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SOME LIE SUPERALGEBRAS ASSOCIATED TO THE WEYL ALGEBRAS

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ABSTRACT. Let $\mathfrak{g}$ be the Lie superalgebra $osp(1,2r)$. We show that there is a surjective homomorphism from $U(\mathfrak{g})$ to the r^{th} Weyl algebra A_r, and we use this to construct an analog of the Joseph ideal. We also obtain a decomposition of the adjoint representation of $\mathfrak{g}$ on A_r and use this to show that if A_r is made into a Lie superalgebra using its natural $\mathbb{Z}_2$-grading, then $A_r = k \oplus [A_r, A_r]$. In addition, we show that if $[A_r, A_r]$ and $[A_s, A_s]$ are isomorphic as Lie superalgebras, then $r = s$. This answers a question of S. Montgomery.

We work throughout over an algebraically closed field k of characteristic zero. If $\mathfrak{g}$ is a simple Lie algebra different from $s\ell(n)$, Joseph shows in [J2], that there is a unique completely prime ideal, J_0 whose associated variety is the closure of the minimal nilpotent orbit in $\mathfrak{g}*$. When $\mathfrak{g}$ is the symplectic algebra $\mathfrak{g} = sp(2r)$, this ideal may be constructed as follows. It is well known that the symmetric elements of degree two in the r^{th} Weyl algebra A_r form a Lie algebra isomorphic to $sp(2r)$ [D, Lemma 4.6.9]. Hence there is an algebra map $\phi : U(\mathfrak{g}) \longrightarrow A_r$ whose kernel is clearly completely prime and primitive. Since the image of ϕ has Gel'fand Kirillov dimension $2r$, and this is the dimension of the minimal nilpotent orbit in $\mathfrak{g}*$ by [CM, Lemma 4.3.5], we have $\ker \phi = J_0$.

Now if $\mathfrak{g}$ is a classical simple Lie superalgebra, and $U(\mathfrak{g})$ contains a completely prime primitive ideal different from the augmentation ideal, then $\mathfrak{g}$ is isomorphic to an orthosymplectic algebra $osp(1,2r)$ (Lemma 1). We observe that if $\mathfrak{g} = osp(1,2r)$, then there is a surjective homomorphism $U(\mathfrak{g}) \longrightarrow A_r$ whose kernel J satisfies $J \cap U(\mathfrak{g}_0) = J_0$. The existence of this homomorphism has previously been shown in the Physics literature; see, for example, [F, pages 55 and 170]. It follows that $\mathfrak{g}$ acts via the adjoint representation on A_r, and we determine the decomposition of this representation explicitly.

This turns out to be a useful setting in which to study the Lie structure of certain associative algebras. A result of Herstein [He] states that if A is a simple algebra with center Z, then $[A,A]/[A,A] \cap Z$ is a simple Lie algebra, unless $[A:Z] = 4$, and Z has characteristic two. Additional results have been obtained for various generalized Lie structures in [BFM] and [Mo].

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Let A_r be the r^{th} Weyl algebra over k with generators $x_1, \dots, x_r, \partial_1, \dots, \partial_r$ such that $\partial_i x_j - x_j \partial_i = \delta_{ij}$.

If A is any $\mathbb{Z}_2$-graded associative algebra, we can regard A as a Lie superalgebra by setting

$$[a, b] = ab - (-1)^{\alpha\beta} ba$$

where a, b are elements of A of degree α, β, respectively. We note that A_r can be made into a $\mathbb{Z}_2$-graded algebra by setting $\deg x_i = \deg \partial_i = 1$.

In [Mo] Montgomery shows that if we consider the r^{th} Weyl algebra A_r as a $\mathbb{Z}_2$-graded algebra, then $[A_r, A_r]/([A_r, A_r] \cap k)$ is a simple Lie superalgebra, and that when $r = 1, A_1 = k \oplus [A_1, A_1]$.

Using the adjoint representation of $\mathfrak{g}$ on A_r we show that $A_r = k \oplus [A_r, A_r]$ for all r. In addition if $r \neq s$, then $[A_r, A_r]$ is not isomorphic to $[A_s, A_s]$ as a Lie superalgebra. This answers a question of Montgomery.

