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EXTENDIBILITY OF HOMOGENEOUS POLYNOMIALS ON BANACH SPACES

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Abstract. We study the n-homogeneous polynomials on a Banach space X that can be extended to any space containing X. We show that there is an upper bound on the norm of the extension. We construct a predual for the space of all extendible n-homogeneous polynomials on X and we characterize the extendible 2-homogeneous polynomials on X when X is a Hilbert space, an $\mathcal{L}_1$-space or an $\mathcal{L}_\infty$-space.

1. Introduction

There is no Hahn-Banach theorem for n-homogeneous polynomials on a Banach space when $n > 1$. For example, the 2-homogeneous polynomial $P(x) = \sum_k x_k^2$ on l_2 cannot be extended to any $C(K)$ space containing l_2, as the presence of the Dunford-Pettis property on $C(K)$ would force P to be weakly sequentially continuous [1]. Even when a homogeneous polynomial can be extended to a larger space, it can happen that the norm cannot be preserved [2, 9]. The Aron-Berner extension process for homogeneous polynomials on a Banach space X [2] avoids these obstacles by requiring that the extension be defined only for a very restricted class of spaces containing X—essentially, the bidual X^{**} and spaces closely related to it (see also [1, 4, 7, 10, 13]). A similar observation applies to the ultrapower method of Dineen-Timoney and Lindström-Ryan [6, 8].

We propose to study those homogeneous polynomials on a Banach space X that extend to *every* space containing X. We do not require that the norm be preserved. We show that, if a homogeneous polynomial, P, on X can be extended to every space containing X, then it is possible to put an upper bound on the norms of the extensions. This enables us to put a natural norm on the space $\mathcal{P}_e(^nX)$ of "extendible" n-homogeneous polynomials. We show that $\mathcal{P}_e(^nX)$ is complete in this norm. We then construct a predual for the Banach space $\mathcal{P}_e(^nX)$. We conclude with some special classes of spaces. We characterize the extendible 2-homogeneous polynomials on X when X is a Hilbert space, an $\mathcal{L}_1$-space or an $\mathcal{L}_\infty$-space.

All the Banach spaces considered can be taken over the real or complex numbers. $\mathcal{P}(^nX;Y)$ denotes the Banach space of bounded, n-homogeneous polynomials from X into Y. Thus if $P \in \mathcal{P}(^nX;Y)$, then there exists a unique bounded symmetric multilinear mapping $A : X^n \to Y$ such that $P(x) = A(x,\dots,x)$ for every $x \in X$.

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When Y is the scalar field we denote this space by $\mathcal{P}(^n X)$. For $P \in \mathcal{P}(^n X)$ we can define a bounded linear operator $T : X \to \mathcal{P}(^{n-1} X)$ by $T(x)(y) = A(x, y, \dots, y)$, where A is the multilinear form on X^n that generates P as described above. We shall refer to T as the linear operator associated with P.

2. Extendibility of linear operators and homogeneous polynomials

We begin with the description of a process that will enable us to "paste together" a family of spaces containing X in a coherent way. Let $j_\alpha : X \to Z_\alpha$ be a family of embeddings, indexed by $\alpha \in A$. By an *amalgamation* of this family we shall mean a triple consisting of a Banach space Z, an embedding $j : X \to Z$ and a family of bounded linear operators $i_\alpha : Z_\alpha \to Z$ with $\|i_\alpha\| \le 1$, such that $i_\alpha \circ j_\alpha = j$ for every $\alpha \in A$. Where there is no danger of confusion, we shall refer to the amalgamation $(Z, j, \{i_\alpha\})$ simply as "the amalgamation Z".

An amalgamation may be constructed as follows: first, form the l_1-sum $(\sum_\alpha Z_\alpha)_1$ and let k_α denote the usual embedding of Z_α into this space. Let

$$N = \left\{ (j_\alpha x_\alpha) \in \left(\sum_\alpha Z_\alpha \right)_1 : x_\alpha \in X,\ \sum_\alpha x_\alpha = 0 \right\}.$$

