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P-CONVEXITY OF ORLICZ-BOCHNER SPACES

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Abstract. A characterization of P-convexity of arbitrary Banach space is given. Moreover, it is proved that the Orlicz-Bochner function space $L_{\Phi}(\mu, X)$ is P-convex if and only if both spaces $L_{\Phi}(\mu)$ and X are P-convex. In particular, the Lebesgue-Bochner space $L^p(\mu, X)$ with $1 < p < \infty$ is P-convex iff X is P-convex.

1. Introduction

Relationships between various kinds of convexity of Banach spaces and reflexivity have been developed by many authors. D. Giesy [5] and R.C. James [11] raised the question whether Banach spaces which are uniformly non-$l_n^{(1)}$ with some positive integer $n \geq 2$ (such spaces are called B-convex) are reflexive. James [11] settled the question affirmatively for $n = 2$ and gave a partial result for $n = 3$. Afterwards, the same author presented in [12] an example of a nonreflexive uniformly non-$l_3^{(1)}$ Banach space. It was natural to ask whether reflexivity is implied by some slightly stronger geometric property. Such a property was introduced by C.A. Kottman [16], and was called P-convexity. Namely, a Banach space $(X, \|\cdot\|_X)$ is said to be *P-convex*, if there exist $\epsilon > 0$ and $n \in \mathcal{N}$ such that for all $x_1, x_2, ..., x_n \in S(X)$

$$\min\left\{\|x_i - x_j\|_X : i, j \leq n,\ i \neq j\right\} \leq 2 - \epsilon,$$

where $S(X)$ denotes the unit sphere of X.

Kottman proved that every P-convex Banach space is reflexive. D. Amir and C. Franchetti [2] showed that in Banach spaces P-convexity follows from uniform convexity as well as from uniform smoothness. In Orlicz and Musielak-Orlicz spaces of real functions P-convexity is equivalent to reflexivity (see [23], [14], [15] and [24]).

In this paper a characterization of P-convexity of an arbitrary Banach space is given. This result enables us to consider P-convexity in Orlicz-Bochner spaces $L_{\Phi}(\mu, X)$. One of the fundamental problems in these spaces is the question of whether or not a geometrical property lifts from X to $L_{\Phi}(\mu, X)$. Although the answer to such a question can often be guessed, the proof of such a response is usually nontrivial. Considerations of that type for various kinds of convexity for $L^p(\mu, X)$ were carried out by many authors (see for instance [6], [7], [17], [18], [20], [21], [22]). We show that $L_{\Phi}(\mu, X)$ is P-convex iff both $L_{\Phi}(\mu)$ and X are P-convex.

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Some similar criteria for B-convexity of Orlicz-Bochner spaces were obtained in [1], [3], [9] and [13].

Denote by $\mathcal{N}$ and $\mathcal{R}$ the sets of natural and real numbers, respectively. Let (T, Σ, μ) be a $\sigma-$finite, complete and non-atomic measure space. Denote by L^0 the set of all μ-equivalence classes of real valued measurable functions defined on T.

A convex, even function $\Phi : \mathcal{R} \longrightarrow [0, \infty)$ is said to be an *Orlicz function* iff Φ vanishes at zero only and $\frac{\Phi(u)}{u} \longrightarrow 0$ as $u \longrightarrow 0$.

For every Orlicz function Φ we define *the complementary function* $\Phi^* : \mathcal{R} \longrightarrow [0, \infty)$ by the formula

$$\Phi^*(v) = \sup_{u>0} \{u\,|v| - \Phi(u)\}$$

for every $v \in \mathcal{R}$.

We say an Orlicz function Φ *satisfies the* Δ_2*-condition for all* u (*for large* u) if there is a constant $k > 2$ (there are constants $u_0 > 0$ and $k > 2$) such that

$$\Phi(2u) \leq k\Phi(u)$$

for every $u \in \mathcal{R}$ (for every $|u| \geq u_0$), respectively. We write $\Phi \in \overline{\Delta}_2$ ($\Phi \in \Delta_2$) if Φ satisfies the Δ_2-condition for all u (for large u), respectively.

It is well known that the Δ_2-condition for all u is equivalent to the following so-called Δ_l-condition:

For every $l > 1$ there exists $k_l > 1$ such that for every $u \in \mathcal{R}$ we have

$$\Phi(lu) \leq k_l \Phi(u). \tag{1}$$

Similarly the Δ_2-condition for large u is equivalent to the fact that for every $w > 0$ and every $l > 1$ there exists $k_{l,w} > 1$ such that $\Phi(lu) \leq k_{l,w}\Phi(u)$ for all $|u| > w$.