Much is known about the enveloping algebras of the Lie superalgebras $osp(1, 2r)$ [F], [M1], [M2], [P]. However, we have tried to keep this paper as self-contained as possible.

Lemma 1. *If $\mathfrak{g}$ is a classical simple Lie superalgebra which is not isomorphic to $osp(1, 2r)$ for any r, then the only completely prime ideal of $U(\mathfrak{g})$ is the augmentation ideal.*

Proof. It is shown in [B, pages 17-20], that if $\mathfrak{g} \neq osp(1, 2r)$, then $\mathfrak{g}$ contains an odd element x such that $[x, x] = 0$. Hence if P is a completely prime ideal, then $x^2 = 0 \in P$ forces $x \in P$. Since $P \cap \mathfrak{g}$ is an ideal of $\mathfrak{g}$, this implies $\mathfrak{g} \subseteq P$.

Lemma 2. *If $\mathfrak{g} = osp(1, 2r)$, there is a surjective homomorphism $U(\mathfrak{g}) \longrightarrow A_r$.*

Proof. Set

$$\mathfrak{g}_1 = \sum_i k x_i + \sum_i k \partial_i$$

and

$$\mathfrak{g}_0 = \sum_{i,j} k x_i x_j + \sum_{i,j} k \partial_i \partial_j + \sum_{i,j} k(x_i \partial_j + \partial_j x_i).$$

We may identify $\mathfrak{g}_0$ with the second symmetric power $S^2 \mathfrak{g}_1$ of $\mathfrak{g}_1$. Then $\mathfrak{g} = \mathfrak{g}_0 \oplus \mathfrak{g}_1$ becomes a Lie superalgebra under the bracket

$$[a, b] = ab - (-1)^{\alpha\beta} ba$$

where $a \in \mathfrak{g}_\alpha$ and $b \in \mathfrak{g}_\beta$. It follows immediately from the description of $osp(m, n)$ given in [K, 2.1.2, supplement] that $\mathfrak{g} \cong osp(1, 2r)$.

Now let $\mathfrak{a}_r$ be the r^{th} Heisenberg Lie algebra with basis $X_1, \dots, X_r, Y_1, \dots, Y_r, Z$ and nonvanishing brackets given by $[X_i, Y_j] = \delta_{ij} Z$. Thus $U(\mathfrak{a}_r)/(Z - 1)$ is isomorphic to A_r via the map sending X_i to x_i and Y_i to y_i. By [D, Lemma 4.6.9], $\mathfrak{g}_0 = sp(2r)$ acts by derivations on $\mathfrak{a}_r$, and hence on $U(\mathfrak{a}_r)$ and on the symmetric algebra $S(\mathfrak{a}_r)$. Therefore by [D, Proposition 2.4.9], the symmetrisation map $w : S(\mathfrak{a}_r) \longrightarrow U(\mathfrak{a}_r)$ is an isomorphism of $\mathfrak{g}_0$-modules. Set $S = S(\mathfrak{a}_r)/(Z - 1)$. Clearly w induces an isomorphism $\overline{w} : S \longrightarrow A_r$ of $\mathfrak{g}_0$-modules. Now S is a polynomial algebra in $2r$ variables, and we let $S(n)$ be the subspace of homogeneous polynomials of degree n. Clearly $S(n)$ is a $\mathfrak{g}_0$-module.Set $A(n) = \overline{w}(S(n))$. Our main result is the following:

Theorem 3. *Under the adjoint action*

1) *$A(n)$ is a simple $\mathfrak{g}_0$-module for all n.*
2) *$A(2n) \oplus A(2n-1)$ is a simple $\mathfrak{g}$-module for all n.*

Remark. When $r = 1$ the decomposition of the adjoint representation is given in [P, Lemma 7.4.1]. Part 1) of the theorem also follows from arguments in [PS, Section 3].

In order to prove the theorem, we need some notation.