It is easy to see that N is a closed subspace of $(\sum_\alpha Z_\alpha)_1$. Let Z be the quotient space:

$$Z = \left(\sum_\alpha Z_\alpha \right)_1 / N,$$

and let π be the quotient mapping of $(\sum_\alpha Z_\alpha)_1$ onto Z. For each $\alpha \in A$ let $i_\alpha = \pi \circ k_\alpha$. Thus, we have $\|i_\alpha\| \le 1$ for every α. To define an embedding of X into Z we note first that for all $\alpha, \beta \in A$, we have $k_\alpha j_\alpha(x) - k_\beta j_\beta(x) \in N$ for every $x \in X$. Therefore $\pi k_\alpha j_\alpha(x) = \pi k_\beta j_\beta(x)$ and we can define

$$j(x) = \pi k_\alpha j_\alpha(x) \quad \text{for any } \alpha \in A.$$

To see that j is an embedding, let $x \in X$. It is clear from the definition of $j(x)$ that $\|j(x)\| \le \|x\|$. On the other hand, if we choose any $\alpha \in A$, we have, by the definition of the quotient norm,

$$\|j(x)\| = \inf\{\|k_\alpha j_\alpha(x) + u\| : u \in N\}.$$

Now if $u \in N$, then $u = (j_\beta x_\beta)$ where $\sum_\beta x_\beta = 0$ and so

$$\begin{aligned} \|k_\alpha j_\alpha(x) + u\| &= \|j_\alpha(x) + j_\alpha(x_\alpha)\| + \sum_{\beta \ne \alpha} \|j_\beta(x_\beta)\| \\ &= \|x + x_\alpha\| + \sum_{\beta \ne \alpha} \|x_\beta\| \ge \left\| x + x_\alpha + \sum_{\beta \ne \alpha} x_\beta \right\| \\ &= \left\| x + \sum_\beta x_\beta \right\| = \|x\|, \end{aligned}$$

since $\sum x_\beta = 0$. Therefore $\|j(x)\| = \|x\|$ for every $x \in X$. Finally, it follows from the definition of j that $i_\alpha \circ j_\alpha = j$ for every α. Thus $(Z, j, (j_\alpha))$ meets all the requirements for an amalgamation of the family of embeddings (j_α).

We shall say that a bounded n-homogeneous polynomial $P : X \to Y$ is *extendible* if, for every embedding of X into a Banach space Z, there exists an extension of P to a bounded n-homogeneous polynomial $Q : Z \to Y$. With the help of amalgamations, we can show that we can control the norm of the extension independently of the space Z:

Proposition 1. *If $P \in \mathcal{P}(^n X; Y)$ is extendible, then there exists $C > 0$ such that for every embedding of X into a Banach space Z, there is an extension $Q \in \mathcal{P}(^n Z; Y)$ of P with $\|Q\| \le C$.*

Proof. If this were false, then for every $n \in \mathbf{N}$ there would exist a Banach space Z_n and an embedding $j_n : X \to Z_n$ such that every extension of P to Z_n has norm at least n. Let $(Z, j, (i_n))$ be an amalgamation of this sequence of embeddings. Now P has a bounded extension, Q, defined on Z. But then $Q \circ i_n$ is an extension of P to Z_n, and hence

$$\|Q\| \ge \|Q \circ i_n\| \ge n \quad \text{for every } n \in \mathbf{N},$$

which is impossible. This concludes the proof.

Let $\mathcal{P}_e(^n X; Y)$ denote the subset of $\mathcal{P}(^n X; Y)$ consisting of all the extendible n-homogeneous polynomials. $\mathcal{P}_e(^n X; Y)$ is a vector space on which we can define a norm by letting $e(P)$ be the smallest positive real number C with the property that for every space Z containing X there is an extension $Q \in \mathcal{P}(^n Z; Y)$ of P with $\|Q\| \le C$. We have $\|P\| \le e(P)$ for every $P \in \mathcal{P}_e(^n X; Y)$.

Proposition 2. *$(\mathcal{P}_e(^n X; Y), e)$ is complete.*

Proof. It suffices to show that every absolutely summable series is convergent. Accordingly, suppose that $\sum_k e(P_k) < \infty$, where $P_k \in \mathcal{P}_e(^n X; Y)$ for every k. Then $\sum P_k$ is absolutely summable in $\mathcal{P}^n(X; Y)$ and hence it converges in this space to a bounded n-homogeneous polynomial P. Let Z be a space that contains X. For each k there exists an extension Q_k of P_k to Z, with $\|Q_k\| \le e(P_k)$. It follows that the series $\sum Q_k$ converges in $\mathcal{P}(^n Z; Y)$ to a polynomial Q. It is clear that Q extends P, and so $P \in \mathcal{P}_e(^n X; Y)$. Furthermore, we have $\|Q\| \le \sum \|Q_k\| \le \sum e(P_k)$. Finally, we have

$$e\left(P - \sum_{k=1}^{N} P_k\right) = e\left(\sum_{k>N} P_k\right) \le \sum_{k>N} e(P_k),$$

and hence $\sum P_k$ converges to P in $(\mathcal{P}_e(^n X; Y), e)$. This concludes the proof.

