PROCEEDINGS OF THE
AMERICAN MATHEMATICAL SOCIETY
Volume 126, Number 1, January 1998, Pages 115–121
S 0002-9939(98)03839-8

MALUTA'S COEFFICIENT IN MUSIELAK-ORLICZ SEQUENCE SPACES EQUIPPED WITH THE ORLICZ NORM

YUNAN CUI, HENRYK HUDZIK, AND HONGWEI ZHU

(Communicated by Palle E. T. Jorgensen)

Abstract. Maluta's coefficient of Musielak-Orlicz sequence spaces equipped with the Orlicz norm is calculated. A sufficient condition for the Schur property of these spaces is given.

1. Introduction

In the sequel $\mathbb{N}, \mathbb{R}$ and $\mathbb{R}_+$ stand for the set of natural numbers, the set of reals and the set of nonnegative reals, respectively. The space of all sequences $x = (x(i))_{i=1}^{\infty}$ of reals is denoted by l^0. A map $\Phi \colon \mathbb{R} \to [0, +\infty)$ is said to be an Orlicz function if Φ is convex, even, vanishing at zero, left continuous on $\mathbb{R}_+$ and not identically equal to zero (see [9, 12, 13, 15] and [16]).

A sequence $\Phi = (\Phi_i)_{i=1}^{\infty}$ of Orlicz functions Φ_i is called a *Musielak-Orlicz function* (see [15]). By $\Psi = (\Psi_i)$ we denote the Musielak-Orlicz function conjugate to $\Phi = (\Phi_i)$ in the sense of Young, i.e.

$$\Psi_i(u) = \sup_{v>0}\{|u|v - \Phi_i(v)\}$$

for each $u \in \mathbb{R}$ and $i \in \mathbb{N}$. Further, $\varphi = (\varphi_i)$ is the right derivative of $\Phi = (\Phi_i)$, i.e. φ_i are the right derivatives of Φ_i for every $i \in \mathbb{N}$.

Given a Musielak-Orlicz function $\Phi = (\Phi_i)$ we define on l^0 a *convex modular* I_Φ by

$$I_\Phi(x) = \sum_{i=1}^{\infty} \Phi_i(x_i) \qquad (\forall x = (x_i) \in l^0),$$

and the *Musielak-Orlicz space* l^Φ by

$$l^\Phi = \{x \in l^0 \colon I_\Phi(\lambda x) < +\infty \quad \text{for some} \quad \lambda > 0\}.$$

Received by the editors July 19, 1995 and, in revised form, March 12, 1996.
1991 *Mathematics Subject Classification.* Primary 46E30.
Key words and phrases. Schur's property, Maluta's coefficient, asymptotic equidistant sequence, reflexivity, weak convergence, the δ_2-condition.
The first and third authors were supported by the Chinese National Science Foundation.
The second author was supported by KBN grant 2 P03A 031 10.

The functional

$$\|x\| = \sup\{\sum_{i=1}^{\infty} x(i)y(i)\colon I_\Psi(y) \le 1\}$$

is a norm in l^Φ (called the *Orlicz norm*) and the couple $(l^\Phi, \|\ \|)$ is a Banach space (see [15]).

Our aim in this paper is to calculate Maluta's coefficient of Musielak-Orlicz spaces equipped with the Orlicz norm. This coefficient is connected with normal structure, which is a very important property of Banach spaces that guarantees the fixed point property for them (see [1, 2, 3, 4, 6, 8, 10, 11, 14] and [19]).

Let in the sequel X denote a reflexive infinite dimensional Banach space (which automatically does not have the Schur property) and let $S(X)$ denote its unit sphere. For each sequence (x_n) in X, we define

$$A((x_n)) = \lim_n\{\sup\{\|x_i - x_j\|\colon i \ne j;\, i,j \ge n\}\},$$
$$A_1((x_n)) = \lim_n\{\inf\{\|x_i - x_j\|\colon i \ne j;\, i,j \ge n\}\}.$$

The *weak uniform normal structure coefficient* of X is defined by (see [4])

$$\begin{aligned} WCS(X) = \sup\{k > 0\colon & \text{ for each weakly convergent sequence } (x_n) \\ & \text{in } S(X) \text{ there is some } y \in \operatorname{conv}(x_n) \text{ such that} \\ & k \limsup_n \|x_n - y\| \le A((x_n))\}. \end{aligned}$$

A sequence (x_n) in X is said to be *an asymptotic equidistant sequence* if $A((x_n)) = A_1((x_n))$. This definition was introduced in [19], where it was proved that $WCS(X) = \inf\{A((x_n))\colon (x_n)$ is an asymptotic equidistant sequence in $S(X)$ and $x_n \to 0$ weakly$\}$.

