

## DOMAINS OF FINITE TYPE AND HÖLDER CONTINUITY OF THE PERRON-BREMERMANN FUNCTION

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ABSTRACT. Let  $\Omega$  be a smoothly bounded domain in  $\mathbb{C}^n$  such that  $0 \in \partial\Omega$ . We give a bound for the type of  $\partial\Omega$  at 0 in terms of the Hölder exponent of its Perron-Bremermann function with simple boundary data. We then use this to show that a smoothly bounded domain in  $\mathbb{C}^2$  is pseudoconvex of finite type if and only if its Perron-Bremermann function corresponding to Hölder continuous boundary data is Hölder continuous.

### 1. INTRODUCTION

Let  $\Omega$  be a bounded domain in  $\mathbb{C}^n$ . A point  $\zeta \in \partial\Omega$  is called a peak point for the class of plurisubharmonic functions if there exists a function  $\rho \in PSH(\Omega) \cap C(\bar{\Omega})$  such that  $\rho(\zeta) = 0$  and  $\rho(z) < 0$  for all  $z \in \bar{\Omega}$ ,  $z \neq \zeta$ . We now let  $f \in C(\partial\Omega)$  and we denote by  $U(\Omega, f)$  the set of plurisubharmonic functions  $v$  on  $\Omega$  satisfying  $\limsup_{z \rightarrow \zeta, z \in \Omega} v(z) \leq f(\zeta)$ , for all  $\zeta \in \partial\Omega$ . The Perron-Bremermann function  $\Psi_{\Omega, f}$  for  $\Omega$  and  $f$  is defined by  $\Psi_{\Omega, f}(z) = \sup\{v(z) : v \in U(\Omega, f)\}$ ,  $z \in \Omega$  (see for example [9, p. 89]). We can extend  $\Psi_{\Omega, f}$  to  $\bar{\Omega}$  by setting  $\Psi_{\Omega, f} = f$  on  $\partial\Omega$ . So, in what follows, whenever  $\Psi_{\Omega, f}$  is referred to as a function defined on  $\bar{\Omega}$  we assume that it is extended to  $\partial\Omega$  in this way. If  $\Omega$  is regular for the Dirichlet problem for the Laplace operator it is easy to see that  $\Psi_{\Omega, f} \in U(\Omega, f)$ . A classical theorem of Walsh [11] states that if  $\lim_{z \rightarrow \zeta, z \in \Omega} \Psi_{\Omega, f}(z) = f(\zeta)$  for all  $\zeta \in \partial\Omega$  then  $\Psi_{\Omega, f}$  is actually continuous on  $\bar{\Omega}$ . This fact implies the following well known theorem (see also Theorem 2.3 in [10]).

**Theorem.** *Let  $\Omega$  be a bounded domain in  $\mathbb{C}^n$ , regular for the Dirichlet problem for the Laplace operator. The following are equivalent:*

- (i) every  $\zeta \in \partial\Omega$  is a peak point for the class of plurisubharmonic functions;
- (ii) if  $f \in C(\partial\Omega)$  then  $\Psi_{\Omega, f} \in C(\bar{\Omega})$ .

In [2] it is shown (Theorem 6.2) that if  $\Omega$  is a strongly pseudoconvex domain in  $\mathbb{C}^n$  and  $f$  is a Hölder continuous function on  $\partial\Omega$  then the function  $\Psi_{\Omega, f}$  is Hölder continuous on  $\bar{\Omega}$ . Moreover, estimates for the modulus of continuity of  $\Psi_{\Omega, f}$  in terms of the modulus of continuity of  $f$  are obtained, via different methods, in [7] and [1]. In this note we prove that a smoothly bounded domain  $\Omega$  in  $\mathbb{C}^2$  is pseudoconvex of finite type if and only if the function  $\Psi_{\Omega, f}$  is Hölder continuous on  $\bar{\Omega}$  whenever  $f$  is a Hölder continuous function on  $\partial\Omega$ .