We shall see that the uniform norm is equivalent to the norm e on $\mathcal{P}_e(^n X)$ if and only if every polynomial in $\mathcal{P}(^n X)$ is extendible.

3. The predual of the space of extendible polynomials

We begin with the construction of a predual for the space $\mathcal{L}_e(^2 X \times Y)$ of extendible bilinear forms on $X \times Y$. We can define a complete norm, e, on this space exactly as in the previous section. We shall denote the projective norm on the tensor product $X \otimes Y$ by $\pi_{X,Y}$. We refer the reader to [3] for further details on tensor products. Now if i, j are embeddings of X, Y into W, Z respectively, then $i \otimes j$ is an algebraic embedding of $X \otimes Y$ into $W \otimes Z$. Thus we may identify $X \otimes Y$ with a vector subspace of $W \otimes Z$. For $u \in X \otimes Y$ we have $\pi_{W,Z}(u) \le \pi_{X,Y}(u)$. The norms $\pi_{W,Z}$ and $\pi_{X,Y}$ are equivalent on $X \otimes Y$ if and only if every bounded bilinear form on $X \times Y$ has a bounded extension to $W \times Z$. For an element u of $X \otimes Y$, we define

$$\eta(u) = \inf\{\pi_{W,Z}(u) : X \subset W, Y \subset Z\},$$

the infimum being taken over all pairs of embeddings $X \hookrightarrow W$, $Y \hookrightarrow Z$. We claim that η is a reasonable crossnorm on $X \otimes Y$. Obviously, we have $\eta(\lambda u) = |\lambda|\eta(u)$. To see that η is subadditive, let $u_1, u_2 \in X \otimes Y$ and let $\varepsilon > 0$. For each $r = 1, 2$ there exists a pair of embeddings $i_r : X \hookrightarrow W_r$, $j_r : Y \hookrightarrow Z_r$, such that $\pi_{W_r,Z_r}(u_r) \leq \eta(u_r) + \varepsilon/2$. Let $(W, i, (k_r))$, $(Z, j, (l_r))$ respectively be amalgamations of these embeddings. Then we have $i \otimes j = (k_r \otimes l_r) \circ (i_r \otimes j_r)$ for each r and since $\|k_r\|$, $\|l_r\| \leq 1$ it follows that $\pi_{W,Z} \leq \pi_{W_r,Z_r}$ for each r. Therefore

$$\eta(u_1 + u_2) \leq \pi_{W,Z}(u_1 + u_2) \leq \pi_{W,Z}(u_1) + \pi_{W,Z}(u_2) \leq \eta(u_1) + \eta(u_2) + \varepsilon,$$

and hence $\eta(u_1 + u_2) \leq \eta(u_1) + \eta(u_2)$. Next, suppose that $\eta(u) = 0$ for some $u \in X \otimes Y$. Then there is a sequence of pairs of embeddings $X \hookrightarrow W_r$, $Y \hookrightarrow Z_r$ such that $\pi_{W_r,Z_r}(u) < 1/r$. Arguing as above, let W, Z be amalgamations of these sequences. Then we have $\pi_{W,Z}(u) \leq \pi_{W_r,Z_r}(u) < 1/r$ for every k and so $\pi_{W,Z}(u) = 0$, whence $u = 0$. This shows that η is a norm. To see that η is a reasonable crossnorm, note first that if $x \in X$, $y \in Y$, then $\pi_{W,Z}(x \otimes y) = \|x\|\,\|y\|$ for all W, Z and so $\eta(x \otimes y) = \|x\|\,\|y\|$. Secondly, let $\varphi \in X^*$, $\psi \in Y^*$. Let $u \in X \otimes Y$ with $\eta(u) < 1$. Choose embeddings $X \hookrightarrow W$, $Y \hookrightarrow Z$ such that $\pi_{W,Z}(u) < 1$ and let φ_W, ψ_Z be Hahn-Banach extensions of φ, ψ to W, Z respectively. Then

$$|\varphi \otimes \psi(u)| = |\varphi_W \otimes \psi_Z(u)| \leq \|\varphi_W\|\,\|\psi_Z\| = \|\varphi\|\,\|\psi\|.$$

It follows that η is a reasonable crossnorm.

