrightarrow (\tau(s), z(s)) \in R \times Z$ is a continously differentiable function near $s = 0$ and $\tau(0) = \tau'(0) = z(0) = z'(0) = 0$.*

While our idea is in the same line with [8], there are some difficulties in higher-dimensional case. We employ various identities to overcome them.

The organization of this paper is the following. In Section 2 we present all technical lemmas. We finish our proof of Theorem 1.1 in Section 3.

2. Technical lemmas

Without loss of generality, we assume that $R = 1$. Let u be a solution of (1.1). By the maximum principle and regularity theory, $u > 0$ and is classical. By the result of Gidas, Ni and Nirenberg [5], u is radial. Moreover $u'(r) < 0$ for $r \neq 0$. Thus $u(r)$ satisfies

$$(r^{n-1}u')' + \lambda r^{n-1} f(u) = 0, u'(0) = 0, u(1) = 0. \tag{2.1}$$

The corresponding linearized problem is

$$(r^{n-1}w')' + \lambda r^{n-1} f'(u)w = 0, w'(0) = 0, w(1) = 0, w(0) = 1. \tag{2.2}$$

We set the following notation:

$$v = ru_r, v_\beta = ru_r + \beta u, \gamma = 1 + \frac{2}{\beta}, \beta > 0, \tag{2.3}$$

$$g(r) = (r^{n-1}w')' r^{n-1}u' - (r^{n-1}u')' r^{n-1}w', \tag{2.4}$$

$$L = \Delta + \lambda f'(u). \tag{2.5}$$

We then have the following simple calculations:

$$Lv = -2f(u), \tag{2.6}$$

$$Lv_\beta = \beta(uf'(u) - \gamma f(u)), \tag{2.7}$$

$$\left(\frac{g}{r^{n-1}}\right)' = -f''(u) r^{n-1} u_r^2 w. \tag{2.8}$$

We first establish the following

Lemma 2.1. *Under the assumptions (f1), (f2) and (f3), let w be a solution of* (2.2). *Then*

$$w(r) > 0 \text{ for } r \in [0, 1). \tag{2.9}$$

Proof. We shall closely follow a technique in Dancer [4] and Kwong and Zhang [10].

Let ξ be the first zero of w. Let $\eta > 0$ be such that $u(\eta) = \phi$. Notice that by $(f3), (y-\phi)f'(y) < f(y)$ on (ϕ, b). Since $u(r) > \phi$ for $r \in [0, \eta)$ and

$$-(r^{n-1}(u-\phi)')' = \lambda f(u) > \lambda f'(u)(u-\phi) \text{ on } [0,\eta),$$

a standard Sturm comparison theorem ensures that $w(r) > 0$ on $[0, \eta]$. Hence $\xi > \eta$.

So $u(\xi) < \phi$. Consider w on $[\xi - \epsilon, 1)$ such that $u(\xi - \epsilon) < \phi$ (by Lemmas 15 and 16 in [10]); we have that $w(r)$ has no zero on $[\xi - \epsilon, 1)$—a contradiction with $w(\xi) = 0$. □

We now prove some useful lemmas.

Lemma 2.2. *We have*

$$\int_{B_1} f'' u_r^2 w = 0. \tag{2.10}$$

Proof. In fact, by (2.8), we have

$$\int_0^1 r^{n-1} f''(u) u_r^2 w = -\int_0^1 \left(\frac{g(r)}{r^{n-1}}\right)' dr = 0,$$

since $\frac{g(r)}{r^{n-1}} = -r^{n-1} f'(u) w u' + r^{n-1} f(u) w'$. □

Lemma 2.3. *We have*

$$\int_{B_1} f(u) w > 0. \tag{2.11}$$

Proof.

$$-2\int_{B_1} f(u)w = \int_{B_1} (Lv)w = \int_{\partial B_1} w\frac{\partial v}{\partial \nu} - v\frac{\partial w}{\partial \nu} = -\int_{\partial B_1} v\frac{\partial w}{\partial \nu} < 0.$$

Hence $\int_{B_1} f(u)w > 0$. □

Lemma 2.4. *Let $u(x)$ be any turning point of* (2.1), *i.e.* (2.2) *has a nontrivial solution. Then*

$$f''(u(0)) < 0. \tag{2.12}$$

Proof. Suppose not, i.e. $f''(u(0)) \geq 0$. Since $f''(u)$ changes sign exactly once in $(0, b)$, we have $f''(u(r)) > 0$ for $r \in (0, 1)$. By (2.8), $(\frac{g}{r^{n-1}})' \leq 0$. Hence $\frac{g}{r^{n-1}}$ is decreasing. Since $\frac{g}{r^{n-1}}(0) = \frac{g}{r^{n-1}}(1) = 0$, we reach a contradiction. □

Lemma 2.5. *$-u_r$ and w intersect exactly once in* $(0, 1)$.

