

## LEVEL ONE REPRESENTATIONS OF $U_q(G_2^{(1)})$

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ABSTRACT. We construct a level one representation of the quantum affine algebra  $U_q(G_2^{(1)})$  by vertex operators from bosonic fields.

### 1. INTRODUCTION

Quantum affine algebras, the quantum groups associated to the affine Kac-Moody Lie algebras, provide an important underlying symmetry for the quantum Yang-Baxter equation [6] and quantum statistical models [11]. Explicit realizations of their representations are much needed in applications of quantum affine algebras. For instance, the Frenkel-Reshetikhin vertex operators [8] associated with the representations can be used to give solutions of the quantum Knizhnik-Zamolodchikov equation.

Lusztig first studied the abstract representations of quantum Kac-Moody algebras [20]. The program of constructing various representations was started in [7] for level one irreducible modules of ADE types, and subsequently twisted types were given in [12] and  $B_n^{(1)}$  in [4]. Recently we have constructed symplectic quantum affine algebras in [17] for level one and in [16] for level  $-1/2$ . The case of  $F_4^{(1)}$  can also be done similarly [15] using the idea of quantum  $Z$ -algebras [13, 15]. Besides the bosonic constructions, fermionic constructions were furnished in [10]. The  $q$ -Wakimoto construction was also known [21, 1, 22] afterwards. Other representations of classical quantum affine algebras have also been constructed [2]. The exceptional case of  $G_2^{(1)}$  was the only case that has not been explicitly constructed.

The purpose of the paper is to give a explicit level one construction of the quantum affine algebra  $U_q(G_2^{(1)})$  by vertex operators. The idea of the construction follows that of quantum  $Z$ -algebras [13, 15], which is a  $q$ -deformation of the classical ( $q = 1$ )  $Z$ -algebras [19, 18]. We construct some auxiliary vertex operators for the short root. This is parallel to the known constructions of the affine Lie algebra  $G_2^{(1)}$  [5, 9, 23], though the specialization of  $q = 1$  in our construction is new even in the classical case.

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The paper is organized as follows. In section two we review the quantum affine algebra  $U_q(G_2^{(1)})$ . Section three gives the Fock space representation of the quantum affine algebra  $U_q(G_2^{(1)})$  stated in Theorem 3.1. Section four uses quantum vertex operator techniques to prove Theorem 3.1. In the proof of Serre relations we have to show a relation about certain symmetric functions, which is characteristic in the quantum affine algebras as noted in [12]. The Serre relations in  $G_2^{(1)}$  turn out to be the most complicated one among both untwisted and twisted cases, and actually capture all the existed phenomena in other types.

## 2. QUANTUM AFFINE ALGEBRA $U_q(G_2^{(1)})$

Let  $\alpha_i$  ( $i = 1, 2$ ) be the simple roots of the simple Lie algebra  $G_2$ , and  $\lambda_i$  be the fundamental weight. Let  $P = \mathbf{Z}\lambda_1 + \mathbf{Z}\lambda_2$  and  $Q = \mathbf{Z}\alpha_1 + \mathbf{Z}\alpha_2$  be the weight and root lattices. We then let  $\Lambda_i, i \in I = \{0, 1, 2\}$ , be the fundamental weights for the affine Lie algebra  $G_2^{(1)}$ , where  $\Lambda_i = \lambda_i + \Lambda_0$ , and  $\lambda_i$  are the fundamental weights for the finite dimensional simple Lie algebra  $G_2$ . The nondegenerate symmetric bilinear form  $(\cdot | \cdot)$  on  $\mathfrak{h}^*$  is given by

$$(2.1) \quad (\alpha_i | \alpha_j) = d_i a_{ij}, \quad (\delta | \alpha_i) = (\delta | \delta) = 0 \quad \text{for all } i, j,$$

where  $(d_0, d_1, d_2) = (1, 1, 1/3)$  and  $A = (a_{ij}) = \begin{pmatrix} 2 & -1 \\ -3 & 2 \end{pmatrix}$ .