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For  $S \subset \mathbb{C}^n$  and  $\alpha$  so that  $0 < \alpha \leq 1$  we denote by  $Lip^\alpha(S)$  the set of functions on  $S$  which are Hölder continuous with exponent  $\alpha$ . If  $f$  is a real valued function defined on  $S$  we set  $H(f) = \sup\{\alpha \in (0, 1] : f \in Lip^\alpha(S)\}$ . Note that if this set is empty then, by definition,  $H(f) = 0$ . We also set, for a fixed  $x_0 \in S$ ,

$$h(f, x_0) = \max \left\{ \liminf_{x \rightarrow x_0, x \in S} \frac{\log |f(x) - f(x_0)|}{\log \|x - x_0\|}, 0 \right\} .$$

It is then obvious that if  $f$  satisfies a Hölder condition at  $x_0$  with exponent  $\alpha > 0$  (i.e.  $|f(x) - f(x_0)| \leq c\|x - x_0\|^\alpha$  for  $x$  in some neighborhood of  $x_0$ ) then  $\alpha \leq h(f, x_0)$ . In particular  $H(f) \leq h(f, x_0)$ . Conversely, if  $h(f, x_0) > 0$  then for any  $\epsilon$ ,  $0 < \epsilon < h(f, x_0)$ ,  $f$  satisfies a Hölder condition at  $x_0$  with exponent  $h(f, x_0) - \epsilon$ .

Let us briefly recall one of the definitions of the notion of finite type. Let  $\Omega$  be a bounded domain in  $\mathbb{C}^n$  with  $C^\infty$  smooth boundary and assume  $0 \in \partial\Omega$ . If  $X$  is a germ of a one-dimensional analytic variety at 0 we set

$$\tau(X) = \sup \left\{ s > 0 : \limsup_{z \rightarrow 0, z \in X \setminus \{0\}} \frac{d(z, \partial\Omega)}{\|z\|^s} < \infty \right\}$$

( $d$ =euclidean distance). If  $r$  is a defining function for  $\partial\Omega$  near 0,  $X$  is irreducible and  $z = \beta(t)$ ,  $t \in \mathbb{C}$ , is a parametrization for  $X$  such that  $\beta(0) = 0$ , then it is easy to see that  $\tau(X) = v(r \circ \beta)/v(\beta)$ , where  $v(r \circ \beta)$  and  $v(\beta)$  are the vanishing orders of  $r \circ \beta$  and  $\beta$  at 0. We say that  $\partial\Omega$  is of type less than  $2k$  at 0 if  $\tau(X) \leq 2k$  for every germ  $X$  of a one-dimensional analytic variety at 0 (for more information see [3]). The domain  $\Omega$  is said to be of finite type if  $\partial\Omega$  is of finite type at all of its points.

## 2. MAIN RESULTS

For  $z \in \mathbb{C}^n$  we write  $z = (z', z_n) \in \mathbb{C}^{n-1} \times \mathbb{C}$ ,  $z_n = x_n + iy_n$ . Let  $\Omega$  be a bounded domain in  $\mathbb{C}^n$  with  $C^\infty$  smooth boundary and let  $\zeta \in \partial\Omega$ . Without loss of generality we may assume that  $\zeta = 0$  and that the boundary of  $\Omega$  near 0 is given by  $y_n = \lambda(z', x_n)$ , where  $\lambda$  is a smooth function satisfying  $\lambda(0, 0) = 0$ ,  $d\lambda(0, 0) = 0$ .

We define  $f_1(z) = -\|(z', x_n)\| = -(\|z'\|^2 + x_n^2)^{1/2}$ ,  $z \in \mathbb{C}^n$ , and we let  $\psi_1(z) = \Psi_{\Omega, f_1}(z)$ ,  $z \in \overline{\Omega}$ , be the Perron-Bremermann function for  $\Omega$  and  $f_1$ . Note that  $H(f_1) = 1$ .

**Theorem 1.** *Let  $\Omega$  and  $\psi_1$  be as above and let  $X$  be a germ of a one-dimensional analytic variety at  $0 \in \partial\Omega$ . Then*

$$\tau(X) \leq 1/h(\psi_1, 0) .$$

**Theorem 2.** *Let  $\Omega$  be a bounded domain in  $\mathbb{C}^2$  with  $C^\infty$  boundary. The following are equivalent:*

- (i)  $\Omega$  is pseudoconvex of finite type;
- (ii) for any  $f \in Lip^\alpha(\partial\Omega)$ ,  $0 < \alpha \leq 1$ , we have  $\Psi_{\Omega, f} \in Lip^\gamma(\overline{\Omega})$ , for some  $\gamma = \gamma(f, \alpha)$ ,  $0 < \gamma \leq \alpha$ .