Recall that *Maluta's coefficient* $M(X)$ of a Banach space X is defined by (see [14])

$$M(X) = \sup\Big\{\frac{\limsup_n d(x_{n+1}, \operatorname{conv}(x_j)_{j=1}^{\infty})}{A((x_n))}\colon (x_n) \text{ is a bounded nonconstant sequence in } X\Big\}.$$

We have $M(X) = 1/WCS(X)$ for each reflexive Banach space X and $M(X) = 1$ for each nonreflexive Banach space X.

To formulate our results, we need to fix some notations. For any $i \in \mathbb{N}$, put

$$b_i = \sup\{v > 0\colon \Psi_i(v) < +\infty\},$$
$$a_i = \begin{cases} b_i & \text{if } \Psi_i(b_i) < 1 \\ \Psi_i^{-1}(1) & \text{if } \Psi_i(b_i) \ge 1, \end{cases}$$
$$N_x = \{i \in \mathbb{N}\colon x(i) \ne 0\}.$$

The result that for every $x \in l^\Phi$ with $\sum_{i \in N_x} \Psi_i(a_i) > 1$, we have $\|x\| = \frac{1}{k}(1 + I_\Phi(kx))$ if and only if $k \in [k_x^*, k_x^{**}]$, where $k_x^* = \inf\{k > 0\colon \sum_{i=1}^{\infty} \Psi_i(\varphi_i(kx(i))) \ge 1\}$, $k_x^{**} = \sup\{k > 0\colon \sum_{i=1}^{\infty} \Psi_i(\varphi_i(kx(i))) \le 1\}$ has been proved in [18]. Moreover, for any

$x \in l^\Phi$ there exists $k > 0$ such that $\|x\| = \frac{1}{k}(1 + I_\Phi(kx))$ whenever $\Phi_i(u)/u \to +\infty$ as $u \to +\infty$ for all $i \in \mathbb{N}$.

We need to define a regularity condition for $\Phi = (\Phi_i)$ called the δ_2^0-condition. A Musielak-Orlicz function $\Phi = (\Phi_i)$ satisfies the δ_2^0-condition if there exist positive constants a and K and a sequence (c_i) in $[0, +\infty]$ such that $\sum_{i=i_0}^{\infty} c_i < +\infty$ for some $i_0 \in \mathbb{N}$ and

$$\Phi_i(2u) \leq k\Phi_i(u) + c_i$$

for each $i \in \mathbb{N}$ and each $u \in \mathbb{R}$ satisfying $\Psi_i(u) \leq a$. In the case when all c_i are in $\mathbb{R}_+$ we say that Φ satsfies the δ_2-condition (see [15]).

2. Results

We start with the following

Lemma 1. *If* $\sum_{i=1}^{\infty} \Psi_i(a_i) \leq 1$ *then* l^Φ *has the Schur property.*

Proof. Suppose $x_n = (x_n(i)) \in S(l^\Phi)$ for each $n \in \mathbb{N}$ and $x_n \to x_0$ weakly. By $\sum_{i=1}^{\infty} \Psi_i(a_i) \leq 1$, we have $\|x_n\| = \sum_{i=1}^{\infty} a_i|x_n(i)|$ $(n = 1, 2, \dots)$. Define $z_n = (a_1x_n(1), a_2x_n(2), \dots)$ and $z_0 = (a_1x_0(1), a_2x_0(2), \dots)$. Then $z_n \in l^1$ for $n = 0, 1, \dots$ and $z_n \to z_0$ weakly in l^1 (because the weak convergence in l^φ implies the weak convergence in $l^1((a_i)))$. Since l^1 has the Schur property, we get $\|z_n - z_0\|_{l^1} \to 0$. Hence, in view of the equality

$$\|x_n - x_0\| = \sum_{i=1}^{\infty} a_i|x_n(i) - x_0(i)| = \|z_n - z_0\|_{l^1},$$

we get $\lim_n \|x_n - x_0\| = 0$, i.e. l^Φ has the Schur property. □

To calculate Maluta's coefficient for l^Φ we need to define some parameter for the generating Musielak-Orlicz function $\Phi = (\Phi_i)$.