Let  $q_i = q^{d_i} = q^{\frac{1}{2}(\alpha_i | \alpha_i)}, i \in I$ . The quantum affine algebra  $U_q(G_2^{(1)})$  is the associative algebra with 1 over  $\mathbf{C}(q^{1/6})$  generated by the elements  $x_{ik}^\pm, a_{il}, K_i^{\pm 1}, \gamma^{\pm 1/2}, q^{\pm d}$  ( $i = 1, 2, \dots, n, k \in \mathbf{Z}, l \in \mathbf{Z} \setminus \{0\}$ ) with the following defining relations [6, 3, 14]:

$$(2.2) \quad [\gamma^{\pm 1/2}, u] = 0 \quad \text{for all } u \in \mathbf{U},$$

$$(2.3) \quad [a_{ik}, a_{jl}] = \delta_{k+l,0} \frac{[(\alpha_i | \alpha_j)k]}{k} \frac{\gamma^k - \gamma^{-k}}{q - q^{-1}},$$

$$(2.4) \quad [a_{ik}, K_j^{\pm 1}] = [q^{\pm d}, K_j^{\pm 1}] = 0,$$

$$(2.5) \quad q^d x_{ik}^\pm q^{-d} = q^k x_{ik}^\pm, \quad q^d a_{il} q^{-d} = q^l a_{il},$$

$$(2.6) \quad K_i x_{jk}^\pm K_i^{-1} = q^{\pm(\alpha_i | \alpha_j)} x_{jk}^\pm,$$

$$(2.7) \quad [a_{ik}, x_{jl}^\pm] = \pm \frac{[(\alpha_i | \alpha_j)k]}{k} \gamma^{\mp |k|/2} x_{j,k+l}^\pm,$$

$$(2.8) \quad (z - q^{\pm(\alpha_i | \alpha_j)} w) X_i^\pm(z) X_j^\pm(w) + (w - q^{\pm(\alpha_i | \alpha_j)} z) X_j^\pm(w) X_i^\pm(z) = 0,$$

$$(2.9) \quad [X_i^+(z), X_j^-(w)] = \frac{\delta_{ij}}{q_i - q_i^{-1}} \left( \psi_i(w\gamma^{1/2}) \delta\left(\frac{w\gamma}{z}\right) - \varphi_i(w\gamma^{-1/2}) \delta\left(\frac{w\gamma^{-1}}{z}\right) \right)$$

where  $X_i^\pm(z) = \sum_{n \in \mathbf{Z}} x_{i,n} z^{-n-1}$ ,  $\psi_{im}$  and  $\varphi_{im}$  ( $m \in \mathbf{Z}_{\geq 0}$ ) are defined by

$$(2.10) \quad \sum_{m=0}^{\infty} \psi_{im} z^{-m} = K_i \exp\left((q_i - q_i^{-1}) \sum_{k=1}^{\infty} a_{ik} z^{-k}\right),$$

$$(2.11) \quad \sum_{m=0}^{\infty} \varphi_{i,-m} z^m = K_i^{-1} \exp\left(-(q_i - q_i^{-1}) \sum_{k=1}^{\infty} a_{i,-k} z^k\right),$$

$$(2.12) \quad \sum_{r=0, \sigma \in S_m}^{m=1-A_{ij}} (-1)^r \begin{bmatrix} m \\ r \end{bmatrix}_i \sigma.X_i^\pm(z_1) \cdots x_i^\pm(z_r) x_j^\pm(w) x_i^\pm(z_{r+1}) \cdots x_i^\pm(z_m) = 0,$$

where the symmetric group  $S_m$  acts on  $z_i$  by permuting their indices.

### 3. FOCK SPACE REPRESENTATIONS

Let  $a_i(m)$  ( $i = 1, 2$ ) be the operators satisfying the Heisenberg relations for  $U_q(G_2^{(1)})$  at  $\gamma = q$ , and let  $b(m)$  and  $c(m)$  be two independent free bosonic operators with the relations

$$(3.1) \quad [a_i(m), a_j(n)] = \delta_{m+l,0} \frac{[(\alpha_i|\alpha_j)m]}{m} [m]$$

$$(3.2) \quad [b(m), b(n)] = -\delta_{m+l,0} \frac{[2m/3]}{m} [m]$$

$$(3.3) \quad [c(m), c(n)] = \delta_{m+l,0} \frac{[2m/3]}{m} [5m/3]$$

$$(3.4) \quad [a_i(m), b(n)] = [a_i(m), c(n)] = [b(m), c(n)] = 0.$$

Let  $\beta_2$  be an auxiliary simple root isomorphic to  $\alpha_2$ . We define the Fock module  $\mathcal{F}$  as the tensor product of the symmetric algebra generated by  $a_i(-n), b(-n), c(-n)$  ( $n \in \mathbb{N}$ ) and the twisted group algebra  $\mathbb{C}\{P + \mathbb{Z}\beta_2\}$  generated by  $e^\alpha, e^\beta$  subject to the relation

$$e^{\alpha_1} e^{\alpha_2} = -e^{\alpha_2} e^{\alpha_1}, \quad e^\alpha e^\beta = e^\beta e^\alpha, \quad e^\alpha e^\alpha = e^\alpha e^\alpha,$$

where  $\alpha \in P$ , and  $\beta$  is an element of the auxiliary lattice  $\mathbb{Z}\alpha_2$  (another copy of the sublattice generated by the short root  $\alpha_2$ ). In the following we reserve  $\beta$  to denote an element from this auxiliary lattice.

