

ON THE TOPOLOGICAL BOUNDARY OF SEMI-FREDHOLM OPERATORS

HAÏKEL SKHIRI

(Communicated by Palle E. T. Jorgensen)

ABSTRACT. We prove several distance formulas from a fixed operator in $B(H)$ to some classes of operators connected with the semi-Fredholm ones. Here H is a separable Hilbert space. In particular, Fredholm and upper and lower semi-Fredholm operators have the same boundary in $B(H)$.

Let H be a separable Hilbert space and let $B(H)$ be the algebra of all bounded linear operators on H . For an operator $T \in B(H)$, we will denote by T^* , $R(T)$, $N(T)$ and $\sigma(T)$ its adjoint, range, kernel and spectrum, respectively. Let $K(H)$ be the ideal of compact operators and $C(H) = B(H)/K(H)$ be the Calkin algebra. Denote by $\pi : B(H) \rightarrow C(H)$ the canonical projection. Endowed with the essential norm $\|T\|_e = \|\pi(T)\|$, $C(H)$ is a C^* -algebra.

The index of an operator $T \in B(H)$ will be denoted by $\text{ind}(T)$ and is defined by $\text{ind}(T) = \dim N(T) - \dim N(T^*)$, with the convention $\infty - \infty = 0$.

We introduce the following notation for several classes of operators :

- $F_+ = \{T \in B(H) : R(T) \text{ is closed, } \dim N(T) < \infty\}$ is the set of all upper semi-Fredholm operators.
- $F_- = \{T \in B(H) : R(T) \text{ is closed, } \dim N(T^*) < \infty\}$ is the set of all lower semi-Fredholm operators.
- $F_{\pm} = F_+ \cup F_-$ is the set of semi-Fredholm operators.
- $F = F_+ \cap F_-$ is the set of Fredholm operators.
- $I_n = \{T \in B(H) : \text{ind}(T) = n\}$ with $n \in \mathbb{Z} = \mathbb{Z} \cup \{-\infty, +\infty\}$.
- $F_{\pm}^n = F_{\pm} \cap I_n$, with $n \in \mathbb{Z}$, the connected component of index n in F_{\pm} .

For a set X in $B(H)$, we will denote by $\text{int}X$, \bar{X} and ∂X the interior, closure and (topological) boundary, respectively.

For a linear operator $T \in B(H)$, we will denote by $\sigma_e(T) = \{\lambda \in \mathbb{C} : T - \lambda I \notin F\}$ the essential spectrum of T . Let

$$m_e(T) = \inf\{\sigma_e(|T|)\}$$

(cf. [1]), where $|T| = (T^*T)^{1/2}$, and

$$M_e(T) = \max\{m_e(T); m_e(T^*)\}.$$

Using Theorem 1.1 of [4] and Theorem 3.1 of [7], we easily obtain the following result:

Received by the editors April 30, 1996 and, in revised form, October 14, 1996.
1991 *Mathematics Subject Classification*. Primary 47A53; Secondary 47A55.

Key words and phrases. Hilbert space operators, semi-Fredholm operators, distance formulas.

Theorem 1. *Let $T \in B(H)$ and $n \in \overline{\mathbb{Z}}$. Then*

$$\text{dist}(T, F_{\pm}^n) = \begin{cases} M_e(T) & \text{if } T \notin I_n, \\ 0 & \text{if not.} \end{cases}$$

We recall the following theorem, which gives a characterization of the boundary of connected components of semi-Fredholm operators.

Theorem 2 ([5], Corollary 1.4). *The boundary of $\overline{F_{\pm}^j}$ does not depend on j , and, if $\Delta = \partial\overline{F_{\pm}^j}$, then $\Delta = \partial F_{\pm}^j$ and $B(H) = \Delta \cup \left(\bigcup_{j \in \overline{\mathbb{Z}}} F_{\pm}^j\right)$.*

Remarks. 1) We have $\Delta = \{T \in B(H) : M_e(T) = 0\}$. Indeed, using [4], Theorem 2.1 and Corollary 2.2, we obtain

$$\Delta = \partial F_{\pm}^0 = \partial\overline{G} = \{T \in B(H) : M_e(T) = 0\},$$

where G is the group of invertible operators in $B(H)$.