Proof. Suppose on the contrary that $-u_r$ and w intersect more than once. Since $-u_r(0) < w(0)$ and $-u_r(1) > w(1)$, we can find $0 < r_1 < r_2 < 1$ such that $-u_r > w$ on (r_1, r_2) and $-u_r(r_1) = w(r_1), -u_r(r_2) = w(r_2)$. Note that $-u_r$ satisfies

$$\Delta(-u_r) + \lambda f'(u)(-u_r) = -\frac{u_r}{r^2}. \tag{2.13}$$

We obtain

$$
\begin{aligned}
&\int_{B_{r_2}\backslash B_{r_1}} (-\frac{u_r}{r^2})w \\
&= \int_{B_{r_2}\backslash B_{r_1}} [\Delta(-u_r - w) + \lambda f'(u)(-u_r - w)]w \\
&= \int_{\partial(B_{r_2}\backslash B_{r_1})} w\frac{\partial(-u_r - w)}{\partial \nu}.
\end{aligned} \tag{2.14}
$$

The right-hand side of (2.14) is negative, while the left-hand side is positive. We reach a contradiction. □

Finally, we recall the following result due to Gardner and Peletier [11] and Dancer [4].

Lemma 2.6. *Under the assumptions (f1), (f2) and (f3), for λ sufficiently large, there exist exactly two solutions with $0 < \|u\|_\infty < b$.*

3. Proof of Theorem 1.1

Proof. First of all, for λ sufficiently small, problem (1.1) has no positive solutions. Indeed, under our assumptions, there is a constant $\gamma > 0$ such that $f(u) \le \gamma u$ for all $u > 0$. Then

$$
\lambda \int_{B_1} u^2 \ge \lambda \int_{B_1} f(u)u = \int_{B_1} |\nabla u|^2 \ge \underline{\lambda} \int_{B_1} u^2,
$$

and the claim follows.

Next by results of Gardner and Peletier [11], Clement and Sweers [3], and Dancer [4], for λ sufficiently large, there exist exactly two solutions $0 < \underline{u}(x,\lambda) < \overline{u}(x,\lambda),\ x \in B_1$. In fact, $\lim_{\lambda\to\infty} \underline{u}(x,\lambda) = 0, x \in \overline{B}_1\backslash\{0\}$, and $\lim_{\lambda\to\infty} \overline{u}(x,\lambda) = b, x \in B_1$.

We begin with $\underline{u}(x,\lambda)$. We continue $\underline{u}(x,\lambda)$ for decreasing λ. If the corresponding linearized equation (2.2) has only the trivial solution $w = 0$, then by the implicit function theorem we can solve (1.1) for $\lambda < \lambda_1$ and λ close to λ_1, obtaining a continuous curve in λ of solutions $u(x,\lambda)$. As in Korman and Ouyang [8], this process of decreasing λ cannot be continued indefinitely, since for sufficiently small $\lambda > 0$, the problem (1.1) has no solution. Let λ_0 be the infimum of λ for which we can continue the curve of solutions to the left. It is easy to see that there is a solution $u(x,\lambda_0) = u_0(x)$. Clearly the linearized equation at $\lambda = \lambda_0$ and $u = u_0$ must have a nontrivial solution and by Lemma 2.1, $w(x) = w(r) > 0$ for $r \in [0,1)$.

We rewrite the equation (1.1) in the operator form

$$
F(\lambda, u) = \Delta u + \lambda f(u) = 0 \tag{3.1}
$$

where $F : R \times C_0^2(B_1) \to C(B_1(0))$. Hence

$$
F_u(\lambda, u)w = \Delta w + \lambda f'(u)w. \tag{3.2}
$$

We show next that at the critical point (λ_0, u_0), the Crandall-Rabinowitz theorem applies. Indeed, $N(F_u(\lambda_0,u_0)) = \mathit{span}\{w(r)\}$ is one dimensional and

$$
\operatorname{codim} R(F_u(\lambda_0,u_0)) = 1
$$

by the Fredholm alternative. It remains to check that $F_\lambda(\lambda_0,u_0) \notin R(F_u(\lambda_0,u_0))$.

Assuming the contrary would imply existence of $v(x) \not\equiv 0$ such that $\Delta v + \lambda f'(u_0)v = f(u_0)$ in B_1 and $v = 0$ on ∂B_1. Hence $0 = \int_{B_1} f(u_0)w$. This is a contradiction to Lemma 2.3.