Proposition 3. *$(X\hat{\otimes}_\eta Y)^*$ is isometrically isomorphic to $\mathcal{L}_e(^2X \times Y)$.*

Proof. Every bounded linear functional on $X\hat{\otimes}_\eta Y$ is the linearization, $\widetilde{T}$, of a bounded bilinear form T on $X \times Y$. Since $\widetilde{T}$ is η-continuous, we have $|\widetilde{T}(u)| \leq \eta^*(\widetilde{T})\eta(u) \leq \eta^*(\widetilde{T})\pi_{W,Z}(u)$ for every $u \in X \otimes Y$ and every pair of embeddings $X \hookrightarrow W$, $Y \hookrightarrow Z$. Hence $\widetilde{T}$ is continuous on $X \otimes Y$ for the norm induced by $W \otimes_\pi Z$ and it follows from the Hahn-Banach theorem that T extends to a bilinear form S on $W \times Z$ with $\|S\| \leq \eta^*(\widetilde{T})$. Therefore T is extendible and $e(T) \leq \eta^*(\widetilde{T})$. Conversely, if $T \in \mathcal{L}_e(^2X \times Y)$ and $X \hookrightarrow W$, $Y \hookrightarrow Z$, then T extends to $S \in \mathcal{L}(^2W \times Z)$ with $\|S\| \leq e(T)$. It follows that $|\widetilde{T}(u)| = |\widetilde{S}(u)| \leq e(T)\pi_{W,Z}(u)$ for $u \in X \otimes Y$ and hence $\widetilde{T}$ is η-continuous, with $\eta^*(\widetilde{T}) \leq e(T)$. Therefore $\eta^*(\widetilde{T}) = e(T)$. This completes the proof.

We now construct a predual for the space of extendible n-homogeneous polynomials. The dual of the n-fold symmetric tensor product $\widehat{\bigotimes}^n_{s,\pi} X$ is the Banach space $\mathcal{L}_s(^nX)$ of symmetric, continuous n-linear forms on X. Since this latter space is isomorphic to $\mathcal{P}(^nX)$, it follows that $\widehat{\bigotimes}^n_{s,\pi} X$ can be considered as an "isomorphic" predual of $\mathcal{P}(^nX)$. We prefer to renorm $\widehat{\bigotimes}^n_{s,\pi} X$ so this becomes an isometry, as follows: for $u \in \widehat{\bigotimes}_{s,\pi} X$, we will define

$$\|u\|_\pi = \sup\{|A(u)| : A \in \mathcal{L}_s(^nX) \text{ and } \|\widehat{A}\| = 1\},$$

where $\widehat{A}$ denotes the n-homogeneous polynomial associated with A. This norm is equivalent to the projective norm on $\widehat{\bigotimes}^n_{s,\pi} X$. Let us write x^n for the tensor

$x \otimes x \otimes \cdots \otimes x$. Then, if u belongs to the uncompleted symmetric tensor product $\bigotimes_{s,\pi}^n X$, it follows from the polarization formula that u can be expressed as a linear combination of tensors of the form x^n. We then have another formula for $\|u\|_\pi$:

$$\|u\|_\pi = \inf \left\{ \sum_{j=1}^{m} |\lambda_j| \, \|x_j\|^n : u = \sum_{j=1}^{m} \lambda_j x_j^n \right\}.$$

We shall denote by $\widehat{X}_\pi^{(n)}$ the space $\widehat{\bigotimes}_{s,\pi}^n X$ with this equivalent norm. Then $\mathcal{P}(^n X)$ is the dual space of $\widehat{X}_\pi^{(n)}$. The mapping $x \mapsto x^n$ is a "universal" continuous n-homogeneous polynomial on X: for every $P \in \mathcal{P}(^n X)$ there is a unique $\widetilde{P} \in (\widehat{X}_\pi^{(n)})^*$ with the same norm, such that

$$P(x) = \widetilde{P}(x^n) \quad \text{for every } x \in X.$$

We refer to [11, 12] for further details.