- 2) The set Δ is stable with respect to compact perturbations: $\Delta + K(H) = \Delta$.
- 3) The set Δ is arcwise connected. Indeed, if $0 \in \Delta$ and $T \in \Delta$, then $tT \in \Delta$ for all $t \in [0, 1]$.

We prove now the following result.

Theorem 3. *Let $T \in B(H)$ and $J \subseteq \overline{\mathbb{Z}}$. Then*

$$(a) \quad \text{dist}\left(T, \bigcup_{j \in J} F_{\pm}^j\right) = \begin{cases} M_e(T) & \text{if } \text{ind}(T) \notin J, \\ 0 & \text{if not,} \end{cases}$$

and

$$(b) \quad \text{dist}\left(T, \bigcup_{j \in J} I_j\right) = \begin{cases} M_e(T) & \text{if } \text{ind}(T) \notin J, \\ 0 & \text{if not.} \end{cases}$$

Proof. (a) Let $n = \text{ind}(T) \in \overline{\mathbb{Z}}$. If $n \in J$, then $F_{\pm}^n \subseteq \bigcup_{j \in J} F_{\pm}^j$. Therefore

$$0 \leq \text{dist}\left(T, \bigcup_{j \in J} F_{\pm}^j\right) \leq \text{dist}(T, F_{\pm}^n).$$

But, using Theorem 1, we obtain $\text{dist}(T, F_{\pm}^n) = 0$. Hence, $\text{dist}\left(T, \bigcup_{j \in J} F_{\pm}^j\right) = 0$.

Suppose now that $n \notin J$ and let $j_0 \in J$. Then, using once again Theorem 1 and the relation $n \neq j_0$, we get

$$\text{dist}\left(T, \bigcup_{j \in J} F_{\pm}^j\right) \leq \text{dist}(T, F_{\pm}^{j_0}) = M_e(T).$$

We now show the converse inequality. For every $S \in \bigcup_{j \in J} F_{\pm}^j$, we have $\text{ind}(S) \neq n = \text{ind}(T)$. Then Theorem 1.1 of [6] implies $\|T - S\| \geq M_e(T)$. Therefore,

$$\text{dist}\left(T, \bigcup_{j \in J} F_{\pm}^j\right) \geq M_e(T),$$

and (a) is proved.

In order to prove (b), it suffices to remark that $\bigcup_{j \in J} F_{\pm}^j \subseteq \bigcup_{j \in J} I_j$ and to use (a). Indeed, if $\text{ind}(T) = n \in J$, then, using (a), we obtain

$$0 \leq \text{dist}\left(T, \bigcup_{j \in J} I_j\right) \leq \text{dist}\left(T, \bigcup_{j \in J} F_{\pm}^j\right) = 0.$$

On the other hand, if $n \notin J$, then for all $S \in \bigcup_{j \in J} I_j$, we have $\|T - S\| \geq M_e(T)$ (cf. [6], Theorem 1.1). This yields

$$\text{dist}\left(T, \bigcup_{j \in J} I_j\right) \geq M_e(T).$$

Using (a) again, we also find that

$$\text{dist}\left(T, \bigcup_{j \in J} I_j\right) \leq \text{dist}\left(T, \bigcup_{j \in J} F_{\pm}^j\right) = M_e(T),$$

which completes the proof. ■

As consequences, the results below easily follow. Assertions (b) and (c) of the following corollary are also in [2, Theorems 12 and 13].

Corollary 4. *Let $T \in B(H)$. Then:*

- (a) $\text{dist}(T, F) = \begin{cases} M_e(T) & \text{if } \text{ind}(T) = \pm\infty, \\ 0 & \text{if not.} \end{cases}$
- (b) $\text{dist}(T, F_+) = \begin{cases} m_e(T^*) & \text{if } \text{ind}(T) = +\infty, \\ 0 & \text{if } \text{ind}(T) \neq +\infty. \end{cases}$
- (c) $\text{dist}(T, F_-) = \begin{cases} m_e(T) & \text{if } \text{ind}(T) = -\infty, \\ 0 & \text{if } \text{ind}(T) \neq -\infty. \end{cases}$

Proof. (a) follows from Theorem 3 above with $J = \mathbb{Z}$, while (b) and (c) follow from [2, Lemma 7] and Theorem 3 above with $J = \mathbb{Z} \cup \{-\infty\}$ and $J = \mathbb{Z} \cup \{+\infty\}$, respectively. ■

Now we can prove the following equalities.