Lemma 1. *For any Orlicz function* Φ *the following assertions hold true:*

a) *If* $\Phi^* \in \overline{\Delta}_2$, *then there exist numbers* $a \in (0,1)$ *and* $\gamma = \gamma(a) \in (0,1)$ *such that*

$$\Phi\left(\frac{u+v}{2}\right) \leq \frac{1}{2}(1-\gamma)\left(\Phi(u) + \Phi(v)\right) \tag{2}$$

for all u, v *satisfying* $\left|\frac{v}{u}\right| \leq a$.

b) *If* $\Phi^* \in \Delta_2$, *then for every* $w > 0$ *there exist numbers* $a = a(w) \in (0,1)$ *and* $\gamma = \gamma(a(w)) \in (0,1)$ *such that the inequality (2) holds true for all* $u \geq w$ *and* v *satisfying* $\left|\frac{v}{u}\right| \leq a$.

Proof. *a*) was proved in [3]. Modifying Lemma 2 in [1] and applying it to our case, we get *b*). □

Define

$$I_\Phi(x) = \int_T \Phi(x(t))\, d\mu$$

for every $x \in L^0$. Then I_Φ is a convex modular on L^0. By the *Orlicz space* $L_\Phi(\mu)$ we mean the space

$$L_\Phi(\mu) = \left\{x \in L^0 : I_\Phi(cx) < \infty \text{ for some } c > 0\right\}$$

equipped with the *Luxemburg norm* defined by

$$\|x\|_L = \inf\left\{\epsilon > 0 : I_\Phi\left(\frac{x}{\epsilon}\right) \leq 1\right\}.$$

For more details we refer to [19].

Now let us define the type of spaces that will be considered in this paper. For a real Banach space $(X, \|\cdot\|_X)$, denote by $M(T,X)$, or simply by $M(X)$, the family of strongly measurable functions $f : T \longrightarrow X$, where functions which are equal μ-almost everywhere are identified. A modular on $M(T,X)$ can be defined by the formula

$$\widetilde{I}_\Phi(f) = I_\Phi\left(\|f(\cdot)\|_X\right)$$

for every $f \in M(T,X)$. Let

$$L_\Phi(\mu, X) = \left\{f \in M(X) : \|f(\cdot)\|_X \in L_\Phi(\mu)\right\}.$$

Then $L_\Phi(\mu, X)$, equipped with the norm

$$\|f\| = \left\|\|f(\cdot)\|_X\right\|_L,$$

becomes a Banach space, called the *Orlicz-Bochner space.*

2. Results

Lemma 2. *A Banach space X is P-convex iff there exist $n \in \mathcal{N}$ and $\delta > 0$ such that for any elements $x_1, x_2, ..., x_n \in X \setminus \{0\}$, two indices i_0, j_0 can be found such that*

$$\left\|\frac{x_{i_0} - x_{j_0}}{2}\right\|_X \le \frac{\|x_{i_0}\|_X + \|x_{j_0}\|_X}{2}\left(1 - \frac{2\delta \min\left\{\|x_{i_0}\|_X, \|x_{j_0}\|_X\right\}}{\|x_{i_0}\|_X + \|x_{j_0}\|_X}\right).$$

Proof. Suppose X is P-convex. Then there exist $\delta > 0$ and $n \in \mathcal{N}$ such that for any $x_1, x_2, ..., x_n \in X \setminus \{0\}$ natural numbers i_0, j_0 can be found such that

$$\frac{1}{2}\left\|\frac{x_{i_0}}{\|x_{i_0}\|_X} - \frac{x_{j_0}}{\|x_{j_0}\|_X}\right\|_X < 1 - \delta.$$

We can assume without loss of generality that $\|x_{i_0}\|_X \ge \|x_{j_0}\|_X$. We have

$$1 - \delta > \frac{1}{2}\left\|\frac{x_{i_0}}{\|x_{i_0}\|_X} - \frac{x_{j_0}}{\|x_{j_0}\|_X}\right\|_X = \left\|\frac{x_{i_0} - x_{j_0}}{2\|x_{j_0}\|_X} + \left(\frac{1}{\|x_{i_0}\|_X} - \frac{1}{\|x_{j_0}\|_X}\right)\frac{x_{i_0}}{2}\right\|_X$$
$$\ge \frac{1}{\|x_{j_0}\|_X}\left\|\frac{x_{i_0} - x_{j_0}}{2}\right\|_X - \frac{1}{2}\|x_{i_0}\|_X\left|\frac{1}{\|x_{i_0}\|_X} - \frac{1}{\|x_{j_0}\|_X}\right|.$$