For $1 \le i \le r-1$, consider the elements of $\mathfrak{g}$ given by

$$e_i = x_{i+1}\partial_i, \quad f_i = x_i\partial_{i+1}$$

and

$$h_i = [e_i, f_i] = x_{i+1}\partial_{i+1} - x_i\partial_i.$$

In addition, set $e_r = \partial_r$, $f_r = x_r$ and $h_r = -[e_r, f_r]/2 = -(x_r\partial_r + \partial_r x_r)/2$. Then $\mathfrak{h} = \operatorname{span}\{h_i | 1 \le i \le r\}$ is a Cartan subalgebra of $\mathfrak{g}$. We let $\alpha_1, \dots, \alpha_r \in \mathfrak{h}^*$ be the positive roots determined by $[h, e_i] = \alpha_i(h)e_i$ for all $h \in \mathfrak{h}$. The values $\alpha_i(h_j)$ are the entries in the (symmetrized) Cartan matrix

$$\begin{bmatrix} 2 & -1 & & & & \\ -1 & 2 & -1 & & & \\ & -1 & 2 & \ddots & & \\ & & \ddots & \ddots & -1 & \\ & & & -1 & 2 & -1 \\ & & & & -1 & 1 \end{bmatrix}.$$

Let $\mathfrak{n}$ be the subalgebra of $\mathfrak{g}$ generated by $e_1, \dots, e_r$ and $\mathfrak{n}_0 = \mathfrak{n} \cap \mathfrak{g}_0$. If L is a $\mathfrak{g}$-module (resp. $\mathfrak{g}_0$-module), we say that $v \in L$ is a highest weight vector for $\mathfrak{g}$ (resp. for $\mathfrak{g}_0$) of weight $\lambda \in \mathfrak{h}^*$ if $hv = \lambda(h)v$ for all $h \in \mathfrak{h}$ and $\mathfrak{n}v = 0$ (resp. $\mathfrak{n}_0 v = 0$).

The bilinear form $(,)$ defined on $\mathfrak{h}^*$ by $(\alpha_i, \alpha_j) = \alpha_i(h_j)$ is invariant under the action of the Weyl group. For later computations involving $(,)$ it is convenient to use the following alternative description [K, 2.5.4]. Identify $\mathfrak{h}^*$ with k^r with standard basis $\epsilon_1, \dots, \epsilon_r$ and $(,)$ with the usual inner product. Then $\alpha_i = \epsilon_i - \epsilon_{i+1}$ for $1 \le i \le r-1$ and $\alpha_r = \epsilon_r$. Let ρ_0 (resp. ρ_1) denote the half-sum of the positive even (resp. odd) roots of $\mathfrak{g}$ and $\rho = \rho_0 - \rho_1$. Under the identification above we have $\rho_0 = \sum_{i=1}^r (r-i+1)\epsilon_i$, $\rho_1 = \frac{1}{2}\sum_{i=1}^r \epsilon_i$ and $\rho = \frac{1}{2}\sum_{i=1}^r (2r-2i+1)\epsilon_i$.

We now return to the homomorphism $\phi : U(\mathfrak{g}) \longrightarrow A_r$. Set $J = \operatorname{Ker}\phi$. Note that $R = \mathbb{C}[x_1, \dots, x_r]$ is a simple A_r-module and hence a faithful simple $U(\mathfrak{g})/J$-module. Also $1 \in R$ is a highest weight vector of weight λ where $\lambda(h_i) = 0$ for $1 \le i \le r-1$, and $\lambda(h_r) = -1/2$. An easy computation shows that $\lambda = -\frac{1}{2}\sum_{i=1}^r i\alpha_i = -\rho_1$. Thus we have shown

Corollary 4. *J is the annihilator of the simple highest weight module with weight $-\rho_1$.*

We comment briefly on the geometric significance of the embedding $U(\mathfrak{g}_0)/J_0 \hookrightarrow U(\mathfrak{g})/J \cong A_r$. Passing to the associated graded rings we have

$$R(\overline{O}_{min}) = gr(U(\mathfrak{g}_0)/J_0) \hookrightarrow gr(A_r) = R(k^{2r})$$

where $R(X)$ denotes the ring of regular functions on X and $\overline{O}_{min}$ is the minimal coadjoint orbit. Let $\mu : k^{2r} \longrightarrow \mathfrak{g}_0^*$ be the moment map for the natural action of $Sp(2r)$ on k^{2r} [CG, 1.4]. Then the image of μ is contained in $\overline{O}_{min}$ and the above inclusion is the comorphism μ^*. All of this is quite well known. The new twist that Lie superalgebras bring to this situation is a consequence of the next result.