Theorem 5. *Let $J \subseteq \overline{\mathbb{Z}}$. Then*

$$(a) \quad \overline{\bigcup_{j \in J} F_{\pm}^j} = \Delta \cup \left(\bigcup_{j \in J} F_{\pm}^j \right) = \bigcup_{j \in J} \overline{F_{\pm}^j} ;$$

$$(b) \quad \partial \left(\bigcup_{j \in J} F_{\pm}^j \right) = \Delta.$$

Proof. (a) We prove the first equality. If $J = \overline{\mathbb{Z}}$, then the result follows from Theorem 2 since $\bigcup_{j \in J} F_{\pm}^j = B(H)$.

Suppose that $J \neq \overline{\mathbb{Z}}$. Then, using Theorem 2 again, we have

$$B(H) = \Delta \cup \left(\bigcup_{j \in J} F_{\pm}^j \right) \cup \left(\bigcup_{j \in \overline{\mathbb{Z}} \setminus J} F_{\pm}^j \right).$$

It follows that $\Delta \cup \left(\bigcup_{j \in J} F_{\pm}^j \right)$ is closed, being the complement of the open set $\left(\bigcup_{j \in \overline{\mathbb{Z}} \setminus J} F_{\pm}^j \right)$ in $B(H)$. Therefore

$$\overline{\bigcup_{j \in J} F_{\pm}^j} \subseteq \Delta \cup \left(\bigcup_{j \in J} F_{\pm}^j \right).$$

In order to show the other inclusion, it suffices to see that $\Delta \subseteq \overline{\bigcup_{j \in J} F_{\pm}^j}$ (see Theorem 3).

The second equality follows from

$$\bigcup_{j \in J} \overline{F_{\pm}^j} = \bigcup_{j \in J} \left(\Delta \cup F_{\pm}^j \right).$$

(b) Since $\bigcup_{j \in J} F_{\pm}^j$ is open in $B(H)$, we have

$$\partial \left(\bigcup_{j \in J} F_{\pm}^j \right) = \left(\overline{\bigcup_{j \in J} F_{\pm}^j} \right) \setminus \left(\bigcup_{j \in J} F_{\pm}^j \right) = \left(\Delta \cup \left(\bigcup_{j \in J} F_{\pm}^j \right) \right) \setminus \left(\bigcup_{j \in J} F_{\pm}^j \right) = \Delta.$$

The proof is complete. ■

This easily implies the following consequence:

Corollary 6. *We have*

$$(a) \quad \overline{F} = F \cup \Delta ; \overline{F_+} = F_+ \cup \Delta \quad \text{and} \quad \overline{F_-} = F_- \cup \Delta ;$$

$$(b) \quad \partial F = \partial F_+ = \partial F_- = \Delta.$$

We also have

Corollary 7. *Let $J \subseteq \overline{\mathbb{Z}}$. Then*

$$\overline{\bigcup_{j \in J} I_j} = \Delta \cup \left(\bigcup_{j \in J} F_{\pm}^j \right) = \Delta \cup \left(\bigcup_{j \in J} I_j \right).$$

Proof. Using Theorems 3 and 5, we obtain

$$\Delta \cup \left(\bigcup_{j \in J} I_j \right) \subseteq \overline{\bigcup_{j \in J} I_j} = \overline{\bigcup_{j \in J} F_{\pm}^j} = \Delta \cup \left(\bigcup_{j \in J} F_{\pm}^j \right) \subseteq \Delta \cup \left(\bigcup_{j \in J} I_j \right).$$

■

For the interior and for the boundary of the closure of sets considered in Theorem 5, we have

Theorem 8. *Let $J \subseteq \overline{\mathbb{Z}}$, $J \neq \overline{\mathbb{Z}}$. Then*

$$(a) \quad \text{int} \left(\overline{\bigcup_{j \in J} F_{\pm}^j} \right) = \bigcup_{j \in J} F_{\pm}^j = \text{int} \left(\bigcup_{j \in J} \overline{F_{\pm}^j} \right);$$

$$(b) \quad \partial \left(\overline{\bigcup_{j \in J} F_{\pm}^j} \right) = \partial \left(\bigcup_{j \in J} \overline{F_{\pm}^j} \right) = \Delta .$$