Moreover, if (i) holds and the type of  $\Omega$  is  $2k$  then  $\gamma$  can be chosen as  $\gamma = \gamma(\alpha) = \alpha/(2k)$ . In addition, if  $\zeta \in \partial\Omega$  (wlog  $\zeta = 0$ ) is a point of the highest type  $2k$  and  $\psi_1$  is as in Theorem 1, then  $H(\psi_1) = 1/(2k)$ .

3. PROOFS

We need the following lemma ([5, Proposition 3.1]):

**Lemma.** *Let  $\Omega$  be a smoothly bounded domain in  $\mathbb{C}^n$  with  $0 \in \partial\Omega$  and let  $X$  be a germ of a one-dimensional analytic variety at 0. Assume that there exists a negative plurisubharmonic function  $\phi$  on  $\Omega$  such that for some  $r > 0$  and for  $z \in \Omega$ , with  $\|z\| < r$ ,  $\phi$  satisfies  $-c_1\|z\|^\gamma \leq \phi(z) \leq -c_2\|z\|^N$ , where  $c_1, c_2 > 0$ ,  $0 < \gamma < N$ . Then  $\tau(X) \leq N/\gamma$ .*

*Proof of Theorem 1.* We may assume that  $h(\psi_1, 0) > 0$ . We fix  $\epsilon$  so that  $0 < \epsilon < h(\psi_1, 0)$ , and set  $\gamma = \gamma(\epsilon) = h(\psi_1, 0) - \epsilon$ . Note that  $\psi_1$  is negative and plurisubharmonic in  $\Omega$  and by the definition of  $h(\psi_1, 0)$  there is a number  $r > 0$  such that  $\psi_1(z) \geq -\|z\|^\gamma$  for  $z \in \Omega$  with  $\|z\| < r$ .

It is enough to prove that for every  $\alpha < 1$  there is a ball  $U_\alpha$  centered at 0 such that

$$(3.1) \quad \psi_1(z) \leq -c_\alpha \|z\|^{1/\alpha}$$

for  $z \in \Omega \cap U_\alpha$ . Indeed, then it would follow from the above lemma that  $\tau(X) \leq 1/(\alpha\gamma(\epsilon))$ , and hence  $\tau(X) \leq 1/h(\psi_1, 0)$  by letting  $\epsilon \rightarrow 0$  and  $\alpha \rightarrow 1$ .

To this end we introduce the functions  $f_\alpha(z) = -\|(z', x_n)\|^\alpha = -(\|z'\|^2 + x_n^2)^{\alpha/2}$ ,  $z \in \mathbb{C}^n$ , and  $\psi_\alpha(z) = \Psi_{\Omega, f_\alpha}(z)$ ,  $z \in \bar{\Omega}$ . Note that  $f_\alpha \in Lip^\alpha(\mathbb{C}^n)$ ,  $H(f_\alpha) = \alpha$  and  $\psi_\alpha$  is negative and plurisubharmonic in  $\Omega$ . As  $\alpha < 1$  the function  $v = -(-\psi_1)^\alpha$  is plurisubharmonic in  $\Omega$ . Also since  $\limsup_{z \rightarrow \zeta} \psi_1(z) \leq f_1(\zeta)$ ,  $\zeta \in \partial\Omega$ , we have  $\limsup_{z \rightarrow \zeta} v(z) \leq -(-f_1(\zeta))^\alpha = f_\alpha(\zeta)$ , so

$$(3.2) \quad v(z) = -(-\psi_1(z))^\alpha \leq \psi_\alpha(z), \quad z \in \Omega.$$

Thus (3.1) follows from (3.2) if we show that

$$(3.3) \quad \psi_\alpha(z) \leq -c_\alpha \|z\|,$$

for some constant  $c_\alpha > 0$  and for  $z \in \Omega \cap U_\alpha$ ,  $U_\alpha$  being a suitable ball centered at 0.