Hence

$$\frac{1}{\|x_{j_0}\|_X}\left\|\frac{x_{i_0} - x_{j_0}}{2}\right\|_X \le 1 - \delta + \frac{1}{2}\|x_{i_0}\|_X\left|\frac{1}{\|x_{i_0}\|_X} - \frac{1}{\|x_{j_0}\|_X}\right|$$
$$= \frac{1}{2} - \delta + \frac{1}{2}\frac{\|x_{i_0}\|_X}{\|x_{j_0}\|_X} = -\delta + \frac{1}{2}\left(1 + \frac{\|x_{i_0}\|_X}{\|x_{j_0}\|_X}\right) = -\delta + \frac{1}{2}\frac{\|x_{i_0}\|_X + \|x_{j_0}\|_X}{\|x_{j_0}\|_X}.$$

Consequently

$$\left\|\frac{x_{i_0} - x_{j_0}}{2}\right\|_X \le -\delta\|x_{j_0}\|_X + \frac{\|x_{i_0}\|_X + \|x_{j_0}\|_X}{2}$$
$$= \frac{\|x_{i_0}\|_X + \|x_{j_0}\|_X}{2}\left(1 - \frac{2\delta\|x_{j_0}\|_X}{\|x_{i_0}\|_X + \|x_{j_0}\|_X}\right),$$

which finishes the proof of the necessity. The sufficiency follows immediately from the definition of P-convexity. □

Lemma 3. *Let X be a P-convex Banach space and $\Phi \in \overline{\Delta}_2$, $\Phi^* \in \overline{\Delta}_2$. Then there exist $n \in \mathcal{N}$ and $r \in (0,1)$ such that for all $x_1, x_2, ..., x_n \in X$ we have*

$$\sum_{i=1}^{n}\sum_{j=i}^{n} \Phi\left(\frac{\|x_i - x_j\|_X}{2}\right) \le \frac{r}{n}\binom{n}{2}\sum_{j=1}^{n}\Phi\left(\|x_j\|_X\right), \tag{3}$$

where $\binom{n}{k} = \frac{n!}{k!(n-k)!}$.

Proof. Let n be the natural number from the definition of P-convexity of X. Take $x_1, x_2, ..., x_n \in X$. Let k_0 be the index such that $\|x_{k_0}\|_X = \max_{1\le i\le n}\|x_i\|_X$. Choose a number $a \in (0,1)$ such that condition (2) from Lemma 1 is satisfied. For clarity, we will divide the proof into two parts.

I. Suppose that there exists an index i_1 such that $\|x_{i_1}\|_X / \|x_{k_0}\|_X < a$. Then

$$\begin{aligned}\Phi\left(\frac{\|x_{i_1} - x_{k_0}\|_X}{2}\right) &\le \Phi\left(\frac{\|x_{i_1}\|_X + \|x_{k_0}\|_X}{2}\right)\\ &\le \frac{1}{2}(1-\gamma)\left(\Phi\left(\|x_{i_1}\|_X\right) + \Phi\left(\|x_{k_0}\|_X\right)\right),\end{aligned}$$

where $0 < \gamma < 1$ and γ depends only on a. Hence, by the convexity of Φ, we have

$$\begin{aligned}\sum_{i=1}^{n}\sum_{j=i}^{n}\Phi\left(\frac{\|x_i - x_j\|_X}{2}\right) &\le \frac{(n-1)}{2}\sum_{i=1}^{n}\Phi\left(\|x_i\|_X\right) - \frac{\gamma}{2}\left(\Phi\left(\|x_{i_1}\|_X\right) + \Phi\left(\|x_{k_0}\|_X\right)\right)\\ &\le \frac{n-1}{2}\sum_{i=1}^{n}\Phi\left(\|x_i\|_X\right) - \frac{\gamma}{2n}\left(n\Phi\left(\|x_{k_0}\|_X\right)\right)\\ &\le \frac{n-1}{2}\sum_{i=1}^{n}\Phi\left(\|x_i\|_X\right) - \frac{\gamma}{2n}\sum_{i=1}^{n}\Phi\left(\|x_i\|_X\right)\\ &= \frac{n-1}{2}\left(1 - \frac{\gamma}{n(n-1)}\right)\sum_{i=1}^{n}\Phi\left(\|x_i\|_X\right).\end{aligned}$$