Proof. (a) We show the first equality. The inclusion

$$\bigcup_{j \in J} F_{\pm}^j \subseteq \text{int} \left(\overline{\bigcup_{j \in J} F_{\pm}^j} \right)$$

is clear. In order to show the other one, let $T \in \text{int} \left(\overline{\bigcup_{j \in J} F_{\pm}^j} \right)$. If T is semi-Fredholm, then $T \in \bigcup_{j \in J} F_{\pm}^j$. Indeed, using Theorem 5, we have

$$\begin{aligned} T \in \text{int} \left(\overline{\bigcup_{j \in J} F_{\pm}^j} \right) &= \text{int} \left(\Delta \cup \left[\bigcup_{j \in J} F_{\pm}^j \right] \right) \\ &\subset \Delta \cup \left(\bigcup_{j \in J} F_{\pm}^j \right) \end{aligned}$$

and, because $T \notin \Delta$, we get $T \in \bigcup_{j \in J} F_{\pm}^j$.

Suppose now that T is not semi-Fredholm. Then $T \in \Delta$. Consider $n \notin J$ (this is always possible since $J \neq \overline{\mathbb{Z}}$). Using Theorem 2, we obtain $T \in \Delta = \partial F_{\pm}^n$. Therefore

$$T \in \text{int} \left(\overline{\bigcup_{j \in J} F_{\pm}^j} \right) \cap \partial F_{\pm}^n,$$

which implies the non-voidness of $\left(\overline{\bigcup_{j \in J} F_{\pm}^j} \right) \cap F_{\pm}^n$. The continuity of the index yields a contradiction.

The second equality follows from Theorem 5.

(b) Using (a) and Theorem 5, we obtain

$$\begin{aligned} \partial\left(\bigcup_{j \in J} \overline{F_{\pm}^j}\right) &= \partial\left(\overline{\bigcup_{j \in J} F_{\pm}^j}\right) \\ &= \left(\overline{\bigcup_{j \in J} F_{\pm}^j}\right) \setminus \text{int}\left(\overline{\bigcup_{j \in J} F_{\pm}^j}\right) \\ &= \left(\Delta \cup \left[\bigcup_{j \in J} F_{\pm}^j\right]\right) \setminus \left(\bigcup_{j \in J} F_{\pm}^j\right) = \Delta . \end{aligned}$$

The proof is complete. ■

The following consequence can be easily obtained.

Corollary 9. *We have*

$$(a) \quad \text{int}(\overline{F}) = F ; \text{int}(\overline{F_+}) = F_+ \text{ and } \text{int}(\overline{F_-}) = F_- ;$$

and

$$(b) \quad \partial\overline{F} = \partial F = \partial\overline{F_+} = \partial F_+ = \partial\overline{F_-} = \partial F_- = \Delta .$$

We also have

Corollary 10. *Let $J \subset \overline{\mathbb{Z}}$, $J \neq \overline{\mathbb{Z}}$. Then*

$$(a) \quad \text{int}\left(\overline{\bigcup_{j \in J} I_j}\right) = \bigcup_{j \in J} F_{\pm}^j = \text{int}\left(\bigcup_{j \in J} I_j\right) ;$$

$$(b) \quad \partial\left(\overline{\bigcup_{j \in J} I_j}\right) = \partial\left(\bigcup_{j \in J} I_j\right) = \Delta .$$

Proof. (a) The first equality follows from Corollary 7 and Theorems 5 and 8.

For the second equality, it is sufficient to note that

$$\bigcup_{j \in J} F_{\pm}^j \subseteq \text{int}\left(\bigcup_{j \in J} I_j\right) \subseteq \text{int}\left(\overline{\bigcup_{j \in J} F_{\pm}^j}\right) = \bigcup_{j \in J} F_{\pm}^j .$$

Indeed, the first two inclusions are obvious and the last equality is (a).

(b) is a direct consequence of (a) and of Corollary 7. ■

We introduce the following notation:

- $G_+ = \{T \in B(H) : T \text{ is left invertible}\}.$
- $G_- = \{T \in B(H) : T \text{ is right invertible}\}.$
- $G_{\pm} = G_+ \cup G_-$: the set of one-sided invertible operators.
- $G = G_+ \cap G_-$: the set of invertible operators.