We fix  $\delta > 0$  small, to be specified later in terms of  $\alpha$  alone, and write  $\Omega = \Omega_1 \cup \Omega_2$  near 0, where  $\Omega_1 = \{z \in \Omega : \|z\| < r(\delta), |y_n| \geq \delta(\|z'\|^2 + x_n^2)^{1/2}\}$ ,  $\Omega_2 = \{z \in \Omega : \|z\| < r(\delta), |y_n| \leq \delta(\|z'\|^2 + x_n^2)^{1/2}\}$  and  $r(\delta)$  is sufficiently small. By Hopf's lemma ([6, p. 57]) there is a constant  $c > 0$  depending on  $\Omega$  and  $\psi_\alpha$  such that  $\psi_\alpha(z) \leq -cd(z, \partial\Omega)$ . But for  $z \in \Omega_1$  we have  $d(z, \partial\Omega) \geq c(\delta)\|z\|$  provided  $r(\delta)$  is small enough; hence (3.3) holds if  $z \in \Omega_1$ .

Now let  $h$  be the harmonic function in  $\Omega$  with boundary value  $f_\alpha$ . Since  $f_\alpha \in Lip^\alpha$  we have as a consequence of the results in [8] that  $h \in Lip^\alpha(\bar{\Omega})$  (note that this is not necessarily true when  $\alpha = 1$ ). Let  $k(\alpha) > 0$  be such that  $|h(z) - h(w)| \leq k(\alpha)\|z - w\|^\alpha$ ,  $z, w \in \bar{\Omega}$ . For  $z = (z', x_n + iy_n) \in \Omega$  near 0 we define  $\tilde{z} = (z', x_n + i\tilde{y}_n) \in \partial\Omega$ . So  $\tilde{y}_n = \lambda(z', x_n)$  and hence  $|\tilde{y}_n| \leq l(\|z'\|^2 + x_n^2)$  for some constant  $l > 0$  depending on  $\lambda$ . For  $z \in \Omega_2$  we have:

$$\begin{aligned} \psi_\alpha(z) &\leq h(z) \leq f_\alpha(\tilde{z}) + k(\alpha)|y_n - \tilde{y}_n|^\alpha \\ &\leq -(\|z'\|^2 + x_n^2)^{\alpha/2} + k(\alpha)|y_n|^\alpha + k(\alpha)|\tilde{y}_n|^\alpha \\ &\leq -(\|z'\|^2 + x_n^2)^{\alpha/2} + k(\alpha)\delta^\alpha(\|z'\|^2 + x_n^2)^{\alpha/2} + k(\alpha)l^\alpha(\|z'\|^2 + x_n^2)^\alpha. \end{aligned}$$

We now choose  $\delta = \delta(\alpha)$  small enough so that  $k(\alpha)\delta^\alpha \leq 1/4$  and also choose  $r(\delta)$  small enough so that  $k(\alpha)l^\alpha(\|z'\|^2 + x_n^2)^{\alpha/2} \leq 1/4$ . Thus by the above

$$\psi_\alpha(z) \leq -\frac{1}{2}(\|z'\|^2 + x_n^2)^{\alpha/2}$$

for  $z \in \Omega_2$ . Also, for  $z \in \Omega_2$  we have

$$\|z\|^2 \leq (1 + \delta^2)(\|z'\|^2 + x_n^2) \leq 2(\|z'\|^2 + x_n^2).$$

Hence  $\psi_\alpha(z) \leq -\frac{1}{4}\|z\|^\alpha \leq -\frac{1}{4}\|z\|$  for  $z \in \Omega_2$ , and (3.3) is proved. □

*Proof of Theorem 2.* The implication (ii)  $\Rightarrow$  (i) follows immediately from Theorem 1. Indeed, we pick a point on  $\partial\Omega$  which without loss of generality may be assumed to be 0. With the notations in Theorem 1 we have  $f_1 \in Lip^1$  so  $\psi_1 \in Lip^\gamma(\overline{\Omega})$ , for some  $\gamma \in (0, 1)$ , and hence  $\tau(X) \leq 1/\gamma$  for any germ  $X$  of a one-dimensional analytic variety at 0. Also, for any  $\zeta \in \partial\Omega$ , we let  $f(z) = -\|z - \zeta\|$  and note that by (ii)  $\Psi_{\Omega,f} \in PSH(\Omega) \cap C(\overline{\Omega})$  peaks at  $\zeta$ , so  $\Omega$  is pseudoconvex.