In this definition there is some analogy to the definition of the parameter d_φ in [7] (see also [17]).

For every $m, n \in \mathbb{N}$ and $k > 1$, we define

$$c(k, m, n) = \inf\{c_{k,x} > 0\colon I_\Phi\left(\frac{kx}{c_{k,x}}\right) = \frac{k-1}{2} \text{ and } x = \sum_{i=m}^{m+n} x(i)e_i \in S(l^\Phi)\}.$$

The sequence $(c(k, m, n))_{n=1}^{\infty}$ is nonincreasing for every $k > 1$ and $m \in \mathbb{N}$. Therefore, the limit

$$d(k, m) = \lim_{n\to\infty} c(k, m, n)$$

exists. Moreover, $d(k, m) \leq c(k, m, n)$ for every $k > 1$ and $m, n \in \mathbb{N}$. The sequence $(d(k, m))_{m=1}^{\infty}$ is nondecreasing. Hence, the limit

$$d_k = \lim_{m\to\infty} d(k, m)$$

exists and $d_k \geq d(k, m)$ for every $k > 1$ and $m \in \mathbb{N}$.

Define

$$d(\Phi) = \inf\{d_k\colon k > 1\}.$$

We are now in a position to give the main result of the paper.

Theorem 1. *Assume that $\Phi = (\Phi_i)$ is a Musielak-Orlicz function such that all Φ_i $(i = 1, 2, \dots)$ are finitely valued and $\Phi_i(u)/u \to +\infty$ as $u \to +\infty$ for all $i \in \mathbb{N}$. Then:* (i) *If l^Φ is nonreflexive, then $M(l^\Phi) = 1$;* (ii) *If l^Φ is reflexive, then $M(l^\Phi) = 1/d(\Phi)$.*

Proof. Statement (i) follows immediately from the fact that $M(X) = 1$ for every nonreflexive Banach space X. So, we only need to prove that $WCS(L^\Phi) = d(\Phi)$ whenever l^Φ is reflexive. It is well known that the reflexivity of l^Φ is equivalent to the fact that both Φ and Ψ satisfy the δ_2^0-condition.

First, we will prove that $WCS(l^\Phi) \le d(\Phi)$. For each $\varepsilon > 0$, by the definition of $d(\Phi)$, there is $k > 1$ such that $d(\Phi) > d_k - \varepsilon$. Recall that $d_k \ge d(k, m)$ for all $k > 1$ and $m \in \mathbb{N}$. By the definition of $d(k, m)$ there is $n(m) \in \mathbb{N}$ such that

$$d(k, m) > c(k, m, n) - \varepsilon \quad \text{whenever} \quad n > n(m).$$

Finally, by the definition of $c(k, m, n)$ there exists $x_{m,n} \in S(l^\Phi)$ such that

$$(1) \quad c_{k,x_{m,n}} - \varepsilon < c(k, m, n), \qquad x_{m,n} = \sum_{i=m}^{m+n} x_{m,n}(i)e_i \in S(l^\Phi),$$

$$(2) \quad I_\Phi\left(\frac{kx_{m,n}}{c_{k,x_{m,n}}}\right) = \frac{k-1}{2}.$$

Take $m_1 = 1$. Then there exist $n_1 \in \mathbb{N}$, $n_1 > n(m_1)$ and x_{m_1,n_1} satisfying (1) and (2) with m_1 and n_1 in place of m and n. Take $m_2 = m_1 + n_1 + 1$. There exists x_{m_2,n_2} satisfying (1) and (2) with m_2, n_2, $n_2 > n(m_2)$, in place of m, n. By induction, we can construct a sequence $(x_{m_i,n_i})_{i=1}^\infty$ in $S(l^\Phi)$ with pairwise disjoint supports and satisfying (1) and (2) with m_i and n_i, $n_i > n(m_i)$ in place of m and n for $i = 1, 2, \dots$.