Now the space $\widehat{X}_\pi^{(n)}$ suffers the same defect as the projective tensor product $X \otimes Y$ in that its norm does not respect embeddings of X into larger spaces. Proceeding as we did for $X \otimes Y$, we can define a norm, η, on $X^{(n)}$ by

$$\eta(u) = \inf \left\{ \sum_{j=1}^{m} |\lambda_j| \, \|w_j\|^n : u = \sum_{j=1}^{m} \lambda_j w_j^n, \ w_j \in W, X \subset W \right\},$$

the infimum being taken over all embeddings $X \hookrightarrow W$. We denote by $X_\eta^{(n)}$ the homogeneous product space $X^{(n)}$ with this norm, and we denote the completion of this space by $\widehat{X}_\eta^{(n)}$. We then have $(\widehat{X}_\eta^{(n)})^* = \mathcal{P}_e(^n X)$:

Proposition 4. *$P \in \mathcal{P}(^n X)$ is extendible if and only if the linear form $\widetilde{P}$ on $X^{(n)}$ is continuous with respect to the norm η. The correspondence $P \leftrightarrow \widetilde{P}$ is an isometric isomorphism of $(\mathcal{P}_e(^n X), e)$ with $(\widehat{X}_\eta^{(n)})^*$.*

The proof is similar to the proof of Proposition 3. D. Cardano and I. Zalduendo have shown that integral polynomials are extendible. This also follows from the above proposition—the n-homogeneous polynomial P is integral precisely when the linear form $\widetilde{P}$ is continuous with respect to the injective norm ε and since η is a reasonable crossnorm, we have $\varepsilon \leq \eta$.

Corollary 5. *The norm e is equivalent to the uniform norm on $\mathcal{P}_e(^n X)$ if and only if every n-homogeneous polynomial on X is extendible.*

Proof. If $\mathcal{P}(^n X) = \mathcal{P}_e(^n X)$, then, since $\|\cdot\| \leq e$, it follows that these norms are equivalent. Conversely, suppose these norms are equivalent on $\mathcal{P}_e(^n X)$. Since the uniform norm is the dual of the projective norm and e is the dual of η, it follows that the projective norm is equivalent to η on $X^{(n)}$. Therefore these norms yield the same dual, hence $\mathcal{P}(^n X) = \mathcal{P}_e(^n X)$. This concludes the proof.

Thus, if every n-homogeneous polynomial on X is extendible, then there exists a positive constant C, depending only on X and n, such that for every embedding $X \hookrightarrow Z$, every $P \in \mathcal{P}(^n X)$ has an extension to Z whose norm is at most $C\|P\|$. On the other hand, if X supports an n-homogeneous polynomial that is not extendible, then, using an amalgamation argument, we see that there is an embedding $X \hookrightarrow Z$ and a sequence of n-homogeneous polynomials P_k on X such that every extension of P_k to Z has norm at least equal to k.

4. Examples

We begin with a simple observation:

Lemma 6. *If $P \in \mathcal{P}(^2X)$ is extendible, then the associated linear operator $T : X \to X^*$ is 2-summing.*

Proof. There exists an indexing set I and an embedding $j : X \to l_\infty^I$. Let $Q \in \mathcal{P}(^2 l_\infty^I)$ be an extension of P and let S be the associated linear operator. Since every bounded linear operator from an $\mathcal{L}_\infty$-space into an $\mathcal{L}_1$-space is 2-summing, S is 2-summing. Hence $T = j^* \circ S \circ j$ is 2-summing. This concludes the proof.

In general, this condition is not sufficient for P to be extendible. However, for $\mathcal{L}_1$-spaces, it is enough:

Proposition 7. *Suppose X is an $\mathcal{L}_1$-space. Then $P \in \mathcal{P}(^2X)$ is extendible if and only if the associated linear operator $T : X \to X^*$ is 2-summing.*

Proof. If T is 2-summing, then there exists a probability measure μ and a factorization of T as $u \circ J_2 \circ v$, where $v \in \mathcal{L}(X, L_\infty(\mu))$, $u \in \mathcal{L}(L_2(\mu), X^*)$ and $J_2 : L_\infty(\mu) \to L_2(\mu)$ is the canonical inclusion. Now let $j : X \to Z$ be an embedding. Since $L_\infty(\mu)$ is injective, v extends to an operator $\tilde{v} : Z \to L_\infty(\mu)$ and then $S = u \circ J_2 \circ \tilde{v}$ extends T. Consider the restriction of S^* to X. We have $S^*|_X = \tilde{v}^* \circ J_2^* \circ u^*|_X$. Now, since X is an $\mathcal{L}_1$-space, the operator $u^*|_X : X \to L_2(\mu)$ is 1-summing, and hence 2-summing. Therefore this operator extends to an operator $w : Z \to L_2(\mu)$. Let $R : Z \to Z^*$ be the composition $\tilde{v}^* \circ J_2^* \circ w$ and let Q be the 2-homogeneous polynomial on l_∞^I associated with R. Then Q extends P. This concludes the proof.