For $n \in \overline{\mathbb{Z}}$, let $G_{\pm}^n = G_{\pm} \cap I_n$. For $n \in \overline{\mathbb{Z}}_- = \mathbb{Z}_- \cup \{-\infty\}$, we denote $G_+^n = G_+ \cap I_n$, while for $n \in \overline{\mathbb{Z}}_+ = \mathbb{Z}_+ \cup \{+\infty\}$, we set $G_-^n = G_- \cap I_n$.

Theorem 11. *Let $T \in B(H)$ and $J \subseteq \overline{\mathbb{Z}}$. Then*

$$\text{dist}\left(T, \bigcup_{j \in J} G_{\pm}^j\right) = \begin{cases} M_e(T) & \text{if } \text{ind}(T) \notin J, \\ 0 & \text{if not.} \end{cases}$$

Proof. Let $n = \text{ind}(T)$. If $n \in J$, then $G_{\pm}^n \subseteq \bigcup_{j \in J} G_{\pm}^j$, so, using Theorem 3.1 of [7], we have

$$0 \leq \text{dist}\left(T, \bigcup_{j \in J} G_{\pm}^j\right) \leq \text{dist}(T, G_{\pm}^n) = 0.$$

Suppose now that $n \notin J$. Let $j_0 \in J$. Using Theorem 3.1 of [7], we get

$$(1) \quad 0 \leq \text{dist}\left(T, \bigcup_{j \in J} G_{\pm}^j\right) \leq \text{dist}(T, G_{\pm}^{j_0}) = M_e(T).$$

On the other hand, for all $L \in \bigcup_{j \in J} G_{\pm}^j$, $\text{ind}(L) \neq n = \text{ind}(T)$. Therefore, using Theorem 1.1 of [6], we have $\|T - L\| \geq M_e(T)$. Thus

$$(2) \quad \text{dist}\left(T, \bigcup_{j \in J} G_{\pm}^j\right) \geq M_e(T).$$

Now (1) and (2) imply the desired equality. ■

We obtain the following consequence.

Corollary 12. *Let $T \in B(H)$, and $J \subseteq \overline{\mathbb{Z}}_+$. Then*

$$\text{dist}\left(T, \bigcup_{j \in J} G_-^j\right) = \begin{cases} M_e(T) & \text{if } \text{ind}(T) \notin J, \\ 0 & \text{if } \text{ind}(T) \in J. \end{cases}$$

A similar result can be stated for $\text{dist}(T, \bigcup_{j \in J} G_+^j)$ if $J \subseteq \overline{\mathbb{Z}}_-$. For the distance of T to the set $G_- \setminus G$ we obtain the following formula:

Corollary 13. *Let $T \in B(H)$. Then*

$$\text{dist}(T, G_- \setminus G) = \begin{cases} M_e(T) & \text{if } \text{ind}(T) \leq 0, \\ 0 & \text{if not.} \end{cases}$$

The following result gives a description of the closure, the interior and the boundary of $\bigcup_{j \in J} G_{\pm}^j$ for $J \subseteq \overline{\mathbb{Z}}$.

Theorem 14. *Let $J \subseteq \overline{\mathbb{Z}}$. Then:*

$$(a) \quad \overline{\bigcup_{j \in J} G_{\pm}^j} = \Delta \cup \left(\bigcup_{j \in J} F_{\pm}^j\right) = \bigcup_{j \in J} \overline{G_{\pm}^j}.$$

If in addition $J \neq \overline{\mathbb{Z}}$, then:

$$(b) \quad \text{int}\left(\overline{\bigcup_{j \in J} G_{\pm}^j}\right) = \bigcup_{j \in J} F_{\pm}^j = \text{int}\left(\bigcup_{j \in J} \overline{G_{\pm}^j}\right),$$

$$(c) \quad \partial\left(\overline{\bigcup_{j \in J} G_{\pm}^j}\right) = \partial\left(\bigcup_{j \in J} \overline{G_{\pm}^j}\right) = \Delta.$$

Proof. (a) We prove the first equality. If $J = \overline{\mathbb{Z}}$, then, using [3, Problem 109], G_{\pm} is dense in $B(H)$ and the result follows from Theorem 2.