II. Suppose that for every $i \in \{1, 2, ..., n\}$ we have $\|x_i\|_X / \|x_{k_0}\|_X \ge a$. Then $x_i \ne 0$ for every i. Let i_0, j_0 be a pair of indices from Lemma 2. Without loss of generality, it can be assumed that

$$a \le \frac{\|x_{i_0}\|_X}{\|x_{j_0}\|_X} \le \frac{1}{a}. \tag{4}$$

Hence

$$\frac{\min\left\{\|x_{i_0}\|_X, \|x_{j_0}\|_X\right\}}{\|x_{i_0}\|_X + \|x_{j_0}\|_X} = \left(1 + \frac{\max\left\{\|x_{i_0}\|_X, \|x_{j_0}\|_X\right\}}{\min\left\{\|x_{i_0}\|_X, \|x_{j_0}\|_X\right\}}\right)^{-1} \ge \frac{1}{1+\frac{1}{a}} = \frac{a}{1+a}.$$

Therefore, using Lemma 2 and inequality (4), we get

$$\begin{aligned}\left\|\frac{x_{i_0} - x_{j_0}}{2}\right\|_X &\le \frac{\|x_{i_0}\|_X + \|x_{j_0}\|_X}{2}\left(1 - \frac{2\delta\min\left\{\|x_{i_0}\|_X, \|x_{j_0}\|_X\right\}}{\|x_{i_0}\|_X + \|x_{j_0}\|_X}\right)\\ &\le \left(1 - \frac{2\delta a}{1+a}\right)\frac{\|x_{i_0}\|_X + \|x_{j_0}\|_X}{2}.\end{aligned}$$

Hence, by the convexity of Φ, we obtain

$$\Phi\left(\left\|\frac{x_{i_0} - x_{j_0}}{2}\right\|_X\right) \le \frac{1}{2}(1-\alpha)\left(\Phi\left(\|x_{i_0}\|_X\right) + \Phi\left(\|x_{j_0}\|_X\right)\right), \tag{5}$$

where $\alpha = \frac{2\delta a}{1+a}$. Putting $l = \frac{1}{a}$ in inequality (1) and denoting $\beta_a = \frac{1}{k_l}$, $v = \frac{1}{a}u$, we get

$$\Phi(av) \geq \beta_a \Phi(v) \tag{6}$$

for every $v \in \mathcal{R}$. Hence, by inequalities (5) and (6), we have

$$\begin{aligned}\sum_{i=1}^{n}\sum_{j=i}^{n}\Phi\left(\frac{\|x_i - x_j\|_X}{2}\right) &\leq \frac{n-1}{2}\sum_{i=1}^{n}\Phi\left(\|x_i\|_X\right) - \frac{\alpha}{2}\left(\Phi\left(\|x_{i_0}\|_X\right) + \Phi\left(\|x_{j_0}\|_X\right)\right)\\ &\leq \frac{n-1}{2}\sum_{i=1}^{n}\Phi\left(\|x_i\|_X\right) - \alpha\Phi\left(a\|x_{k_0}\|_X\right)\\ &\leq \frac{n-1}{2}\sum_{i=1}^{n}\Phi\left(\|x_i\|_X\right) - \alpha\beta_a\Phi\left(\|x_{k_0}\|_X\right)\\ &= \frac{n-1}{2}\sum_{i=1}^{n}\Phi\left(\|x_i\|_X\right) - \frac{\alpha\beta_a}{n}\left(n\Phi\left(\|x_{k_0}\|_X\right)\right)\\ &\leq \frac{n-1}{2}\left(1 - \frac{2\alpha\beta_a}{n(n-1)}\right)\sum_{i=1}^{n}\Phi\left(\|x_i\|_X\right).\end{aligned}$$

Finally, combining the considerations from Parts **I** and **II** and denoting

$$r = \max\left\{1 - \frac{2\alpha\beta_a}{n(n-1)},\ 1 - \frac{\gamma}{n(n-1)}\right\},$$

we get inequality (3), which finishes the proof.