Applying the Crandall-Rabinowitz theorem, we conclude that (λ_0, u_0) is a bifurcation point, near which the solutions of (1.1) form a curve $(\lambda_0+\tau(s), u_0+sw+z(s))$ with s near $s = 0$ and $\tau(0) = \tau'(0) = 0, z(0) = z'(0) = 0$.

We claim that

$$\tau''(0) > 0, \tag{3.3}$$

i.e. only "turns to the left" in the (λ, u) "plane" are possible. In fact, for $u_0(x) = u(x, \lambda_0)$,

$$\tau''(0) = -\lambda_0 \frac{\int_{B_1} f''(u)w^3}{\int_{B_1} f(u)w}. \tag{3.4}$$

For completeness, we include a proof of (3.4) here.

Differentiate (1.1) in s twice,

$$\Delta u^{ss} + \lambda f''(u)(u^s)^2 + \lambda f'(u)u^{ss} + 2\tau' f' u^s + \tau'' f(u) = 0. \tag{3.5}$$

At $s = 0, \tau'(0) = 0, u^s|_{s=0} = w(x)$, we obtain

$$\Delta u^{ss} + \lambda f'(u)u^{ss} + \lambda f''(u)w^2 + \tau''(0)f(u) = 0. \tag{3.6}$$

Multiplying (3.6) by w and integrating by parts, we obtain (3.4).

By Lemma 2.3, $\int_{B_1} f(u)w > 0$. We just need to show that

$$\int_{B_1} f''(u)w^3 < 0. \tag{3.7}$$

By Lemma 2.2, we just need to show that

$$\int_{B_1} f''(u)w^3 < \int_{B_1} f''(u)(u_r^2)w. \tag{3.8}$$

To show (3.8), we first notice that $f''(u_0(x))$ changes sign exactly once on $(0, 1)$. By Lemma 2.4, $f''(u_0(0)) < 0$ and $f''(u_0(1)) > 0$. Since $u_0(r)$ is decreasing on $(0, 1)$, the claim follows.

Let $\overline{x}$ be such that $f''(u_0(\overline{x})) = 0$. By Lemma 2.5, $-u_r$ and w intersect exactly once on $(0, 1)$. By multiplying some constants, we can assume that $-u_r$ and w intersect at $\overline{x}$. Hence we see that on the interval $(0, \overline{x}), f''(u_0) < 0, w^2 > u_r^2$; on the interval $(\overline{x}, 1), f''(u_0) > 0, w^2 < u_r^2$. Hence we have

$$f''(u)w^3 < f''(u)u_r^2 w \text{ in } B_1(0). \tag{3.9}$$

Therefore (3.8) follows.

It follows that at any critical point (λ_0, u_0) the curve of solutions turns to the "right" in the (λ, u) plane. After the curve we can continue this curve of solutions for increasing λ, using the implicit function theorem, so long as (λ, u) is not a singular point of $F(\lambda, u)$. However, there can be no critical points on the lower branch, since we know precisely the structure of solutions at any critical point, namely a turn to the right always occurs, which is impossible at the lower branch. Hence the lower branch can be continued for all $\lambda > \lambda_0$. The same is true for the upper branch and we obtain a parabolic-like curve of solutions. It remains to show that there is exactly one such curve and to prove the monotonicity properties of its branches.

We claim that the upper branch is increasing for all $\lambda > \lambda_0$. For λ close to λ_0 this follows from the Crandall-Rabinowitz theorem ($u_\lambda(x,\lambda) \simeq w(x) > 0$ for all x).

Assuming the claim to be false, denote by λ_1 the first λ where $u_\lambda > 0$ is violated, i.e. $u_\lambda(r,\lambda_1) \geq 0$ for all $r \in [0,1)$, and $u_\lambda(\overline{r},\lambda_1) = 0$ for some $\overline{r} \in (0,1)$. Since $\overline{r}$ is a point of minimum, $u_r(\overline{r},\lambda_1) = 0$ and $u_{rr}(\overline{r},\lambda_1) \geq 0$. It follows from the equation that

$$0 < u(\overline{r},\lambda_1) \leq a. \tag{3.10}$$

Differentiating (2.1) in λ, we have

$$\Delta u_\lambda + \lambda f'(u) u_\lambda + f(u) = 0. \tag{3.11}$$

By (2.6), $\frac{v}{2}$ satisfies

$$\Delta(\frac{v}{2}) + \lambda f'(u)(\frac{v}{2}) + f(u) = 0. \tag{3.12}$$

Multiplying (3.11) by $\frac{v}{2}$ and (3.12) by u_λ and subtracting, we obtain

$$\int_{\partial B_1 \setminus B_{\overline{r}_2}} \frac{v}{2}\frac{\partial u_\lambda}{\partial \nu} - u_\lambda \frac{\partial v/2}{\partial \nu} + \int_{B_1 \setminus B_{\overline{r}_2}} f(u)(\frac{v}{2} - u_\lambda) = 0. \tag{3.13}$$