Lemma 5. *Suppose $k = \mathbb{C}$, let $\mathfrak{g} = \mathfrak{g}_0 \oplus \mathfrak{g}_1$ be any Lie superalgebra over $\mathbb{C}$ and define $\pi : \mathfrak{g}_1 \longrightarrow \mathfrak{g}_0$ by $\pi(y) = y^2$ $(= \frac{1}{2}[y, y])$. Then*

$$(\exp \operatorname{ad} x)(\pi(y)) = \pi(\exp \operatorname{ad} x(y))$$

for all $x \in \mathfrak{g}_0$ and $y \in \mathfrak{g}_1$.

In the above situation, we can identify $\mathfrak{g}_0^*$ with $\mathfrak{g}_0$ and $\mathbb{C}^{2r}$ with $\mathfrak{g}_1$ in such a way that $\mu = \pi$.

Proof. The second claim follows from the first and the uniqueness of the moment map. The formula involving π is proved by formally expanding both sides. Define $[x, y]_0 = y$ and $[x, y]_n = (\operatorname{ad} x)^n(y) = [x, [x, y]_{n-1}]$ for $n > 0$. Similarly we define $[x, y^2]_n$. Then $(\exp \operatorname{ad} x)\pi(y) = \sum_{n\geq 0}[x, y^2]_n/n!$. To show that this equals $\pi(\exp \operatorname{ad} x(y))$ we use the identity

$$\begin{aligned} [x, y^2]_{2m} &= \binom{2m}{m}[x, y]_m^2 \\ &+ \sum_{j=0}^{m-1}\binom{2m}{j}[[x, y]_j, [x, y]_{2m-j}] \end{aligned}$$

and a similar identity for $[x, y^2]_{2m+1}$. The identities are easily proved by induction.

Lemma 6. *Under the adjoint action of $\mathfrak{g}_0$ or $\mathfrak{g}$ on A_r,*
1) *∂_1^n is a highest weight vector for $\mathfrak{g}_0$ of weight $n\epsilon_1$.*
2) *If n is even, ∂_1^n is a highest weight vector for $\mathfrak{g}$.*

Proof. A simple computation.

If $\lambda \in \mathfrak{h}^*$, we denote the simple $\mathfrak{g}_0$-module with highest weight λ by $L(\lambda)$.

Lemma 7. *We have* $\dim L(n\epsilon_1) = \binom{2r+n-1}{n}$ *for all n.*

Proof. By Weyl's dimension formula

$$\dim L(\lambda) = \Pi_{\alpha>0}\frac{(\lambda + \rho_0, \alpha)}{(\rho_0, \alpha)}$$

where the product is taken over all positive even roots α. The even roots α for which $(\epsilon_1, \alpha) > 0$ are listed in the first column of the table below. The other columns give the information we need.

α	(ρ_0, α)	$(n\epsilon_1, \alpha)$
$\epsilon_1 - \epsilon_{i+1},\ 1 \leq i \leq r-1$	i	n
$\epsilon_1 + \epsilon_j,\ 2 \leq j \leq r$	$2r - j + 1$	n
$2\epsilon_1$	$2r$	$2n$

Therefore

$$\dim L(n\epsilon_1) = \prod_{i=1}^{r} \frac{n+i}{i} \prod_{j=2}^{r} \frac{2r+n-j+1}{2r-j+1} = \binom{2r+n-1}{n}.$$

Proof of Theorem 3. Set $A = A_r$. Part 1) of the theorem follows from Lemmas 6 and 7, since $dimA(n) = \binom{2r+n-1}{n}$. Thus $B(n) = A(2n) \oplus A(2n-1)$ is a direct sum of two nonisomorphic simple $\mathfrak{g}_0$-modules. Also the highest weight vectors ∂_1^{2n} and ∂_1^{2n-1} for these $\mathfrak{g}_0$-modules satisfy

$$[x_1, \partial_1^{2n}] = -2n\partial_1^{2n-1},$$
$$[\partial_1, \partial_1^{2n-1}] = 2\partial_1^{2n}.$$

Let M be the ad$\mathfrak{g}$-submodule of A generated by ∂_1^{2n}. It follows that $B(n) \subseteq M$. Also M is a finite dimensional image of a Verma module (which has a unique simple quotient). On the other hand all finite dimensional simple $\mathfrak{g}$-modules are completely reducible by [DH]. It follows that M is a simple ad$\mathfrak{g}$-module. (cf. the argument in [Jan, Lemma 5.14]).