Lemma 4. *Let X be a P-convex Banach space, $\Phi \in \Delta_2$ and $\Phi^* \in \Delta_2$. Then there exists $n \in \mathcal{N}$ such that for every $w > 0$ there is a number $r = r(w) \in (0,1)$ such that inequality* (3) *holds true for all $x_1, x_2, ..., x_n \in X$ satisfying $\sum_{j=1}^{n}\Phi\left(\|x_j\|_X\right) \geq n\Phi(w)$.*

Proof. Let n be the natural number from the definition of P-convexity of X. Fix $w > 0$, and take $x_1, x_2, ..., x_n \in X$ satisfying $\sum_{j=1}^{n}\Phi\left(\|x_j\|_X\right) \geq n\Phi(w)$. Let k_0 be the index such that $\|x_{k_0}\|_X = \max_{1\leq i\leq n}\|x_i\|_X$. Obviously, there exists $i \in \{1, 2, ..., n\}$ that satisfies $\|x_i\|_X \geq w$. Hence $\|x_{k_0}\|_X \geq w$. Now, using Lemma 1*b*) and repeating the same argumentation as in the proof of Lemma 3, we obtain inequality (3) with r depending on w only.

Theorem 1. *The following statements are equivalent:*

(*a*) *$L_\Phi(\mu, X)$ is P-convex.*

(*b*) *Both X and $L_\Phi(\mu)$ are P-convex.*

(*c*) *X is P-convex, and either $\Phi \in \overline{\Delta}_2$ and $\Phi^* \in \overline{\Delta}_2$ if μ is infinite, or $\Phi \in \Delta_2$ and $\Phi^* \in \Delta_2$ if μ is finite.*

Proof. (*a*) ⇒ (*b*). Since the spaces $L_\Phi(\mu)$ and X are embedded isometrically into $L_\Phi(\mu, X)$ and P-convexity is inherited by subspaces, $L_\Phi(\mu)$ and X are P-convex.

(*b*) ⇒ (*c*). Every P-convex Banach space is reflexive. Therefore, by the reflexivity of $L_\Phi(\mu)$, we have $\Phi \in \overline{\Delta}_2$ and $\Phi^* \in \overline{\Delta}_2$ if μ is infinite, or $\Phi \in \Delta_2$ and $\Phi^* \in \Delta_2$ if μ is finite.

(*c*) ⇒ (*a*). Assume that X is P-convex. Consider the following two cases:

I. Suppose (T,Σ,μ) is an infinite measure space, $\Phi\in\overline{\Delta}_2$ and $\Phi^*\in\overline{\Delta}_2$. Take $f_1,f_2,...,f_n\in S(L_\Phi(\mu,X))$. By Lemma 3, there exists $r\in(0,1)$ such that

$$\sum_{i=1}^{n}\sum_{j=i}^{n}\Phi\left(\frac{\|f_i(t)-f_j(t)\|_X}{2}\right)\le\frac{r}{n}\binom{n}{2}\sum_{j=1}^{n}\Phi\left(\|f_j(t)\|_X\right)$$

for μ-a.e. $t\in T$. Integrating both sides of this inequality over T, we get

$$\sum_{i=1}^{n}\sum_{j=i}^{n}\widetilde{I}_\Phi\left(\frac{1}{2}(f_i-f_j)\right)=\sum_{i=1}^{n}\sum_{j=i}^{n}\int_T\Phi\left(\frac{\|f_i(t)-f_j(t)\|_X}{2}\right)dt\le\binom{n}{2}r.$$

Hence there exist i_0,j_0 such that

$$\widetilde{I}_\Phi\left(\frac{1}{2}(f_{i_0}-f_{j_0})\right)\le r$$

and consequently, by $\Phi\in\overline{\Delta}_2$,

$$\|f_{i_0}-f_{j_0}\|\le 2-\epsilon$$

for some $\epsilon>0$ depending on r only (see [9]), i.e. $L_\Phi(\mu,X)$ is P-convex.