Define $y_k = x_{m_k,n_k}$. Then, we have $y_n \in S(l^\Phi)$ for every $n \in \mathbb{N}$. Moreover, $y_n \to 0$ weakly and for every $\nu, l \in \mathbb{N}$ there holds

$$\begin{aligned}
\left\|\frac{y_\nu - y_l}{d(\Phi) + 2\varepsilon}\right\| &\le \frac{1}{k}(1 + I_\Phi(k\frac{y_\nu - y_l}{d(\Phi) + 2\varepsilon})) = \frac{1}{k}(1 + I_\Phi(\frac{y_\nu}{d(\Phi) + 2\varepsilon}) + I_\Phi(\frac{y_l}{d(\Phi) + 2\varepsilon})) \\
&\le \frac{1}{k}(1 + I_\Phi(\frac{kx_{m_\nu,n_\nu}}{c_{k,x_{m_\nu,n_\nu}}}) + I_\Phi(\frac{kx_{m_l,n_l}}{c_{k,x_{m_l,n_l}}})) \\
&= \frac{1}{k}(1 + \frac{k-1}{2} + \frac{k-1}{2}) = 1.
\end{aligned}$$

Hence $A((y_n)) \le d(\Phi) + 3\varepsilon$. By the arbitrariness of $\varepsilon > 0$, we have $A((y_n)) \le d(\Phi)$. By virtue of Proposition 2 in [19] which says that for each weakly convergent sequence on the unit sphere of a Banach space X there exists an asymptotic equidistant subsequence, we can now conclude that $WCS(l^\Phi) \le d(\Phi)$.

Next, we will prove that $WCS(l^\Phi) \ge d(\Phi)$. First, we will show the equality

$$WCS(l^\Phi) = \inf\{A((x_n))\colon x_n = \sum_{i=l_{n-1}+1}^{l_n} x_n(i)e_i \text{ and } (x_n) \text{ is an asymptotic equidistant sequence in } S(l^\Phi)\} =: d.$$

It is obvious that $WCS(l^\Phi) \le d$, so we only need to prove that $WCS(l^\Phi) \ge d$. For any $\varepsilon > 0$, by the definition of $WCS(l^\Phi)$, there exists a sequence (x_n) in $S(l^\Phi)$ being an asymptotic equidistant sequence, weakly convergent to 0 and such that

$$A((x_n)) < WCS(l^\Phi) + \varepsilon.$$

Let $v_1 = x_1$. Take $l_1 \in \mathbb{N}$ satisfying $\|\sum_{i=l_1+1}^{\infty} v_1(i)e_i\| < \varepsilon$. Such a number l_1 exists since by the reflexivity of l^Φ, the generating function $\Phi = (\Phi_i)$ satisfies the δ_2^0-condition. By $x_n(i) \to 0$ as $n \to \infty$, $(i = 1, 2, \ldots, l_1)$ there is $n_0 \in \mathbb{N}$ such that

$$\|\sum_{i=1}^{l_1} x_n(i)e_i\| < \varepsilon \text{ whenever } n > n_0.$$

Let us fix $N_1 > n_0$ and set $v_2 = x_{N_1}$. Then

$$\|\sum_{i=1}^{l_1} v_2(i)e_i\| < \varepsilon.$$

Take $l_2 > l_1$ such that $\|\sum_{i=l_2+1}^{\infty} v_2(i)e_i\| < \varepsilon$. By $x_n(i) \to 0$ as $n \to \infty$ for $i = 1, 2, \ldots$, we can find $N_2 > N_1$ such that

$$\|\sum_{i=1}^{l_2} x_n(i)e_i\| < \varepsilon \text{ whenever } n > N_2.$$

Let us choose $N_3 > N_2$ and set $v_3 = x_{N_3}$. Then

$$\|\sum_{i=1}^{l_2} v_3(i)e_i\| < \varepsilon.$$

Take $l_3 > l_2$ such that

$$\|\sum_{i=l_3+1}^{\infty} v_3(i)e_i\| < \varepsilon.$$

In such a way we can construct by induction a sequence (l_n) of natural numbers with $l_1 < l_2 < \ldots$ and a subsequence (v_n) of (x_n) satisfying $A((v_n)) = A((x_n))$ and

$$\|\sum_{i=1}^{l_{n-1}} v_n(i)e_i\| < \varepsilon, \quad \|\sum_{i=l_n+1}^{\infty} v_n(i)e_i\| < \varepsilon,$$

where $l_0 = 0$ by definition. Put

$$u_n = \sum_{i=l_{n-1}+1}^{l_n} v_n(i)e_i / \|\sum_{i=l_{n-1}+1}^{l_n} v_n(i)e_i\| \quad (n = 1, 2, \ldots).$$