The element 1 is the vacuum state. We define the action by

$$a_i(n).1 = 0 \quad (n > 0), \quad b_i(n).1 = 0 \quad (n > 0),$$

The elements  $a_i(0)$  and  $b(0)$  act as differential operators by

$$a_i(0)e^\alpha = (\alpha_i|\alpha)e^\alpha, \quad b(0)e^\beta = -\frac{2}{3}e^\beta.$$

As usual we define the normal product as the ordered product by moving annihilation operators  $a_i(n), b(n), a_i(0), b(0)$  to the left.

Let's introduce the following operators:

$$Y_1^\pm(z) = \exp(\pm \sum_{n=1}^{\infty} \frac{a_1(-n)}{[n]} q^{\mp \frac{n}{2}} z^n) \exp(\mp \sum_{n=1}^{\infty} \frac{a_1(n)}{[n]} q^{\mp \frac{n}{2}} z^{-n}) e^{\alpha_1} z^{\mp a_1(0)},$$

$$Y_2^\pm(z) = \exp(\pm \sum_{n=1}^{\infty} \frac{a_1(-n) + b(-n)}{[n]} q^{\mp \frac{n}{2}} z^n) \exp(\mp \sum_{n=1}^{\infty} \frac{a_2(n) + b(n)}{[n]} q^{\mp \frac{n}{2}} z^{-n}) \\ \cdot e^{\alpha + b} z^{\mp a_2(0) + b(0)},$$

$$U_\pm(z) = \exp(\mp \sum_{n=1}^{\infty} \frac{[n/3]}{[2n/3]} b(\pm n) z^{\mp n}) q^{\mp b(0)/2},$$

$$W_\pm(z) = \exp(\mp \sum_{n=1}^{\infty} \frac{[n/3]}{[2n/3]} c(\pm n) z^{\mp n}).$$

**Theorem 3.1.** *The space  $\mathcal{F}$  is a  $U_q(G_2^{(1)})$ -module of level one under the action defined by  $\gamma \mapsto q, K_i \mapsto q^{a_i(0)}, a_{im} \mapsto a_i(m), q^d \mapsto q^{\bar{d}}$ , and*

$$\begin{aligned} X_1^\pm(z) &\mapsto Y_1^\pm(z), \\ X_2^\pm(z) &\mapsto \frac{\pm Y_2^\pm(z)}{q_2 - q_2^{-1}} \left( U_\pm(q^{\mp 5/6}z)W_\pm(q^{\mp 1/2}z)^{\pm 1} - U_\pm(q^{\pm 5/6}z)W_\pm(q^{\pm 1/2}z)^{\mp 1} \right). \end{aligned}$$

#### 4. PROOF OF THE THEOREM

We now prove the theorem by checking that the action satisfies the Drinfeld relations. It is clear that (2.2-2.6) are true by the construction. The relation (2.7) follows from the definition of  $Y_i^\pm(z)$  and the commutativity among  $\alpha_i(n), b(n)$  and  $c(n)$ . So we only need to show (2.8) and (2.9).

We first compute the operator product expansions for  $Y_i^\pm(z)$ :

$$\begin{aligned} (4.1) \quad Y_i^\pm(z)Y_j^\pm(w) &= : Y_i^\pm(z)Y_j^\pm(w) : \\ &\quad \cdot \exp\left(-\sum_{n=1}^{\infty} \frac{[(\alpha_i|\alpha_j)n]}{n[n]} q^{\mp n} \left(\frac{w}{z}\right)^n z^{(\alpha_i|\alpha_j)}\right), \\ Y_i^\pm(z)Y_j^\mp(w) &= : Y_i^\pm(z)Y_j^\mp(w) : \\ &\quad \cdot \exp\left(\sum_{n=1}^{\infty} \frac{[(\alpha_i|\alpha_j)n]}{n[n]} \left(\frac{w}{z}\right)^n z^{-(\alpha_i|\alpha_j)}\right). \end{aligned}$$

For  $\epsilon = \pm = \pm 1$  we define

$$\begin{aligned} X_{2\epsilon}^+(z) &= Y_2^+(z)U_\epsilon(q^{-5\epsilon/6}z)W_\epsilon(q^{-1\epsilon/2}z), \\ X_{2\epsilon}^-(z) &= Y_2^-(z)U_\epsilon(q^{5\epsilon/6}z)W_\epsilon(q^{1\epsilon/2}z)^{-1}, \end{aligned}$$

so that  $X_2^\pm(z) = \frac{1}{q_2 - q_2^{-1}} (X_{2+}^\pm(z) - X_{2-}^\pm(z))$ .