II. Now let (T,Σ,μ) be a finite measure space, $\Phi\in\Delta_2$ and $\Phi^*\in\Delta_2$. Take $f_1,f_2,...,f_n\in S(L_\Phi(\mu,X))$ and define

$$E=\left\{t\in T:\sum_{i=1}^{n}\Phi\left(\|f_i(t)\|_X\right)\ge\frac{1}{\mu(T)}\right\}.$$

Since

$$\sum_{i=1}^{n}\widetilde{I}_\Phi\left(f_i\chi_{T\setminus E}\right)\le\int_{T\setminus E}\left(\sum_{i=1}^{n}\Phi\left(\|f_i(t)\|_X\right)\right)d\mu\le 1,$$

we have

$$\sum_{i=1}^{n}\widetilde{I}_\Phi\left(f_i\chi_E\right)\ge n-1.$$

Putting now $w=\Phi^{-1}\left(\frac{1}{n\mu(T)}\right)$, we have $\frac{1}{\mu(T)}=n\Phi(w)$, whence

$$\sum_{i=1}^{n}\Phi\left(\|f_i(t)\|_X\right)\ge n\Phi\left(w\right)$$

for μ-a.e. $t\in E$. Applying now Lemma 4 just with this w, we conclude that there exists $r=r(w)\in(0,1)$ for which

$$\begin{aligned}&\sum_{i=1}^{n}\sum_{j=i}^{n}\widetilde{I}_\Phi\left(\frac{1}{2}(f_i-f_j)\right)\\
&=\sum_{i=1}^{n}\sum_{j=i}^{n}\widetilde{I}_\Phi\left(\frac{1}{2}(f_i-f_j)\chi_{T\setminus E}\right)+\sum_{i=1}^{n}\sum_{j=i}^{n}\widetilde{I}_\Phi\left(\frac{1}{2}(f_i-f_j)\chi_E\right)\\
&\le\frac{1}{n}\binom{n}{2}\sum_{i=1}^{n}\widetilde{I}_\Phi\left(f_i\chi_{T\setminus E}\right)+\frac{r}{n}\binom{n}{2}\sum_{i=1}^{n}\widetilde{I}_\Phi\left(f_i\chi_E\right)\\
&=\frac{1}{n}\binom{n}{2}\left(\sum_{i=1}^{n}\widetilde{I}_\Phi\left(f_i\right)-\sum_{i=1}^{n}\widetilde{I}_\Phi\left(f_i\chi_E\right)\right)+\frac{r}{n}\binom{n}{2}\sum_{i=1}^{n}\widetilde{I}_\Phi\left(f_i\chi_E\right)\end{aligned}$$

$$= \binom{n}{2}\left(1 - \frac{1-r}{n}\sum_{i=1}^{n}\widetilde{I}_{\Phi}\left(f_i\chi_E\right)\right)$$
$$\leq \binom{n}{2}\left(1 - \frac{(1-r)(n-1)}{n}\right).$$

Consequently

$$\sum_{i=1}^{n}\sum_{j=i}^{n}\widetilde{I}_{\Phi}\left(\frac{1}{2}\left(f_i - f_j\right)\right) \leq \binom{n}{2}(1-d),$$

where $d = \frac{(1-r)}{2} \in (0,1)$. Therefore, there are i_0 and j_0 such that $i_0 \neq j_0$ and

$$\widetilde{I}_{\Phi}\left(\frac{1}{2}\left(f_{i_0} - f_{j_0}\right)\right) \leq (1-d).$$

So, by the Δ_2-condition for Φ, we have

$$\|f_{i_0} - f_{j_0}\| \leq 2 - \epsilon$$

for some $\epsilon > 0$ depending on w only (see [9]), i.e. $L_{\Phi}(\mu, X)$ is P-convex.

Corollary 1. *The Lebesgue-Bochner space* $L^p(\mu, X)$ $(1 < p < \infty)$ *is* P*-convex iff* X *is* P*-convex.*

Proof. The Lebesgue space $L^p(\mu)$ is an Orlicz space generated by the Orlicz function $\Phi(u) = |u|^p$ satisfying all the assumptions of Theorem 1.

The following characterization of P-convexity proved directly in [23] (by a long proof), is an immediate consequence of Theorem 1.

Corollary 2. *The following statements are equivalent:*

(*a*) $L_{\Phi}(\mu)$ *is* P*-convex.*

(*b*) $L_{\Phi}(\mu)$ *is reflexive.*

(*c*) *Either* $\Phi \in \overline{\Delta}_2$ *and* $\Phi^* \in \overline{\Delta}_2$ *if* μ *is infinite, or* $\Phi \in \Delta_2$ *and* $\Phi^* \in \Delta_2$ *if* μ *is finite.*

Proof. It is enough to apply Theorem 1 with $X = \mathcal{R}$.