Suppose now that $J \neq \overline{\mathbb{Z}}$. Using Theorem 5, we obtain that $\Delta \cup \left(\bigcup_{j \in J} F_{\pm}^j\right)$ is closed. But $\bigcup_{j \in J} G_{\pm}^j \subseteq \Delta \cup \left(\bigcup_{j \in J} F_{\pm}^j\right)$. Thus

$$(1) \quad \overline{\bigcup_{j \in J} G_{\pm}^j} \subseteq \Delta \cup \left(\bigcup_{j \in J} F_{\pm}^j\right).$$

On the other hand, using Theorem 11, we have:

$$(2) \quad \Delta \cup \left(\bigcup_{j \in J} F_{\pm}^j\right) \subseteq \overline{\bigcup_{j \in J} G_{\pm}^j}.$$

The first equality follows from (1) and (2).

The second equality follows from $\overline{G_{\pm}^j} = \overline{F_{\pm}^j}$, for all $j \in \overline{\mathbb{Z}}$. This follows easily from Theorem 1 and Theorem 11.

(b) is a direct consequence of (a), Theorem 5 and Theorem 8.

(c) Since the first equality follows from (a) and (b), it sufficient to show the second one. Using (a) and (b), we have

$$\partial\left(\bigcup_{j \in J} \overline{G_{\pm}^j}\right) = \left(\Delta \cup \left[\bigcup_{j \in J} F_{\pm}^j\right]\right) \setminus \left(\bigcup_{j \in J} F_{\pm}^j\right) = \Delta.$$

The proof is complete. ■

We obtain as consequences the following formulas :

Corollary 15. 1) a) $\overline{G_+} = \Delta \cup \left(\bigcup_{j < 0} F_{\pm}^j\right),$

b) $\overline{G_-} = \Delta \cup \left(\bigcup_{j \geq 0} F_{\pm}^j\right);$

2) a) $\text{int}(\overline{G_+}) = \bigcup_{j \leq 0} F_{\pm}^j,$

b) $\text{int}(\overline{G_-}) = \bigcup_{j \geq 0} F_{\pm}^j;$

3) $\partial \overline{G_+} = \partial \overline{G_-} = \Delta.$

Corollary 16. 1) a) $\overline{G_+ \setminus G} = \Delta \cup \left(\bigcup_{j < 0} F_{\pm}^j\right),$

b) $\overline{G_- \setminus G} = \Delta \cup \left(\bigcup_{j > 0} F_{\pm}^j\right);$

2) a) $\text{int}(\overline{G_+ \setminus G}) = \bigcup_{j < 0} F_{\pm}^j,$

b) $\text{int}(\overline{G_- \setminus G}) = \bigcup_{j > 0} F_{\pm}^j;$

3) $\partial(\overline{G_+ \setminus G}) = \partial(\overline{G_- \setminus G}) = \Delta.$

Similar formulas can be given for $\bigcup_{j \in J} G_+^j$, $J \subseteq \overline{\mathbb{Z}}_-$, and for $\bigcup_{j \in J} G_-^j$, $J \subseteq \overline{\mathbb{Z}}_+.$

ACKNOWLEDGEMENT

I would like to thank my supervisor, M. Mbekhta, for his guidance, and C. Badea for his help.

REFERENCES

- [1] R. Bouldin, *The essential minimum modulus*, Indiana. Univ. Math. J. **30** (1981), 513–517. MR **82i**:47001
- [2] R. Bouldin, *Approximation by semi-Fredholm operators with fixed nullity*, Rocky Mountain J. Math. **20** (1990), 39–50. MR **91m**:47021
- [3] P.R. Halmos, *A Hilbert space problem book*, D. Van Nostrand, 1967. MR **34**:8178
- [4] S. Izumino & Y. Kato, *The closure of invertible operators on a Hilbert space*, Acta. Sci. Math. (Szeged) **49** (1985), 321–327. MR **87h**:47101
- [5] M. Mbekhta, *Sur la structure des composantes connexes semi-Fredholm de $B(H)$* , Proc. Amer. Math. Soc. **116** (1992), 521–524. MR **92m**:48021
- [6] D.D. Rogers, *Approximation by unitary and essentially unitary operators*, Acta. Sci. Math. (Szeged) **39** (1977), 141–151. MR **56**:6439
- [7] P. Y. Wu, *Approximation by invertible and noninvertible operators*, J. Approx. Theory, **56** (1989), 267–276. MR **90c**:47003

DÉPARTEMENT DE MATHÉMATIQUES, BÂT. M2, UNIVERSITÉ DE LILLE I, F-59655 VILLENEUVE D'ASCQ, FRANCE

E-mail address: `skhiri@gat.univ-lille1.fr`