We do not know yet that $B(n)$ is an ad$\mathfrak{g}$-module. This can be seen as follows. We define a filtration $\{B_n\}$ on A by setting $B_n = \bigoplus_{m \leq n} B(m)$. Note that this filtration is the image of the filtration $\{U_n\}$ of $U(\mathfrak{g})$ defined by $U_n = U_1^n$ where $U_1 = k \oplus \mathfrak{g}$. Hence the associated graded ring $\bigoplus_{n \geq 0} B_n/B_{n-1}$ is supercommutative. It follows that $[\mathfrak{g}, B_n] \subseteq B_n$ and so $M \subseteq B_n$. If M strictly contains $B(n)$, we would have $M \cap (B(n-1) \oplus \ldots \oplus B(1) \oplus k) \neq 0$. By induction, the $B(i)$ with $i < n$ are simple ad$\mathfrak{g}$-modules, so M would contain ∂_1^{2i} for some $i < n$. However a simple $U(\mathfrak{g})$-module cannot contain more than one highest weight vector. This contradiction shows that $M = B(n)$ and completes the proof.

Theorem 8. *We have $[A_r, A_r] = \bigoplus_{n>0} A(n)$. In particular $A_r = k \oplus [A_r, A_r]$.*

Proof. Note that if $a, b, c \in A$ have degrees α, β and γ, then as noted in [Mo, Lemma 1.4 (3)]

$$[ab, c] = [a, bc] + (-1)^{\alpha(\beta+\gamma)}[b, ca].$$

Therefore, since A_r is generated by the image of $\mathfrak{g}$, we have $[A_r, A_r] = [A_r, \mathfrak{g}]$. The result now follows from Theorem 3.

Remark. From [Mo, Theorem 4.1] it follows that $[A_r, A_r]$ is a simple Lie superalgebra for all r.

A question raised in [Mo] is whether, for different r, the $[A_r, A_r]$ are all nonisomorphic. We show that this is the case by finding the largest rank of a finite dimensional simple Lie subalgebra of $[A_r, A_r]$. Note that $sp(2r) \cong A(2) \subseteq [A_r, A_r]$. On the other hand we have

Lemma 9. *If L is a finite dimensional simple Lie subalgebra of $[A_r, A_r]$, then* $\text{rank}(L) \leq r$.

Proof. Note that under the stated hypothesis, L is a Lie subalgebra of A_r with the usual Lie bracket $[a, b] = ab - ba$. Now in [J1], Joseph investigates for each simple Lie algebra L, the least integer $n = n_A(L)$ such that L is isomorphic to a Lie subalgebra of A_n. (The integer $n_A(L)$ is determined to within one for all classical Lie algebras.) In particular it follows from Lemma 3.1 and Table 1 of [J1] that $n_A(L) \geq \operatorname{rank}(L)$.

Corollary 10. *If $[A_r, A_r] \cong [A_s, A_s]$ as Lie superalgebras, then $r = s$.*

For the sake of completeness, we give a proof of Corollary 10 which is independent of [J1]. It is enough to show that if $\mathfrak{g}_0 = sp(2r)$ is a Lie subalgebra of a Weyl algebra A_n, then $n \geq r$. The elements $x_1 x_i, x_1 \partial_i$, with $2 \leq i \leq r$ and x_1^2 span a Heisenberg subalgebra $\mathfrak{a} = \mathfrak{a}_{r-1}$ of $\mathfrak{g}_0$ with center spanned by x_1^2. The inclusion $\mathfrak{g}_0 \subseteq A_n$ induces a homomorphism $\phi : U(\mathfrak{g}_0) \longrightarrow A_n$. If $I = \ker\phi \cap U(\mathfrak{a}) = 0$, then we have $GK(U(\mathfrak{a})) = 2r - 1 \leq GK(A_n) = 2n$, where $GK(\;)$ denotes Gel'fand-Kirillov dimension, and so $r \leq n$. However if $I \neq 0$, then since the localization of $U(\mathfrak{a})$ at the nonzero elements of $k[x_1^2]$ is a simple ring, we would have $x_1^2 - \alpha \in I$ for some scalar α. This would imply that x_1^2 is central in $\mathfrak{g}_0$, a contradiction.