References

[1] G. Alherk and H. Hudzik, *Uniformly non-*$l_n^{(1)}$ *Musielak-Orlicz spaces of Bochner type,* Forum Math. **1** (1989), 403-410. MR **90k:**46081

[2] D. Amir and C. Franchetti, *The radius ratio and convexity properties in normed linear spaces*, Trans. Amer. Math. Soc. **282** (1984), 275-291. MR **85h:**46026

[3] S. Chen and H. Hudzik, *On some convexities of Orlicz and Orlicz-Bochner spaces*, Comm.Math.Univ.Carolinae **29.1** (1988). MR **89g:**46054

[4] M. Denker and R. Kombrink, *On B-convex Orlicz spaces,* Proc. Second Internat. Conf., Oberwolfach (1978), 87-95. MR **80k:**46030

[5] D.P. Giesy, *On a convexity condition in normed linear spaces.* Trans. Amer. Math. Soc. **125** (1966), 114-146. MR **34:**4866

[6] P. Greim, *A note on strong extreme and strongly exposed points in Bochner* L^p *spaces,* Proc. Amer. Math. Soc. **93** (1985), 65-66. MR **86g:**46050

[7] P. Greim, *Strongly exposed points in Bochner* L^p *spaces,* Proc. Amer. Math. Soc. **88** (1983), 81-84. MR **84f:**46044

[8] H. Hudzik, *Uniformly non-*$l_n^{(1)}$ *Orlicz spaces with Luxemburg norm,* Studia Math, **81** (1985), 271-283. MR **87f:**46048

[9] H. Hudzik, *Some classes of uniformly non-square Orlicz-Bochner spaces*, Comm. Math. Univ. Carolinae **26** (1985), 269-274. MR **87f:**46060

[10] H. Hudzik and A. Kamińska, *On uniformly convexifiable and B-convex Musielak-Orlicz spaces,* Commentat. Math. **25** (1985), 59-75. MR **87a:**46043

[11] R.C. James, *Uniformly non-square Banach spaces*, Ann. of Math. **2** 80 (1964), 542-550. MR **30:**4139

[12] R.C. James, *A nonreflexive Banach space that is uniformly nonoctathedral,* Israel J. Math. **18 (**1974), 145-155. MR **50:**8012

[13] A. Kamińska and B. Turett, *Uniformly non-$l_n^{(1)}$ Orlicz-Bochner spaces,* Bull. Acad. Polon. Sci. Math. **35**, 3-4 (1987), 211-218. MR **88j:**46036

[14] P. Kolwicz, *A note on P-convex Orlicz function spaces*, Comm. Math. **35** (1995), 163-170.

[15] P. Kolwicz and R. Płuciennik, *On P-convex Musielak-Orlicz spaces,* Comment. Math. Univ. Carolinae **36.4** (1995), 655–672. MR **96m:**46048

[16] C.A. Kottman, *Packing and reflexivity in Banach spaces,* Trans. Amer. Math. Soc. **150** (1970), 565-576. MR **42:**827

[17] B.L. Lin and P.K. Lin, *Denting points in Bochner L^p spaces,* Proc. Amer. Math. Soc. **97.4** (1986), 629-633. MR **88i:**46030

[18] B.L. Lin, P.K. Lin and S.L. Troyanski, *Property (**H**) in Lebesgue-Bochner function spaces,* Proc. Amer. Math. Soc. **95.4** (1985), 581-584. MR **87a:**46060

[19] J. Musielak, *Orlicz spaces and modular spaces.* Lecture Notes in Math. **1034** (1983), 1-222. MR **85m:**46028

[20] M.A. Smith, *Strongly extreme points in $L^p(\mu, X)$*, Rocky Mnt J. Math. **16** (1986), 1-5. MR **87d:**46043

[21] M.A. Smith and B. Turett, *Rotundity in Lebesgue-Bochner function spaces,* Trans. Amer. Math. Soc. **257** (1980), 105-118. MR **80m:**46031

[22] K. Sundaresan, *Extreme points of the unit cell in Lebesgue-Bochner function spaces,* Colloq. Math. **22** (1970), 111-119. MR **43:**2493

[23] Y. Ye, M. He and R. Płuciennik, *P-convexity and reflexivity of Orlicz spaces,* Comment. Math. **31** (1991), 203-216. MR **92m:**46043

[24] Y. Ye and Y. Huang , *P-convexity property in Musielak-Orlicz sequence spaces*, Collect. Math. **44** (1993), 307-325. MR **95g:**46036

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