To prove that (i)  $\Rightarrow$  (ii) we follow the method of Bedford and Taylor ([2, Theorem 6.2]), and use a theorem of Fornaess and Sibony ([5, Theorem A]) on the existence of plurisubharmonic peak functions. Precisely, if  $\Omega$  is a bounded pseudoconvex domain in  $\mathbb{C}^2$  with smooth boundary and of finite type  $2k$  then there exist a neighborhood  $U$  of  $\overline{\Omega}$  and a constant  $C$  such that for each  $\zeta \in \partial\Omega$  there is a plurisubharmonic function  $\phi_\zeta(z)$  on  $U$  satisfying  $|\phi_\zeta(z) - \phi_\zeta(w)| \leq C\|z - w\|$  for  $z, w \in U$ ,  $\phi_\zeta(z) \leq -\|z - \zeta\|^{2k}$  for  $z \in \Omega$ , and  $\phi_\zeta(\zeta) = 0$ .

Now let  $f \in Lip^\alpha(\partial\Omega)$ ,  $0 < \alpha \leq 1$ , and let  $c_f$  be the smallest constant for which  $|f(\zeta) - f(\xi)| \leq c_f\|\zeta - \xi\|^\alpha$ ,  $\zeta, \xi \in \partial\Omega$ . We fix  $\zeta \in \partial\Omega$  and consider the function  $v_\zeta(z) = -c_f(-\phi_\zeta(z))^{\alpha/(2k)} + f(\zeta)$ . Since  $\phi_\zeta$  is negative and plurisubharmonic in  $\Omega$  and  $\alpha/(2k) < 1$  the function  $v_\zeta$  is plurisubharmonic in  $\Omega$  and continuous on  $\overline{\Omega}$ . For  $\xi \in \partial\Omega$  we have

$$v_\zeta(\xi) \leq -c_f\|\xi - \zeta\|^\alpha + f(\zeta) \leq f(\xi),$$

so  $v_\zeta(z) \leq \Psi_{\Omega,f}(z)$ , for  $z \in \overline{\Omega}$ . Note that  $v_\zeta$  satisfies a Hölder condition of the form  $|v_\zeta(z) - v_\zeta(w)| \leq C'c_f\|z - w\|^{\alpha/(2k)}$ ,  $z, w \in \overline{\Omega}$ , where  $C' = C^{\alpha/(2k)}$ .

We next define  $u(z) = \sup\{v_\zeta(z) : \zeta \in \partial\Omega\}$ ,  $z \in \overline{\Omega}$ . Then  $u$  satisfies the same Hölder condition as  $v_\zeta$  so, in particular,  $u$  is continuous on  $\overline{\Omega}$  and hence plurisubharmonic in  $\Omega$ . Also  $u(z) \leq \Psi_{\Omega,f}(z)$  for  $z \in \overline{\Omega}$ . As  $v_\zeta(\xi) \leq f(\xi)$ ,  $\xi \in \partial\Omega$ , and  $v_\zeta(\xi) = f(\xi)$  we see that  $u = \Psi_{\Omega,f} = f$  on  $\partial\Omega$ . So if  $z \in \overline{\Omega}$  and  $\zeta \in \partial\Omega$  we have  $-C'c_f\|z - \zeta\|^{\alpha/(2k)} \leq u(z) - u(\zeta) \leq \Psi_{\Omega,f}(z) - f(\zeta)$ . Replacing  $f$  by  $-f$  and proceeding in the same way we construct a function  $\tilde{u}$  which satisfies the same Hölder condition as  $u$  in  $\overline{\Omega}$ , is plurisubharmonic in  $\Omega$  and  $\tilde{u} = -f$  on  $\partial\Omega$ . By the maximum principle  $\Psi_{\Omega,f} + \tilde{u} \leq 0$  on  $\overline{\Omega}$  so for  $z \in \overline{\Omega}$  and  $\zeta \in \partial\Omega$  we get  $\Psi_{\Omega,f}(z) - f(\zeta) \leq -\tilde{u}(z) + \tilde{u}(\zeta) \leq C'c_f\|z - \zeta\|^{\alpha/(2k)}$ . We conclude that

$$(3.4) \quad |\Psi_{\Omega,f}(z) - \Psi_{\Omega,f}(\zeta)| \leq C'c_f\|z - \zeta\|^{\alpha/(2k)},$$

for  $z \in \overline{\Omega}$  and  $\zeta \in \partial\Omega$ .