Remark. It is shown in [Mo, Proposition 4.2] that if A_r is isomorphic to A_s, then $r = s$. Corollary 10 also follows from this and Theorem 8.

Finally, we note that the proof of Theorem 8 works for certain other algebras.

Theorem 11. *Let $\mathfrak{g}$ be a semisimple Lie algebra, and A a primitive factor algebra of $U(\mathfrak{g})$, then $A = k \oplus [A, A]$.*

Proof. As before we have $[A, A] = [A, \mathfrak{g}]$. Also $A = \bigoplus V$, a direct sum of finite dimensional simple submodules under the adjoint representation. Since $[V, \mathfrak{g}]$ is a submodule of V for any such V, and the center of A equals k, we obtain $[A, A] = \bigoplus_{V \neq k} V$, and the result follows.

References

[B] E. Behr, Enveloping algebras of Lie superalgebras, Pacific J. Math 130 (1987), 9-25. MR **89b:**17023

[BFM] Yu. Bahturin, D. Fischman and S. Montgomery, On the generalized Lie structure of associative algebras, Israel J. Math., 96 (1996), 27-48. MR **98d:**16048

[CG] N. Chriss and V. Ginzburg, Representation Theory and Complex Geometry, Birkhäuser, Boston 1997. CMP 9:08

[CM] D. H. Collingwood and W. M. McGovern, Nilpotent orbits in semisimple Lie algebras, Van Nostrand Reinhold, New York, 1993. MR **94j:**17001

[D] J. Dixmier, Enveloping Algebras, North Holland, Amsterdam 1977. MR **58:**16803b

[DH] D. Ž. Djoković and G. Hochschild, Semisimplicity of 2-graded Lie algebras, II, Illinois J. Math., 20 (1976) 134-143. MR **52:**8206

[F] C. Fronsdal (editor), Essays on Supersymmetry, Mathematical Physics Studies 8, Reidel Publ. Comp., Dordrecht, 1986. MR **88a:**81004

[H] J. E. Humphreys, Introduction to Lie Algebras and Representation Theory, Graduate Texts in Mathematics 9, Springer-Verlag, New York, 1972. MR **48:**2197

[He] I. N. Herstein, On the Lie and Jordan rings of a simple associative ring, Amer. J. Math. 77 (1955), 279-285. MR **16:**789e

[Jan] J. C. Jantzen, Lectures on Quantum Groups, American Math. Society, 1996. MR **96m:**17029

[J1] A. Joseph, Minimal realizations and spectrum generating algebras, Comm. Math. Phys. 36 (1974), 325-338. MR **49:**6795

[J2] A. Joseph, The minimal orbit in a simple Lie algebra and its associated maximal ideal, Ann. scient. Éc. Norm. Sup. 9 (1976), 1-29. MR **53:**8168

[K] V. Kac, Lie Superalgebras, Adv. in Math. 16 (1977), 8-96. MR **58:**5803

[Mo] S. Montgomery, Constructing simple Lie superalgebras from associative graded algebras, J. of Algebra, 195 (1997) 558-579. CMP 98:01

[M1] I. M. Musson, On the center of the enveloping algebra of a classical simple Lie superalgebra, J. Algebra, 193 (1997), 75-101. CMP 97:14

[M2] I. M. Musson, The enveloping algebra of the Lie superalgebra, $osp(1, 2r)$, Representation Theory, an electronic journal of the AMS, 1 (1997), 405-423. CMP 98:04

[P] G. Pinczon, The enveloping algebra of the Lie superalgebra $osp(1, 2)$, J. Alg. 132 (1990), 219-242. MR **91j:**17014

[PS] G. Pinczon and J. Simon, Nonlinear representations of inhomogeneous groups, Lett. Math. Phys. 2 (1978), 499-504. MR **80a:**22018

[Sch] M. Scheunert, The theory of Lie superalgebras. An introduction, Lecture Notes in Mathematics, 716, Springer-Verlag, Berlin, 1979. MR **80i:**17005

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