Note that for  $i = j = 1$  the relation (2.8) follows from the  $sl(2)$  case. For  $(\alpha_i|\alpha_j) = -1$  (i.e.  $i \neq j$ ) equations (4.1) become

$$(4.2) \quad Y_i^\pm(z)Y_j^\pm(w) = : Y_i^\pm(z)Y_j^\pm(w) : (z - q^{\mp 1}w)^{-1},$$

$$(4.3) \quad Y_i^\pm(z)Y_j^\mp(w) = : Y_i^\pm(z)Y_j^\mp(w) : (z - q^{\mp 1}w),$$

which implies that for  $i \neq j$

$$(4.4) \quad (z - q^{\mp 1}w)X_i^\pm(z)X_j^\pm(w) = (q^{\mp 1}z - w)X_j^\pm(w)X_i^\pm(z),$$

$$(4.5) \quad [X_1^+(z), X_2^-(w)] = 0,$$

where the latter is one case of relation (2.9).

To prove the remaining case of (2.8) we compute

$$(4.6) \quad X_{2\epsilon}^+(z)X_{2\epsilon}^+(w) = : X_{2\epsilon}^+(z)X_{2\epsilon}^+(w) : \frac{z-w}{z-q^{2/3}w} q^{(1+\epsilon)/6}.$$

Then we immediately get the “+” case of relation (2.8) for  $i = j = 2$ . The “-” case is shown similarly.

In relation (2.9), again we only need to show the cases involved with the short root  $\alpha_2$ , since the proof of  $[X_1^+(z), X_1^-(w)]$  is quite similar to that of type A in [12]. Observe that

$$(4.7) \quad X_{2\epsilon}^+(z)X_{2,-\epsilon}^-(w) = : X_{2\epsilon}^+(z)X_{2,-\epsilon}^-(w) : .$$

Thus we reduce the relation to the commutators  $[X_{2\epsilon}^+(z), X_{2\epsilon}^-(w)]$ . We compute that

$$\begin{aligned} & [X_{2+}^+(z), X_{2+}^-(w)] \\ &= : X_{2+}^+(z)X_{2+}^-(w) : \left( \frac{z - q^{5/3}w}{z - qw} q^{-1/3} - \frac{w - q^{-5/3}z}{w - q^{-1}z} q^{1/3} \right) \\ &= : X_{2+}^+(z)X_{2+}^-(w) : \frac{z - q^{5/3}w}{z} q^{-1/3} \delta\left(\frac{qw}{z}\right) \\ &= (q^{-1/3} - q^{1/3}) \psi_2(zq^{1/2}) \delta\left(\frac{qw}{z}\right) \end{aligned}$$

Similarly we can prove that

$$[X_{2-}^+(z), X_{2-}^-(w)] = (q^{1/3} - q^{-1/3}) \phi_2(zq^{-1/2}) \delta\left(\frac{q^{-1}w}{z}\right).$$

Finally we use the quantum vertex operator calculus [12] to prove the Serre relations. The case  $(a_{12} = -1)$  is similar to that of  $U_q(A_n^{(1)})$  [12]. We only check the other one  $(a_{ij} = -3)$  in the “+” case:

$$\begin{aligned} (4.8) \quad & \sum_{\sigma \in S_4} \sigma. (X_1^+(w)X_2^+(z_1)X_2^+(z_2)X_2^+(z_3)X_2(z_4) \\ & - [4]_2 X_2^+(z_1)X_1^+(w)X_2^+(z_2)X_2^+(z_3)X_2^+(z_4) \\ & + \frac{[4]_2[3]_2}{[2]_2} X_2^+(z_1)X_2^+(z_2)X_1^+(w)X_2^+(z_3)X_2^+(z_4) \\ & - [4]_2 X_2^+(z_1)X_2^+(z_2)X_2^+(z_3)X_1^+(w)X_2^+(z_4) \\ & \quad + X_2^+(z_1)X_2^+(z_2)X_2^+(z_3)X_2(z_4)X_1^+(w)) = 0 \end{aligned}$$