Then $u_n \in S(l^\Phi)$ for each $n \in \mathbb{N}$. Moreover, for every $m, n \in \mathbb{N}, n < m$, we have

$$\begin{aligned}
\|v_n - v_m\| &= \|\sum_{i=1}^{l_{n-1}}(v_n(i) - v_m(i))e_i + \sum_{i=l_{n-1}+1}^{l_n}(v_n(i) - v_m(i))e_i \\
&\quad + \sum_{i=l_n+1}^{l_{m-1}}(v_n(i) - v_m(i))e_i + \sum_{i=l_{m-1}+1}^{l_m}(v_n(i) - v_m(i))e_i \\
&\quad + \sum_{i=l_m+1}^{\infty}(v_n(i) - v_m(i))e_i\| \\
&\geq \|\sum_{i=l_{n-1}+1}^{l_n} v_n(i)e_i - \sum_{i=l_{m-1}+1}^{l_m} v_m(i)e_i\| - 4\varepsilon \\
&\geq \|(u_n - u_m)(1 - 2\varepsilon)\| - 4\varepsilon.
\end{aligned}$$

Therefore

$$A((u_n)) \leq \frac{A((v_n))}{1-2\varepsilon} + \frac{4\varepsilon}{1-2\varepsilon} = \frac{A((x_n)) + 4\varepsilon}{1-2\varepsilon} \leq \frac{WCS(l^\Phi) + 5\varepsilon}{1-2\varepsilon}.$$

In view of the arbitrariness of $\varepsilon > 0$, we have $d \leq WCS(l^\Phi)$. Finally, we will prove that $d \geq d(\Phi)$. For any asymptotic equidistant sequence

$$x_n = \sum_{i=l_{n-1}+1}^{l_n} x_n(i)e_i \in S(l^\Phi) \quad (n = 1, 2, \dots),$$

there are $k_{m,n} > 0$ such that

$$\|(x_m - x_n)/d(\Phi)\| = \frac{1}{k_{m,n}}(1 + I_\Phi(k_{m,n}\frac{x_m - x_n}{d(\Phi)}))$$

for all $m, n \in \mathbb{N}, m \neq n$. Let us assume in the following that $m, n \in \mathbb{N}$ and $m \neq n$. We will consider now two cases.

I. If $k_{m,n} \leq 1$, then $\|x_m - x_n\| \geq d(\Phi)$.

II. If $k_{m,n} > 1$, then

$$\begin{aligned}
\|(x_m - x_n)/d(\Phi)\| &= \frac{1}{k_{m,n}}(1 + I_\Phi(\frac{k_{m,n}x_m}{d(\Phi)}) + I_\Phi(\frac{k_{m,n}x_n}{d(\Phi)})) \\
&\geq \frac{1}{k_{m,n}}(1 + \frac{k_{m,n}-1}{2} + \frac{k_{m,n}-1}{2}) \\
&= 1,
\end{aligned}$$

whence we get again $\|x_m - x_n\| \geq d(\Phi)$. Consequently $A((x_n)) \geq d(\Phi)$. By the arbitrariness of (x_n) being an asymptotic equidistant sequence in $S(l^\Phi)$ it follows that $WCS(l^\Phi) \geq d(\Phi)$. □

The following example shows how to compute $d(\Phi)$ in some concrete cases.

Example 1. Let $\Phi_i(u) = |u|^p$ for all $i \in \mathbb{N}$ and $u \in \mathbb{R}$, where $1 < p < \infty$. If $\Phi = (\Phi_i)_{i=1}^\infty$, then $d(\Phi) = 2^{1/p}$.

Proof. It is obvious that $l^\Phi = l^p$. Moreover, $\|x\| = p^{1/p}q^{1/q}\|x\|_p$ for any $x \in l^\Phi$, where $1/p + 1/q = 1$ and $\|x\|_p = (\sum_{i=1}^\infty |x_i|^p)^{1/p}$ (see [9]). Take arbitrary $k > 1$ and $x \in S(l^\Phi)$ with finite support. It is easy to see that the number $c = c(k, x) > 0$

satisfying the equality $I_\Phi(\frac{kx}{c}) = \frac{k-1}{2}$ is equal to $2^{1/p}k(k-1)^{-1/p}p^{-1/p}q^{-1/q}$. Therefore, $d(\Phi) = \inf\{2^{1/p}k(k-1)^{-1/p}p^{-1/p}q^{-1/q} \colon k > 1\}$. To calculate this infimum it is enough to find $\inf\{k(k-1)^{-1/p} \colon k > 1\}$. Using the standard method, we get that this infimum is attained at $k_0 = q$. Since $k_0 - 1 = q/p$, we get $d(\Phi) = 2^{1/p}$. □

Acknowledgement

The authors thank the referee for valuable remarks.