We now show that the above inequality holds for all  $z, w \in \overline{\Omega}$ . For  $y \in \mathbb{C}^n$  we set  $\Omega_y = \Omega \cap (-y + \Omega)$  and consider the function

$$F_y(z) = \begin{cases} \max\{\Psi_{\Omega,f}(z), \Psi_{\Omega,f}(z + y) - C'c_f\|y\|^{\alpha/(2k)}\}, & \text{if } z \in \Omega_y, \\ \Psi_{\Omega,f}(z), & \text{if } z \in \Omega \setminus \Omega_y. \end{cases}$$

By (3.4) we have  $\limsup_{z \rightarrow \zeta, z \in \Omega} F_y(z) \leq f(\zeta)$  for all  $\zeta \in \partial\Omega$ . Also if  $z \in \partial\Omega_y \cap \Omega$  then  $z + y \in \partial\Omega$  so by (3.4)  $\Psi_{\Omega, f}(z) \geq \Psi_{\Omega, f}(z + y) - C'c_f\|y\|^{\alpha/(2k)}$  and  $F_y$  is plurisubharmonic. Hence  $F_y(z) \leq \Psi_{\Omega, f}(z)$  for all  $z \in \Omega$ . If  $w \in \Omega$  then  $z \in \Omega_{w-z}$  so

$$\Psi_{\Omega, f}(w) - C'c_f\|w - z\|^{\alpha/(2k)} \leq F_{w-z}(z) \leq \Psi_{\Omega, f}(z)$$

and (3.4) holds for all  $z, w \in \overline{\Omega}$ .

Finally, we assume that  $\zeta = 0 \in \partial\Omega$  is a point of type  $2k$  and we let  $\psi_1$  be the function in Theorem 1. Then  $1/(2k) \leq H(\psi_1) \leq h(\psi_1, 0) \leq 1/(2k)$ , the last inequality following from Theorem 1. Hence  $H(\psi_1) = 1/(2k)$  and the proof is complete.  $\square$

*Remark.* If  $f \in C(\partial\Omega)$  then it is easy to see that  $\|\Psi_{\Omega, f}\|_{\infty} \leq \|f\|_{\infty}$  (indeed,  $-2\|f\|_{\infty} \leq \Psi_{\Omega, f} - \|f\|_{\infty} = \Psi_{\Omega, f} - \|f\|_{\infty} \leq 0$ , by the maximum principle). So if  $\Omega$  is a bounded pseudoconvex domain in  $\mathbb{C}^2$  of finite type  $2k$  it follows from the proof of the above theorem that there exists a constant  $C$  depending only on  $\Omega$  such that, for  $f \in Lip^{\alpha}(\partial\Omega)$ ,  $0 < \alpha \leq 1$ , we have  $\|\Psi_{\Omega, f}\|_{\frac{\alpha}{2k}} \leq C\|f\|_{\alpha}$ . Here  $\|\cdot\|_{\alpha}$  denotes the standard norm on  $Lip^{\alpha}(S)$ ,  $\|f\|_{\alpha} = \|f\|_{\infty} + \sup\{|f(x) - f(y)|/|x - y|^{\alpha} : x \neq y \in S\}$ .

We finally consider the case when  $\Omega$  is a bounded pseudoconvex domain in  $\mathbb{C}^n$  with real analytic boundary. It follows from Theorems 2 and 3 in [4] that there are constants  $C, \gamma$  and  $N$  depending only on  $\Omega$  such that  $C > 0, 0 < \gamma < 1 < N$ , and such that for every  $\zeta \in \partial\Omega$  there is a function  $\phi_{\zeta} \in PSH(\Omega) \cap C(\overline{\Omega})$  satisfying  $|\phi_{\zeta}(z) - \phi_{\zeta}(w)| \leq C\|z - w\|^{\gamma}$  for  $z, w \in \overline{\Omega}$ ,  $\phi_{\zeta}(z) \leq -\|z - \zeta\|^N$  for  $z \in \Omega$  and  $\phi_{\zeta}(\zeta) = 0$ . Using these plurisubharmonic peak functions one can proceed as in the proof of Theorem 2 to obtain the following

**Theorem 3.** *Let  $\Omega$  be a bounded pseudoconvex domain in  $\mathbb{C}^n$  with real analytic boundary. Then there exists a constant  $m > 1$  depending only on  $\Omega$  ( $m = N/\gamma$ , where  $\gamma$  and  $N$  are as above) such that if  $f \in Lip^{\alpha}(\partial\Omega)$ ,  $0 < \alpha \leq 1$ , then  $\Psi_{\Omega, f} \in Lip^{\alpha/m}(\overline{\Omega})$ .*

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