Next, we consider linear operators and 2-homogeneous polynomials on a Hilbert space H. The situation concerning extendibility of linear operators on Hilbert spaces is particularly simple. In general, 2-summing operators are extendible, since they factor through $L_\infty(\mu)$ spaces [5]. Conversely, suppose that $T\colon H \to H$ is extendible. Let $j\colon H \to l_\infty^I$ be an embedding. T extends to an operator $S : l_\infty^I \to H$. But every such operator is 2-summing, and hence T is 2-summing. Therefore, the spaces of extendible and 2-summing operators on H coincide.

The situation with 2-homogeneous polynomials is different. Let P be an extendible 2-homogeneous polynomial on H and let $T\colon H \to H$ be the associated linear operator. Let $j : H \to l_\infty^I$ be an embedding. P extends to a 2-homogeneous polynomial Q on l_∞^I which has an associated linear operator $S\colon l_\infty^I \to (l_\infty^I)^*$ with the property that $j^* \circ S \circ j = T$. Since $(l_\infty^I)^*$ is an $\mathcal{L}_1$-space, $S\colon l_\infty^I \to (l_\infty^I)^*$ and $j^*\colon (l_\infty^I)^* \to H$ are both 2-summing, and hence $j^* \circ S$ is nuclear [5, p. 119]. Therefore T is nuclear and it follows that P is also nuclear. Thus every extendible 2-homogeneous polynomial on H is nuclear. On the other hand, it follows easily from the Hahn-Banach theorem that nuclear polynomials are extendible. In summary, we have:

Proposition 8. *Let H be a Hilbert space.*

(a) *A bounded linear operator from H into H is extendible if and only if it is 2-summing.*
(b) *A bounded 2-homogeneous polynomial on H is extendible if and only if it is nuclear.*

Finally, let X be an $\mathcal{L}_\infty$-space. Then X^{**} is injective and so, by the Aron-Berner extension theorem [2] we have:

Proposition 9 (Aron and Berner). *Suppose X is an $\mathcal{L}_\infty$-space. Then every $P \in \mathcal{P}(^n X)$ is extendible.*

References

1. R. Aron, *Extension and lifting theorems for analytic mappings*, Functional Analysis: Surveys and Recent Results II, Math. Stud. 38, North-Holland, 1980, 257–267. MR **81i:**46006
2. R. Aron and P. Berner, *A Hahn-Banach extension theorem for analytic mappings*, Bull. Soc. Math. France **106** (1978), 3–24. MR **80e:**46029
3. A. Defant and K. Floret, *Tensor Norms and Operator Ideals*, North-Holland Math. Studies 176, 1993. MR **94e:**46130
4. A. M. Davie and T. W. Gamelin, *A theorem on polynomial-star approximation*, Proc. Amer. Math. Soc. **106** (1989), 351–356. MR **89k:**46023
5. J. Diestel, H. Jarchow and A. Tonge, *Absolutely Summing Operators*, Cambridge University Press, 1995. MR **96i:**46001
6. S. Dineen and R. Timoney, *Complex geodesics on convex domains*, Progress in Functional Analysis (ed. K. Bierstedt, J. Bonet, J. Horvath and M. Maestre), Math. Studies 170, North-Holland, 1992, 333–365. MR **92m:**46066
7. P. Galindo, D. García, M. Maestre and J. Mujica, *Extension of multilinear mappings on Banach spaces*, Studia Math. **108** (1994), 55–76. MR **95f:**46072
8. M. Lindström and R. A. Ryan, *Applications of ultraproducts to infinite dimensional holomorphy*, Math. Scand. **71** (1992), 229–242. MR **94c:**46090
9. P. Mazet, *A Hahn-Banach theorem for quadratic forms*, preprint.
10. L. Moraes, *A Hahn-Banach extension theorem for some holomorphic functions*, Complex Analysis, Functional Analysis and Approximation Theory (ed. J. Mujica), Math. Studies 125, North-Holland, 1986, 205–220. MR **88f:**46094
11. R. A. Ryan, *Applications of Topological Tensor Products to Infinite Dimensional Holomorphy*, Ph.D. Thesis, Trinity College, Dublin, 1980.
12. R. A. Ryan and J. B. Turret, *Products of linear functionals*, Preprint, 1995.
13. I. Zalduendo, *A canonical extensions for analytic functions on Banach spaces*, Trans. Amer. Math. Soc. **320** (1990), 747–763. MR **90k:**46108

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