Recalling that  $X_2^\pm(z) = \frac{1}{q_2 - q_2^{-1}} (X_{2+}^\pm(z) - X_{2-}^\pm(z))$  and using (4.1-4.6) and Wick's theorem, we can reduce the left-hand side of (4.8) to a linear combination of operator product terms :  $X_1^+(w)X_{2\epsilon_1}^+(z_1)X_{2\epsilon_2}^+(z_2)X_{2\epsilon_3}^+(z_3)X_{2\epsilon_4}(z_4)$  :, where  $\epsilon_i = \pm$ . Thus the Serre relation is equivalently reduced to four Serre-like relations grouped by the number of appearances of  $X_{2+}^+(z_i)$  in the product. Due to (4.7) the most complicated contraction functions comes from the case when all  $\epsilon_i = +$ . All four subcases can be treated similarly. In the following we will only prove the case when  $\epsilon_i = +$ . Ignoring the factor  $(q_2 - q_2^{-1})^{-4}$ , the left-hand side of this Serre-like relation is  $q^2 : X_1^+(w)X_{2+}^+(z_1)X_{2+}^+(z_2)X_{2+}^+(z_3)X_{2+}(z_4) :$  times the following expression:

$$\begin{aligned} & \sum_{\sigma \in S_4} \sigma. \prod_i \frac{1}{(w - q^{-1}z_i)(z_i - q^{-1}w)} \prod_{i < j} \frac{z_i - z_j}{z_i - q^{2/3}z_j} \\ & \cdot [(z_1 - q^{-1}w) \cdots (z_4 - q^{-1}w) + [4]_2 (w - q^{-1}z_1)(z_2 - q^{-1}w) \cdots (z_4 - q^{-1}w) \\ & + \left[ \begin{matrix} 4 \\ 2 \end{matrix} \right]_2 (w - q^{-1}z_1)(w - q^{-1}z_2)(z_3 - q^{-1}w)(z_4 - q^{-1}w) \\ & + [4]_2 (w - q^{-1}z_1) \cdots (w - q^{-1}z_3)(z_4 - q^{-1}w) + (w - q^{-1}z_1) \cdots (w - q^{-1}z_4)], \end{aligned}$$

where the symmetric group  $S_4$  acts on the ring of rational functions in  $z_i$  by permutations on the indices. The  $q$ -binomial identity implies that the coefficients of 1

and  $w^4$  are zero, and the expression in the brackets is then simplified to

$$\begin{aligned} & q_2^4(q_2^{-6} - 1) [q_2^{-1}w^3 (q_2^{-12}z_1 - q_2^{-6}(1 + q_2^{-2} + q_2^{-4})z_2 + q_2^{-2}(1 + q_2^{-2} + q_2^{-4})z_3 - z_4) \\ & + w^2(1 + q_2^{-2}) (q_2^{-12}z_1z_2 - q_2^{-6}(1 + q_2^{-2})z_1z_3 + q_2^{-4}(1 + q_2^{-2} + q_2^{-4})z_1z_4 \\ & + q_2^{-4}(1 + q_2^{-2} + q_2^{-4})z_2z_3 + q_2^{-4}(1 + q_2^{-2})z_2z_4 - z_3z_4) \\ & + q_2^{-1}w (q_2^{-12}z_1z_2z_3 - q_2^{-6}(1 + q_2^{-2} + q_2^{-4})z_1z_2z_3 \\ & + q_2^{-2}(1 + q_2^{-2} + q_2^{-4})z_1z_3z_4 - z_2z_3z_4)]. \end{aligned}$$

Let  $f(z_1, z_2, z_3, z_4)$  denote the above expression. Since  $\prod_{i < j} (z_i - q_2^2 z_j)(z_i - q_2^{-2} z_j)$  is symmetric, we see that the Serre relation is equivalent to the following identity:

$$(4.9) \quad \sum_{\sigma \in S_4} \text{sgn}(\sigma) \sigma \left( f(z_1, z_2, z_3, z_4) \prod_{i < j} (z_i - q_2^{-2} z_j) \right) = 0.$$

We claim that (4.9) is true. Notice that it is enough to check only the coefficients of  $w$  and  $w^2$ , and even these two are quite similar. The tedious checking of the coefficient of  $w$  shows that it is zero indeed. Thus the Serre relation is proved.

The constructed level one representation is reducible due to the presence of auxiliary bosons  $b(m)$  and  $c(m)$ . All integrable irreducible level one modules are contained in the Fock representation and can be recovered by the technique of the screening operators.

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