Hence

$$\int_{\partial B_1 \setminus B_{\overline{r}_2}} \frac{v}{2}\frac{\partial u_\lambda}{\partial \nu} + \int_{B_1 \setminus B_{\overline{r}_2}} f(u)(\frac{v}{2} - u_\lambda) = 0. \tag{3.14}$$

Moreover,

$$\int_{\partial B_1 \setminus B_{\overline{r}_2}} \frac{v}{2}\frac{\partial u_\lambda}{\partial \nu} = \int_{\partial B_1} \frac{v}{2}\frac{\partial u_\lambda}{\partial \nu} - \int_{\partial B_{\overline{r}_2}} \frac{v}{2}\frac{\partial u_\lambda}{\partial \nu} \geq 0.$$

Hence the left-hand side of (3.14) is positive, a contradiction.

Therefore, the upper branch is increasing for all $\lambda > \lambda_0$. Moreover by Dancer [4], the upper branch tends to b as $\lambda \to \infty$.

Assume now that there is another curve of solutions, denoted by $v(x,\lambda)$. By the result of Dancer [4], $v(x,\lambda) \equiv u(x,\lambda)$ for λ large.

It is easy to see that $v(x,\lambda) \equiv u(x,\lambda)$ for $\lambda \geq \lambda_0$ where λ_0 is a turning point. Hence the upper branch of $v(x,\lambda)$ coincides with the upper branch of $u(x,\lambda)$. Similarly lower branches of v and u are the same.

Finally we prove that $u(0,\lambda)$ is decreasing on the lower branch. By the Crandall-Rabinowitz theorem, we know that $u_\lambda(r,\lambda) < 0$ for λ close to λ_0 and all $r \in [0,1)$ on the lower branch.

Let λ_1 be the first λ where $u_\lambda(r,\lambda_1) < 0$ is violated, i.e. $u_\lambda(r,\lambda_1) \leq 0$ for all $r \in [0,1)$ and $u_\lambda(\overline{r},\lambda_1) = 0$ for some $\overline{r} \in [0,1)$.

Notice that u_λ satisfies

$$\Delta u_\lambda + \lambda f'(u) u_\lambda + f(u) = 0. \tag{3.15}$$

If $\overline{r} = 0$, then since $\frac{v}{2}$ and u_λ satisfy the same equation and boundary conditions, we have $\frac{v}{2} = u_\lambda(r,\lambda_1)$.

But $u_\lambda(1,\lambda_1) = 0, \frac{v}{2}(1) < 0$, a contradiction!

Hence we may suppose $\overline{r} \neq 0$, i.e. $u_\lambda(0,\lambda_1) < 0$; then $f(u(\overline{r})) \geq 0$. So $u(\overline{r}) \geq a$. Over $B_{\overline{r}}, f(u(r)) > 0$.

Let $\overline{r}$ be the first r such that $u_\lambda(r,\lambda_1) = 0$. Then $u_\lambda(r,\lambda_1)$ satisfies

$$\begin{cases} \Delta u_\lambda + \lambda f'(u) u_\lambda + f(u) = 0, \\ u_\lambda(\overline{r},\lambda_1) = 0, u'_\lambda(\overline{r},\lambda_1) = 0. \end{cases} \tag{3.16}$$

Since $\overline{r} \neq 1, u_\lambda(\overline{r}, \lambda_1) = 0, u'_\lambda(\overline{r}, \lambda_1) = 0$. Notice that in $B_{\overline{r}}, f(u(\overline{r})) \geq 0, u(\overline{r}) \geq a$. On the other hand, by the same proof of Lemma 2.1, let w be the solution for the equation

$$(r^{n-1}w')' + \lambda r^{n-1} f'(u)w = 0, w'(0) = 0, w(0) = 1. \tag{3.17}$$

Then $w > 0$ over $(0, 1)$. We obtain

$$\begin{aligned} 0 &= \int_{B_{\overline{r}}} (\Delta u_\lambda + \lambda f'(u)u_\lambda + f(u))w - (\Delta u + \lambda f(u))u_\lambda \\ &= \int_{\partial B_{\overline{r}}(0)} w\frac{\partial u_\lambda}{\partial \nu} - u_\lambda \frac{\partial w}{\partial \nu} + \int_{B_{\overline{r}}} f(u)w \\ &= \int_{B_{\overline{r}}} f(u)w > 0, \end{aligned}$$

a contradiction. □

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Note added in proof

After the paper was accepted for publication, I was informed by Professors Korman, Yi Li and Ouyang that they obtained similar results in the case $n = 1, 2$.

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