References

1. T. Dominguez Benavides, *Weak uniform normal structure in direct-sum spaces*, Studia Math. **103** (1992), 283-290. MR **94c:**46024
2. T. Dominguez Benavides and G. Lopez Acedo, *Lower bounds for normal structure coefficients*, Proc. Roy. Soc. Edinburgh 121A (1992), 245-252. MR **93i:**46025
3. T. Dominguez Benavides, G. López Acedo and Hong-Kun Xu, *Weak uniform normal structure and iterative fixed points of nonexpansive mappings*, Colloquium Math. **68** (1) (1995), 17-23. MR **95k:**47086
4. W.L. Bynum, *Normal structure coefficients for Banach spaces*, Pacific J. Math. **86** (1980), 427-436. MR **81m:**46030
5. Yu.A. Cui, *Midpoint locally uniform rotundity of Musielak-Orlicz sequence space with Orlicz norm*, Northwestern J. Math. **9**(4) (1993), 561-565. MR **96d:**46017
6. K. Goebel and W.A. Kirk, *Topics in Metric Fixed Point Theory*, Cambridge University Press, 1990. MR **92c:**47070
7. H. Hudzik, C. Wu and Yi.N. Ye, *Packing constant in Musielak-Orlicz sequence spaces equipped with the Luxemburg norm*, Revista Math. **7** (1) (1994), 13-26. MR **95h:**46013
8. W.A. Kirk, *A fixed point theorem for mappings which do not increase distances*, Amer. Math. Monthly **72** (1965), 1004-1006. MR **32:**6436
9. M.A. Krasnoselskiĭ and Ya.B. Rutickiĭ, *Convex Functions and Orlicz Spaces* (translation), Groningen 1961. MR **23:**A4016
10. T. Landes, *Permanence properties of normal structure*, Pacific J. Math. **110** (1) (1984), 125-143. MR **86e:**46014
11. T.C. Lim, *On the normal structure coefficient and the bounded sequence coefficient*, Proc. Amer. Math. Soc. **88** (1983), 262-264. MR **85g:**46021
12. W.A.J. Luxemburg, *Banach Function Spaces*, Thesis, Delft 1955. MR **17:**285a
13. L. Maligranda, *Orlicz Spaces and Interpolation*, Seminars in Math. **5**, Campinas 1989.
14. E. Maluta, *Uniformly normal structure and related coefficients for Banach spaces*, Pacific J. Math. **111** (1984), 357-369. MR **85j:**46023
15. J. Musielak, *Orlicz Spaces and Modular Spaces*, Lecture Notes in Math. **1034**, Springer-Verlag 1983. MR **85m:**46028
16. M.M. Rao and Z.D. Ren, *Theory of Orlicz Spaces*, Marcel Dekker Inc., New York, Basel, Hong Kong 1991.
17. T.F. Wang, *Packing sphere in Orlicz sequence spaces*, Chinese Ann. Math. **6A**: 5 (1985), 567-574.
18. C.X. Wu and H.Yi. Sun, *Norm calculation and complex convexity on sequence Musielak-Orlicz spaces*, Chinese Ann. Math. **12**A (1991), 98-102. MR **92h:**46012
19. G.L. Zhang, *Weakly convergent sequence coefficient of product space*, Proc. Amer. Math. Soc. **117** (3) (1992), 637-643. MR **93d:**46037

(Yunan Cui and Hongwei Zhu) Harbin University of Science and Technology, Department of Mathematics, Harbin (150080), China
E-mail address, Y. Cui: `cuiya@hkd.hrbust.edu.cn`

(Henryk Hudzik) Adam Mickiewicz University, Faculty of Mathematics and Computer Science, Matejki 48/49, 60-769 Poznań, Poland
E-mail address: `hudzik@math.amu.edu.pl`